Series ONS

SET-1

Roll No. □□□□□□□

Candiates must write code on the title page of the answer – book

- Please check that this question paper contains 12 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 26 questions.
- Please write down the Serial Number of the question before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS

Time allowed : 3 hours

Maximum Marks : 100

General Instructions :

(i) ***All questions are compulsory.***

(ii) *Please check that this question paper contains 26 questions.*

(iii) *Questions 1 to 6 Section-A are very short-answer type questions carrying 1 mark each.*

(iv) *Questions 7 to 19 Section-B are long-answer I type questions carrying 4 mark each.*

(v) *Questions 20 to 26 Section-C are long-answer II type questions carrying 6 mark each.*

(vi) *Pleace write down the serial number of the question before attempting it.*

SECTION - A

Question number 1 to 6 carry 1 mark each.

1. If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, find α satisfying $0 < \alpha < \frac{\pi}{2}$ when $A + A^T = \sqrt{2}\, I_2$; where A^T is transpose of A.

2. If A is a 3 × 3 matrix and $|3A| = k|A|$, then write the value of k.

3. For what values of k, the system of linear equations
$x+y+z=2$
$2x+y-z=3$
$3x+2y+kz=4$
has a unique solution ?

4. Write the sum of intercepts cut off by the plane $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)-5=0$ on the three axes.

5. Find λ and μ if
$\left(\hat{i}+3\hat{j}+9\hat{k}\right)\times\left(3\hat{i}-\lambda\,\hat{j}+\mu\,\hat{k}\right)=\vec{0}.$

6. If $\vec{a} = 4\hat{i}-\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}-2\hat{j}+\hat{k}$, then find a unit vector parallel to the vector $\vec{a}+\vec{b}$.

SECTION -B

Question numbers 7 to 19 carry 4 marks each.

7. Solve for x : $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$.

OR

Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) - \tan^{-1}\left(\frac{4x}{1-4x^2}\right) = \tan^{-1}2x$; $|2x| < \frac{1}{\sqrt{3}}$.

8. A typist charges Rs. 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are Rs. 180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only Rs. 2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy ? Which values are reflected in this problem ?

9. If $f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x} & , \ x<0 \\ 2 & , \ x=0 \\ \dfrac{\sqrt{1+bx}-1}{x} & , \ x>0 \end{cases}$

is continuous at x=0, then find the values of a and b.

10. If $x\cos(a+y) = \cos y$ then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

Hence show that $\sin a \frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0$.

OR

Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$

11. Find the equation of tangents to the curve $y = x^3 + 2x - 4$, which are perpendicular to line $x + 14y + 3 = 0$.

12. Find : $\int \frac{(2x-5)\, e^{2x}}{(2x-3)^3}\, dx$

OR

Find : $\int \frac{x^2 + x + 1}{(x^2+1)\,(x+2)}\, dx$

13. Evaluate: $\int_{-2}^{2} \frac{x^2}{1+5^x}\, dx$.

14. Find : $\int (x+3)\sqrt{3-4x-x^2}\, dx.$

15. Find the particular solution of differential equation : $\frac{dy}{dx} = -\frac{x + y\cos x}{1+\sin x}$ given that $y = 1$ when $x = 0$.

16. Find the particular solution of the differential equation
$2y\, e^{x/y}\, dx + (y - 2x\, e^{x/y})dy = 0$
given that $x = 0$ when $y = 1$.

17. Show that the four points A(4, 5, 1), B(0, –1, –1), C (3, 9, 4) and D (–4, 4, 4) are coplanar.

18. Find the coordinates of the foot of perpendicular drawn from the point A(–1, 8, 4) to the line joining the points B(0, –1, 3) and C (2, –3, –1). Hence find the image of the point A in the line BC.

19. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

OR

A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

SECTION - C

Question numbers 20 to 26 carry 6 marks each.

20. Three numbers are selected at random (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean and variance of the distribution.

21. Let A = R × R and * be a binary operation on A defined by
(a, b) * (c, d) = (a+c, b+d)
Show that * is commutative and associative. Find the identity element for * on A. Also find the inverse of every element (a, b) ∈ A.

22. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ on $\left[0, \frac{\pi}{2}\right]$.

OR

Show that semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

23. Using the method of integration, find the area of the triangular region whose vertices are (2, –2), (4, 3) and (1, 2).

24. Find the equation of the plane which contains the line of intersection of the planes
$\vec{r}.\left(\hat{i} - 2\hat{j} + 3\hat{k}\right) - 4 = 0$ and
$\vec{r}.\left(-2\hat{i} + \hat{j} + \hat{k}\right) + 5 = 0$
and whose intercept on x-axis is equal to that of on y-axis.

25. A retired person wants to invest an amount of Rs. 50, 000. His broker recommends investing in two type of bonds 'A' and 'B' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least Rs. 20,000 in bond 'A' and at least Rs. 10,000 in bond 'B'. He also wants to invest at least as much in bond 'A' as in bond 'B'. Solve this linear programming problem graphically to maximise his returns.

26. Using properties of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz\,(x+y+z)^3$$

OR

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = O$ find k.