Stage II

Sample QUESTION PAPER

Fully Solved **(Questions-Solutions)**

MATHEMATICS

A Highly Simulated Practice Question Papers for **CBSE Class X**

Term I *Examination* (SA I)

Time : 3 hrs **Max. Marks :** 90

General Instructions

1. All questions are compulsory.
2. Draw neat labelled diagram whenever necessary to explain your answer.
3. Question Numbers 1- 8 are multiple choice questions, carrying 1 mark each.
4. Question Numbers 9-14 are short answer type questions, carrying 2 marks each.
5. Question Numbers 15-24 are short answer type questions, carrying 3 marks each.
6. Question Numbers 25-34 are long answer type questions, carrying 4 marks each.

Section A

Que 1. After how many decimal places will the decimal expansion of the number $\frac{53}{2^2 5^3}$ terminate?

(a) 4 (b) 3

(c) 2 (d) 1

Que 2. The largest number which divides 318 and 739 leaving remainder 3 and 4 respectively, is

(a) 110 (b) 7

(c) 35 (d) 105

Que 3. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then

(a) c and a have opposite signs

(b) c and b have opposite signs

(c) c and a have same sign

(d) c and d have the same sign

Que 4. If $\Delta ABC \sim \Delta PQR$, $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{9}{4}$, cm, then AB is equal to

(a) 14 cm (b) 8 cm

(c) 10 cm (d) 12 cm

Que 5. $\sin 2A = 2 \sin A$ is true when $A = ?$

(a) 0° (b) 30° (c) 45° (d) 60°

Que 6. If $\sin \alpha = \frac{1}{\sqrt{2}}$ and $\cos \beta = \frac{\sqrt{3}}{2}$, then the value of $(\alpha + \beta)$ is

(a) 90° (b) 60° (c) 75° (d) 45°

Que 7. The value of $(\sin 60^\circ + \cos 60^\circ) - (\sin 30^\circ + \cos 30^\circ)$ is

(a) -1 (b) 0 (c) 1 (d) 2

Que 8. For the following distribution

Class interval	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

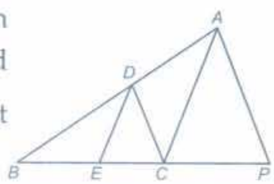
The sum of lower limit of the median class and modal class is

(a) 15 (b) 25
(c) 30 (d) 35

Section B

Que 9. Use Euclid's division algorithm to find the HCF of 867 and 255.

Que 10. In the given figure, $DE \parallel AC$ and $\frac{BE}{EC} = \frac{BC}{CP}$. Prove that $DC \parallel AP$.



Que 11. Write a quadratic polynomial whose sum of the zeroes is $2\sqrt{3}$ and their product is 2.

OR

If α and β are zeroes of the polynomial $3x^2 + 5x + 2$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

Que 12. If $3 \cot \theta = 4$, then find the value of $\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 3 \cos \theta}$.

Que 13. Find the mean of first five prime numbers.

Que 14. For what value of p, the following system of equation have no solution?

$$(2p-1)x + (p-1)y = 2p+1$$
$$y + 3x - 1 = 0$$

Section C

Que 15. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9 m, $9m+1$ or $9m+8$.

Que 16. Evaluate

$$\frac{\sec 29^\circ}{\operatorname{cosec} 61^\circ} + 2 \cot 8^\circ \cot 17^\circ \cot 45^\circ \cot 73^\circ \cot 82^\circ - 3(\sin^2 38^\circ + \sin^2 52^\circ).$$

Que 17. Gaura went to a 'sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, 'the number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased.' Help her friend to find.

(i) How many pants and skirts Gaura bought?
(ii) Which mathematical concept is used to solve the above question?
(iii) Which value(s) are hidden behind conductivity in the question?

Que 18. Represent the following system of linear equation graphically $3x + y - 5 = 0$; $2x - y - 5 = 0$.
From the graph, find the points where the lines intersect the y-axis.

Que 19. In ΔABC, if AD is the median, show that $AB^2 + AC^2 = 2(AD^2 + BD^2)$.

Que 20. Two ΔABC and ΔDBC are on the same base BC and on the same side of BC in which $\angle A = \angle D = 90^\circ$. If CA and BD meet each other at E, then show that

$$AE \cdot EC = BE \cdot ED.$$

Que 21. Find the value of $\sin 45^\circ$ geometrically.

Que 22. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

OR

If $\angle A$ or $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Que 23. The following table gives the distribution of the life time of 400 neon lamps

Life time (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Que 24. The given distribution shows the number of runs scored by some top batsmen of the world in one day international cricket matches.

Runs scored	Number of batsmen
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	7
7000-8000	6
8000-9000	3
9000-10000	1
10000-11000	1

Find the mode of the data.

Section D

Que 25. Find all the zeroes of $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeroes are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

Que 26. Sum of two zeroes of a polynomial of degree 4 is -1 and their product is -2. If other two zeroes are $\sqrt{3}$ and $-\sqrt{3}$. Find the polynomial.

Que 27. Formulate the following problems as a pair of equations and hence find their solutions.

2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work and also that taken by 1 man alone.

Que 28. Prove that, if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, the other two sides are divided in the same ratio.

Que 29. Prove the following

$(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2.$

Que 30. Using Euclid's division algorithm, find the HCF of 56,96 and 404.

Que 31. Following distribution shows the marks obtained by 100 students in a class.

Marks	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	10	15	30	32	8	5

Draw a less than ogive for the given data and hence obtain the median marks from the graph.

Que 32. State whether the following are true or false. Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$.

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) $\sin\theta = \cos\theta$ for all values of θ.

(iv) $\cot A$ is not defined for $A = 0°$.

Que 33. Prove that the area of the semi-circle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the semi-circles drawn on the other two sides of the triangle.

Que 34. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows

Number of letters	1-4	4-7	7-10	10-13	13-16	16-19
Number of surnames	6	30	40	16	4	4

Determine the median number of letter in the surnames. Also, find the modal size of the surnames.

Solutions

1. (*b*) We have, $\frac{53}{2^2 \cdot 5^3} = \frac{53}{2^2 \times 5^3} \times \frac{2}{2}$

$$= \frac{106}{2^3 \times 5^3} = \frac{106}{(2\times5)^3} = \frac{106}{10^3}$$

$$= \frac{106}{1000} = 0.106$$

2. (*d*) We have,

3	735
5	245
7	49
7	7
	1

3	315
3	105
5	35
7	7
	1

$318 - 3 = 315$; $739 - 4 = 735$

$$315 = 3 \times 3 \times 5 \times 7$$
$$= 3^2 \times 5 \times 7$$

and $$735 = 3 \times 5 \times 7 \times 7$$
$$= 3 \times 5 \times 7^2$$

$$\text{HCF } (315, 735) = 3 \times 5 \times 7$$
$$= 105$$

3. (*c*) Given, the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal *i.e.*,

discriminant $(D) = 0$

$$b^2 - 4ac = 0$$

$$\Rightarrow \quad b^2 = 4ac$$

which is possible only when a and c have the same sign.

4. (*d*) As $\Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \quad \frac{9}{4} = \frac{AB^2}{PQ^2} \quad \left[\because \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{9}{4}\right]$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{AB}{8} = \frac{3}{2} \quad [\because PQ = 8]$$

$$\Rightarrow \quad AB = \frac{8 \times 3}{2} = 12 \text{ cm}$$

5. (*a*) Given, $\sin 2A = 2\sin A$

when $A = 0°$, $\sin(2 \times 0°) = 2\sin 0°$

$$\Rightarrow \quad \sin 0° = 2(0)$$

$$\Rightarrow \quad 0 = 0, \text{ true}$$

6. (*c*) We have, $\sin\alpha = \frac{1}{\sqrt{2}}$

$$\Rightarrow \quad \sin\alpha = \sin 45° \quad \left[\because \sin 45° = \frac{1}{\sqrt{2}}\right]$$

$$\therefore \quad \alpha = 45°$$

and $$\cos\beta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \cos\beta = \cos 30° \quad \left[\because \cos 30° = \frac{\sqrt{3}}{2}\right]$$

$$\therefore \quad \beta = 30°$$

Now, $\alpha + \beta = 45° + 30° = 75°$

7. (*b*) We have,

$$(\sin 60° + \cos 60°) - (\sin 30° + \cos 30°)$$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}+1}{2} - \frac{1+\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}+1-1-\sqrt{3}}{2} = 0$$

$$\left[\begin{array}{l} \because \sin 30° = \frac{1}{2}, \cos 30° = \frac{\sqrt{3}}{2} \\ \sin 60° = \frac{\sqrt{3}}{2}, \cos 60° = \frac{1}{2} \end{array}\right]$$

8. (*b*)

Class interval	Frequency	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66
Total	66	

Median lies in the class 10-15 and mode lies in the class 15-20.

Sum of the lower limit of median and the modal class $= 10 + 15 = 25$

9. By Euclid's division algorithm, we have

Divident = (Divisor × Quotient + Remainder)

$867 = 255 + 3 + 102$ **[1/2]**

$255 = 102 \times 2 + 51$ **[1/2]**

$102 = 51 \times 2 + 0$ **[1/2]**

Hence, HCF (867, 255) = 51 **[1/2]**

10. In ΔABC,

$$DE \parallel AC$$

[by basic proportionality theorem]

$$\therefore \quad \frac{BD}{DA} = \frac{BE}{EC} \quad \text{...(i) [1/2]}$$

Now, $$\frac{BE}{EC} = \frac{BC}{CP} \quad \text{[given] ...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{BD}{DA} = \frac{BC}{CP} \quad \text{[1/2]}$$

So, in ΔABP,

$$\frac{BD}{DA} = \frac{BC}{CP} \quad \text{[from above]}$$

$$\Rightarrow \quad DC \parallel AP \quad \text{[1]}$$

[using converse of basic proportionality theorem]

11. Given, Sum of the zeroes $= 2\sqrt{3}$

Product of the zeroes $= 2$ **[1]**

$\therefore$ Required quadratic polynomial

$= x^2 -$ (sum of the zeroes) $x +$ (product of the zeroes) **[1]**

$= x^2 - 2\sqrt{3}x + 2$

OR

Sol. Since, α and β are the zeroes of $3x^2 + 5x + 2$.

$$\Rightarrow \quad \alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= -\frac{5}{3} \quad \text{[1]}$$

and $$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{2}{3} \quad \text{[1/2]}$$

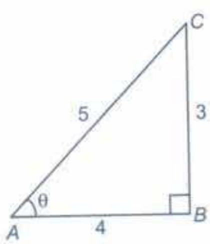
$$\therefore \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{-5/3}{2/3} = -\frac{5}{3} \times \frac{3}{2} = -\frac{5}{2} \quad \text{[1/2]}$$

12. Given, $3\cot\theta = 4$

$$\Rightarrow \quad \cot\theta = \frac{4}{3} = \frac{B}{P}$$

$$= \frac{AB}{BC}$$



In ΔABC, using Pythagoras theorem,

$$AC^2 = AB^2 + BC^2 \quad \text{[1/2]}$$

$$\Rightarrow \quad AC = \sqrt{4^2 + 3^2}$$

$$\Rightarrow \quad AC = \sqrt{16 + 9}$$

$$\Rightarrow \quad AC = \sqrt{25} \Rightarrow AC = 5 \quad \text{[1/2]}$$

$$\therefore \quad \sin\theta = \frac{3}{5} \text{ and } \cos\theta = \frac{4}{5}$$

Now, $$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta} = \frac{5 \times \frac{3}{5} - 3 \times \frac{4}{5}}{5 \times \frac{3}{5} + 3 \times \frac{4}{5}}$$

$$= \frac{\frac{15}{5} - \frac{12}{5}}{\frac{15}{5} + \frac{12}{5}} = \frac{\frac{15-12}{5}}{\frac{15+12}{5}} = \frac{3}{5} \times \frac{5}{27} = \frac{1}{9} \quad \text{[1]}$$

13. Since, the first five prime numbers are 2, 3, 5, 7 and 11.

$\therefore$ Sum of the numbers $= 2 + 3 + 5 + 7 + 11 = 28$ **[1]**

$$N = 5$$

$$\therefore \quad \text{Mean} = \frac{28}{5} = 5.6 \quad \text{[1]}$$

14. The given system of equations is

$$(2p-1)x + (p-1)y = 2p + 1$$

and $$y + 3x - 1 = 0.$$

On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 2p - 1, b_1 = (p-1), c_1 = -(2p+1)$$

and $$a_2 = 3, b_2 = 1, c_2 = -1 \quad \text{[1]}$$

The given system of equation has no solution.

If $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

i.e., $$\frac{2p-1}{3} = \frac{p-1}{1}$$

$$\Rightarrow \quad 2p - 1 = 3p - 3$$

$$\Rightarrow \quad 2p - 3p = -3 + 1$$

$$\Rightarrow \quad -p = -2$$

$$\therefore \quad p = 2 \quad \text{[1]}$$

15. Let q be any positive integer. Then, it is of the form $3q, 3q+1$ or $3q+2$.

Now, we have to prove that the cube of each of these can be rewritten in the form $9m, 9m+1$ or $9m+8$.

Now, $(3q)^3 = 27q^3 = 9(3q^3) = 9m$, where $m = 3q^3$ **[1]**

$$(3q+1)^3 = (3q)^3 + 3(3q)^2 \cdot 1 + 3(3q) \cdot 1^2 + 1$$

$$= 27q^3 + 27q^2 + 9q + 1$$

$$= 9(3q^3 + 3q^2 + q) + 1$$

$$= 9m + 1, \text{ where } m = 3q^3 + 3q^2 + q \quad \text{[1]}$$

and $$(3q+2)^3 = (3q)^3 + 3(3q)^2 \cdot 2 + 3(3q) \cdot 2^2 + 8$$

$$= 27q^3 + 54q^2 + 36q + 8$$

$$= 9(3q^3 + 6q^2 + 4q) + 8$$

$$= 9m + 8, \text{ where } m = 3q^3 + 6q^2 + 4q \quad \text{[1]}$$

Hence proved.

16. We have,

$$\frac{\sec 29^\circ}{\operatorname{cosec} 61^\circ}+2\cot 8^\circ \cot 17^\circ \cot 45^\circ \cot 73^\circ \cot 82^\circ -3(\sin^2 38^\circ+\sin^2 52^\circ)$$

$$=\frac{\sec(90^\circ-61^\circ)}{\operatorname{cosec} 61^\circ}+2\cot(90^\circ-82^\circ)\cot(90^\circ-73^\circ)\cdot\cot 45^\circ\cdot\cot 73^\circ\cdot\cot 82^\circ -3[\sin^2 38^\circ+\sin^2(90^\circ-38^\circ)]$$ **[1]**

$$=\frac{\operatorname{cosec} 61^\circ}{\operatorname{cosec} 61^\circ}+2\tan 82^\circ\cdot\tan 73^\circ\cdot\cot 45^\circ\cdot\cot 73^\circ \cot 82^\circ -3(\sin^2 38^\circ+\cos^2 38^\circ)$$

$$\left[\because \sec(90^\circ-\theta)=\operatorname{cosec}\theta,\ \cot(90^\circ-\theta)=\tan\theta \quad \sin(90^\circ-\theta)=\cos\theta\right]$$

$$=\frac{\operatorname{cosec} 61^\circ}{\operatorname{cosec} 61^\circ}+2\tan 82^\circ\cdot\tan 73^\circ\cdot\cot 45^\circ\cdot\frac{1}{\tan 73^\circ}\cdot\frac{1}{\tan 82^\circ} -3(\sin^2 38^\circ+\cos^2 38^\circ)$$

$$\left[\because \cot\theta=\frac{1}{\tan\theta}\right] \text{ and } [\because \sin^2\theta+\cos^2\theta=1]$$ **[1]**

$=1+2\times1-3\times1$

$=3-3$

$=0$ **[1]**

17. (i) Let the number of pants be x and the number of skirts be y.

According to the question,

$y=2x-2$ ...(i)

$y=4x-4$ ...(ii) **[1]**

From Eqs. (i) and (ii), we get

$4x-4=2x-2$

$\Rightarrow \quad 4x-2x=-2+4$

$\Rightarrow \quad 2x=2$

$\Rightarrow \quad x=1$

Putting the value of x in Eq. (i),

$y=2\times1-2$ **[1]**

$=2-2=0$

Hence, the number of pants, she purchased is 1 and she did not buy any skirt.

(ii) Polynomial **[1/2]**

(iii) Friendly nature and fond of shopping. **[1/2]**

18. Given equations are

$3x+y-5=0$ ...(i)

and $2x-y-5=0$ ...(ii)

From Eq. (i),

$y=5-3x$

If $x=0$, then $y=5-3\times0=5$

If $x=1$, then $y=5-3\times1=2$

If $x=2$, then $y=5-3\times2=-1$

x	0	1	2
y	5	2	-1

[1/2]

From Eq. (ii),

$y=2x-5$

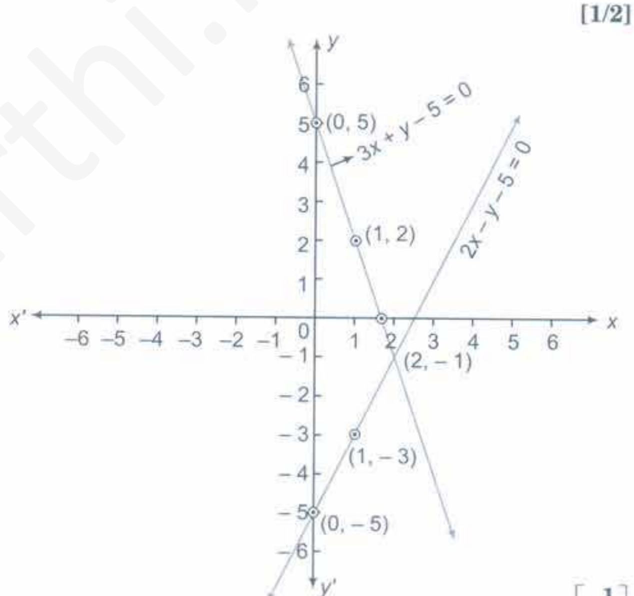
If $x=0$, then $y=2\times0-5=-5$

If $x=1$, then $y=2\times1-5=-3$

If $x=2$, then $y=2\times2-5=-1$

x	0	1	2
y	-5	-3	-1

[1/2]



$\left[1\frac{1}{2}\right]$

Here, $3x+y-5=0$ cuts y-axis at $(0, 5)$ and $2x-y-5=0$ cuts y-axis at $(0,-5)$. **[1/2]**

19. Draw $AE \perp BC$

In right ΔAED, using Pythagoras theorem

$AD^2=AE^2+ED^2$ ...(i)

In right ΔAEB, using Pythagoras theorem

$AB^2=AE^2+BE^2=AE^2+(BD-ED)^2$

$=AE^2+BD^2+ED^2-2BD\cdot ED$

$=(AE^2+ED^2)+BD^2-2BD\cdot ED$

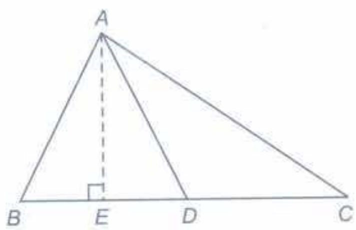
$= AD^2 + BD^2 - 2BD \cdot ED$

[from Eq. (i)] ...(ii) [1]

In right ΔAEC,

$$AC^2 = AE^2 + EC^2$$

[using Pythagoras theorem]



$= AE^2 + (ED + DC)^2$

$= AE^2 + ED^2 + DC^2 + 2ED \cdot DC$

$= (AE^2 + ED^2) + DC^2 + 2ED \cdot DC$

$= AD^2 + BD^2 + 2ED \cdot BD$

[$\because$ AD is the median] ...(iii) [1]

On adding Eqs. (ii) and (iii), we get

$AB^2 + AC^2 = AD^2 + BD^2 - 2BD \cdot ED$

$+ AD^2 + BD^2 + 2ED \cdot BD$

$\Rightarrow \quad AB^2 + AC^2 = 2AD^2 + 2BD^2$

$\Rightarrow \quad AB^2 + AC^2 = 2(AD^2 + BD^2)$ [1]

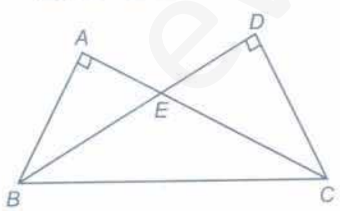
Hence proved.

20. Given Two ΔABC and ΔDBC are on the same base BC and on the same side of BC in which $\angle A = \angle D = 90^\circ$ such that CA and BD meet each other at E. [1]

To prove $AE \cdot EC = BE \cdot ED$

Proof In ΔAEB and ΔDEC,

$$\angle BAC = \angle BDC = 90^\circ$$



$\angle AEB = \angle DEC$ [vertically opposite angles] [1]

$\therefore \quad \Delta AEB \sim \Delta DEC$ [by AA similarity]

$\Rightarrow \quad \dfrac{AE}{ED} = \dfrac{BE}{EC}$

$\therefore \quad AE \cdot EC = BE \cdot ED$ [1]

21. In ΔABC, right angled at B, if one angle is 45°, then other angle is also 45°.

i.e., $\angle A = \angle C = 45^\circ$

So, $BC = AB$ [side opposite to equal angles]

Let $BC = AB = a$ [1]

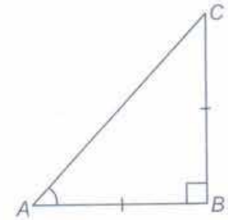
Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = a^2 + a^2$

$\Rightarrow \quad AC^2 = 2a^2$

$\Rightarrow \quad AC = \sqrt{2}a$



[1]

Now, $\sin 45^\circ = \dfrac{\text{Side opposite to } \angle 45^\circ}{\text{Hypotenuse}}$

$= \dfrac{BC}{AC} = \dfrac{a}{\sqrt{2}a}$

$\therefore \quad \sin 45^\circ = \dfrac{1}{\sqrt{2}}$ [1]

22. $\sin\theta + \cos\theta = \sqrt{3}$ [given]

On squaring both sides, we get

$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$

$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

[using $(a+b)^2 = a^2 + b^2 + 2ab$]

$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 3$ [$\because \sin^2\theta + \cos^2\theta = 1$] [1]

$\Rightarrow \quad 2\sin\theta\cos\theta = 3 - 1$

$\Rightarrow \quad 2\sin\theta\cos\theta = 2 \Rightarrow \sin\theta\cos\theta = 1$

$\Rightarrow \quad \sin\theta\cos\theta = \sin^2\theta + \cos^2\theta$ [1]

On dividing by $\sin\theta\cos\theta$ in both sides, we get

$\Rightarrow \quad 1 = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta}$

$\Rightarrow \quad 1 = \dfrac{\sin^2\theta}{\sin\theta \cdot \cos\theta} + \dfrac{\cos^2\theta}{\sin\theta \cdot \cos\theta}$

$\Rightarrow \quad 1 = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

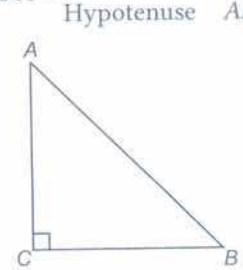
Therefore, $1 = \tan\theta + \cot\theta$

$\Rightarrow \quad \tan\theta + \cot\theta = 1$ [1]

OR

Sol. In right ΔACB, $\angle C = 90^\circ$, we have

$\cos A = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{AC}{AB}$ [1]



$\cos B = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{BC}{AB}$ [1]

We have, $\cos A = \cos B$ [given]

$\Rightarrow \quad \dfrac{AC}{AB} = \dfrac{BC}{AB}$

$\Rightarrow \quad AC = BC \Rightarrow \angle B = \angle A$ **[1]**

[$\because$ angles opposite to equal sides are equal]

23.

Life time (in hours)	Number of lamps (f_i)	Cumulative frequency (cf)
1500-2000	14	14 = 14
2000-2500	56	(14 + 56) = 70
2500-3000	60	(70 + 60) = 130 = cf
(Median class) 3000-3500	86 = f	(130 + 86) = 216
3500-4000	74	(216 + 74) = 290
4000-4500	62	(290 + 62) = 352
4500-5000	48	(352 + 48) = 400
Total	$N = 400$	

$\because \quad \dfrac{N}{2} = \dfrac{400}{2} = 200$ **[1]**

Since, cumulative frequency 20 lies in the interval 3000-3500.

Here, (Lower median class) $l = 3000$, $f = 86$, $cf = 130$, (Class width) $h = 500$, (Total observations) $N = 400$

$$\text{Median} = l + \left\{\frac{\frac{N}{2} - cf}{f}\right\} \times h \quad \textbf{[1]}$$

$$= 3000 + \frac{(200 - 130)}{86} \times 500$$

$$= 3000 + \frac{70 \times 500}{86}$$

$$= 3000 + \frac{35000}{86} = 3000 + 406.98$$

$$= 3406.98 \text{ h}$$

Hence, median life time of a lamp is 3406.98 h. **[1]**

24. Since, maximum frequency is $f_m = 18$, so its modal class (4000-5000).

$\therefore$ (Lower modal class)

$l = 4000$, $f_m = 18$, $f_1 = 4$, $f_2 = 9$, (Class width) $h = 1000$ **[1]**

$$\text{Mode} = l + \left\{\frac{f_m - f_1}{2f_m - f_1 - f_2}\right\} \times h \quad \textbf{[1]}$$

$$= 4000 + \left\{\frac{18 - 4}{36 - 4 - 9}\right\} \times 1000$$

$$= 4000 + \frac{14000}{23} = 4000 + 608.7$$

$$= 4608.7 \text{ runs}$$ **[1]**

25. Given, two zeroes of $2x^4 - 9x^3 + 5x^2 + 3x - 1$ are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

$\therefore$ Sum of the zeroes $= 2+\sqrt{3}+2-\sqrt{3} = 4$

and product of zeroes $= (2+\sqrt{3})(2-\sqrt{3})$

$= 4 - 3 = 1$ **[1]**

A polynomial whose roots are $(2+\sqrt{3})$ and $(2-\sqrt{3})$ is given by

$x^2 - (\text{sum of the zeroes})x + (\text{product of zeroes})$

$= x^2 - 4x + 1$ **[1/2]**

So, $x^2 - 4x + 1$ is a factor of given polynomial.

Divide $2x^4 - 9x^3 + 5x^2 + 3x - 1$ by $x^2 - 4x + 1$

$$x^2 - 4x + 1 \overline{\big)\, 2x^4 - 9x^3 + 5x^2 + 3x - 1 \,\big(}\, 2x^2 - x - 1$$
$$\begin{array}{r} 2x^4 - 8x^3 + 2x^2 \\ - \quad + \quad - \\ \hline -x^3 + 3x^2 + 3x - 1 \\ -x^3 + 4x^2 - x \\ + \quad - \quad + \\ \hline -x^2 + 4x - 1 \\ -x^2 + 4x - 1 \\ + \quad - \quad + \\ \hline 0 \end{array}$$ **[1]**

Now, $2x^2 - x - 1 = 2x^2 - 2x + x - 1$

$= 2x(x-1) + 1(x-1)$

$= (x-1)(2x+1)$ **[1/2]**

$\therefore 2x^4 - 9x^3 + 5x^2 + 3x - 1 = (x^2 - 4x + 1)(x-1)(2x+1)$

So, the other zeroes are 1 and $-\dfrac{1}{2}$.

Thus, all zeroes of given polynomial are $(2+\sqrt{3})$, $(2-\sqrt{3})$, 1 and $-\dfrac{1}{2}$. **[1]**

26. Two zeroes of polynomial $p(x)$ are $\sqrt{3}$ and $-\sqrt{3}$.

$\therefore \ (x-\sqrt{3})$ and $(x+\sqrt{3})$ are two factors of $p(x)$. **[1]**

$\Rightarrow \quad (x-\sqrt{3})(x+\sqrt{3})$ is a factor of $p(x)$.

$\Rightarrow \quad (x^2 - 3)$ is a factor of $p(x)$. **[1]**

Polynomial with sum of zeroes -1 and product -2 is $x^2 + x - 2$.

$\therefore x^2 + x - 2$ is also a factor of $p(x)$. **[1]**

Since, degree of $p(x)$ is 4.

$\therefore p(x) = (x^2 - 3)(x^2 + x - 2)$

$= x^4 + x^3 - 5x^2 - 3x + 6$ **[1]**

27. Let 1 woman finish the work in x days and let 1 man finish the work in y days.

Work of 1 woman in 1 day $= \frac{1}{x}$

Work of 1 man in 1 day $= \frac{1}{y}$

Work of 2 women and 5 men in one day

$$= \frac{2}{x} + \frac{5}{y} = \frac{5x+2y}{xy}$$

The number of days required for complete work

$$= \frac{xy}{5x+2y} \qquad [1]$$

We are given that, $\frac{xy}{5x+2y} = 4$

Similarly, in second case

$$\frac{xy}{6x+3y} = 3 \qquad \text{[given]}$$

Then, $\frac{5x+2y}{xy} = \frac{1}{4}$ and $\frac{6x+3y}{xy} = \frac{1}{3}$

$\Rightarrow \quad \frac{20}{y} + \frac{8}{x} = 1$ and $\frac{18}{y} + \frac{9}{x} = 1$ **[1]**

On putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$

$\therefore \quad 20v + 8u = 1 \qquad$ ...(i)

and $\quad 18v + 9u = 1 \qquad$ ...(ii)

On multiplying Eq. (i) by 9 and Eq. (ii) by 8, then subtracting later from first, we get

$$180v - 144v = 9 - 8$$

$\Rightarrow \quad 36v = 1 \Rightarrow v = \frac{1}{36}$ **[1]**

On substituting $v = \frac{1}{36}$ in Eq. (ii), we get

$$18 \times \frac{1}{36} + 9u = 1$$

$\Rightarrow \quad 9u = 1 - \frac{1}{2} \Rightarrow u = \frac{1}{18}$

Now, $\quad u = \frac{1}{18}$ and $v = \frac{1}{36}$

$\Rightarrow \quad \frac{1}{x} = \frac{1}{18}$ and $\frac{1}{y} = \frac{1}{36}$

$\Rightarrow \quad x = 18$ and $y = 36$

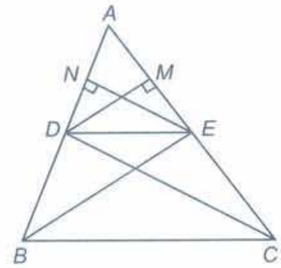
Hence, single woman finishes the work in 18 days and single man finishes the work in 36 days. **[1]**

28. Given A ΔABC in which a line parallel to side BC intersects other two sides AB and AC at D and E, respectively. **[1/2]**

To prove $\frac{AD}{DB} = \frac{AE}{EC}$

Construction Join BE and CD and then draw $DM \perp AC$ and $EN \perp AB$. **[1/2]**

Proof In ΔADE and ΔBDE,



Area of $\Delta ADE = \frac{1}{2} \times AD \times EN \qquad$ ...(i)

Area of $\Delta BDE = \frac{1}{2} \times DB \times EN \qquad$ ...(ii)**[1]**

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \qquad \text{...(iii)}$$

Similarly, Area of $\Delta ADE = \frac{1}{2} \times AE \times DM \qquad$ ...(iv)

Area of $\Delta DEC = \frac{1}{2} \times EC \times DM \qquad$ ...(v)**[1]**

On dividing Eq. (iv) by Eq. (v), we get

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \qquad \text{...(vi)}$$

Now, ΔBDE and ΔDEC are on the same base DE and between the same parallel lines BC and DE.

So, Area of ΔBDE = Area of $\Delta DEC \qquad$ ...(vii)

From Eq. (vi),

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{AE}{EC} \quad \text{[from Eq. (vii)]} \ \text{...(viii)}$$

From Eqs. (iii) and (viii), we get

$$\frac{AD}{DB} = \frac{AE}{EC} \qquad [1]$$

Hence proved.

29. LHS $= (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) \qquad [1]$$

$$\left[\because \cot A = \frac{\cos A}{\sin A}, \operatorname{cosec} A = \frac{1}{\sin A}, \tan A = \frac{\sin A}{\cos A} \text{ and } \sec A = \frac{1}{\cos A}\right]$$

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right) \qquad [1]$$

]

$$= \frac{(\sin A + \cos A)^2 - 1^2}{\sin A \cos A}$$

$$[\because (a+b)(a-b) = a^2 - b^2]$$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}{\sin A \cos A} \quad \textbf{[1]}$$

$$[\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$= \frac{1 + 2\sin A \cos A - 1}{\sin A \cos A} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{2\sin A \cos A}{\sin A \cos A} = 2$$

$\therefore$ LHS = RHS **[1]**

30. Given, integers are 56, 96 and 404.

First, we will find the HCF of 56 and 96.

On applying Euclid's division algorithm, we get

$$96 = 56 \times 1 + 40$$

Since, remainder, $40 \neq 0$, so we apply Euclid's division algorithm to 56 and 40.

$$56 = 40 \times 1 + 16 \quad \textbf{[1]}$$

$\because$ Remainder, $16 \neq 0$, so we apply Euclid's division algorithm to 40 and 16.

$$40 = 16 \times 2 + 8$$

$\because$ Remainder, $8 \neq 0$, so again apply Euclid's division algorithm, we get $16 = 8 \times 2 + 0$

Clearly, HCF of 56 and 96 is 8. **[1]**

Now, we find the HCF of 8 and third number 404.

On applying Euclid's division, we get

$$404 = 50 \times 8 + 4 \quad \textbf{[1]}$$

Since, remainder, $4 \neq 0$, so we apply Euclid's division algorithm to 8 and 4.

$$8 = 4 \times 2 + 0$$

Since, remainder is 0.

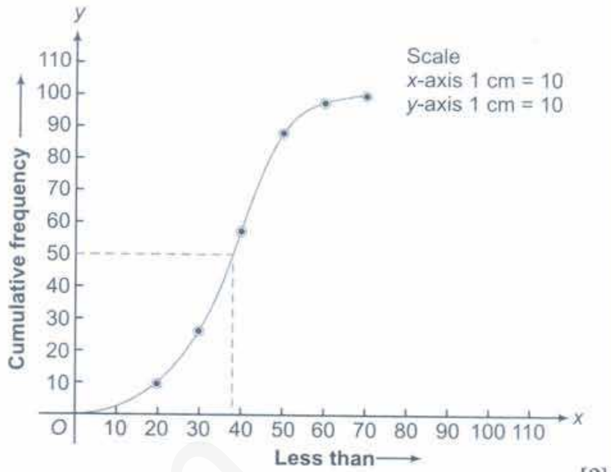
i.e., HCF of 56, 96 and 404 is 4. **[1]**

31. We have,

Marks	Number of students	Marks less than	cf
10-20	10	less than 20	10
20-30	15	less than 30	25
30-40	30	less than 40	55
40-50	32	less than 50	87
50-60	8	less than 60	95
60-70	5	less than 70	100
Total	N=100 (even)		

[1]

Now, plot the points (20, 10), (30, 25), (40, 55), (50, 87), (60, 95) and (70, 100).



[2]

$$\therefore \text{Median} = \text{Size of} \left(\frac{N}{2}\right)\text{th term}$$

$$= \text{Size of} \left(\frac{100}{2}\right)\text{th term}$$

$$= \text{Size of 50th term}$$

$$= 38.3 \quad \textbf{[1]}$$

32. (i) False; because when $A = 60^\circ$ and $B = 30^\circ$.

$$\sin(A + B) = \sin(60^\circ + 30^\circ)$$

$$= \sin 90^\circ = 1$$

and $\sin A + \sin B = \sin 60^\circ + \sin 30^\circ$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= \frac{\sqrt{3} + 1}{2}$$

So, $\sin(A + B) \neq \sin A + \sin B$ **[1]**

(ii) True; because it is clear from the table below that the $\sin\theta$ increase as θ increase.

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

[1]

(iii) False; because it is true only for $\theta = 45^\circ$

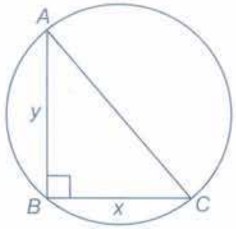
$$\left[\because \sin 45^\circ = \frac{1}{\sqrt{2}} = \cos 45^\circ\right] \quad \textbf{[1]}$$

(iv) True; since for $A = 0^\circ$, $\tan A = \tan 0^\circ = 0$

$$\therefore \cot A = \frac{1}{\tan A} = \frac{1}{0} = \text{not defined.} \quad \textbf{[1]}$$

33. Let ABC be a right angled triangle at B.

Let $AB = y$, $BC = x$



A semi-circle is drawn all three sides of a triangle.

In ΔABC, by Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$
$$= y^2 + x^2$$
$$\Rightarrow \quad AC = \sqrt{y^2 + x^2} \qquad [1]$$

$\therefore$ Area of semi-circle drawn on AC, is

$$A_1 = \frac{\pi r_1^2}{2}$$
$$= \frac{\pi}{2}\left(\frac{\sqrt{y^2 + x^2}}{2}\right)^2$$
$$= \pi\left(\frac{y^2 + x^2}{8}\right) \quad \left(\because r = \frac{AC}{2}\right) \quad \text{...(i) [1]}$$

Now, area of semi-circle drawn on BC, is

$$A_2 = \frac{\pi r_2^2}{2} = \frac{\pi}{2}\left(\frac{x}{2}\right)^2 \qquad \text{...(ii)}$$

Also, area of semi-circle drawn on AB, is

$$A_3 = \frac{\pi r_3^2}{2} = \frac{\pi}{2}\left(\frac{y}{2}\right)^2$$
$$= \frac{\pi y^2}{8} \qquad \text{...(iii) [1]}$$

$\therefore$ From Eq. (i), we get

$$A_1 = \pi\left(\frac{y^2}{8} + \frac{x^2}{8}\right)$$
$$= \frac{\pi y^2}{8} + \frac{\pi x^2}{8} = A_3 + A_2 \qquad [1]$$

[from Eqs. (ii) and (iii)]

Hence proved.

34. (i)

Number of letters	Number of surnames	Cumulative frequency
1-4	6	6 = 6
4-7	30	6 + 30 = 36 = *cf*
(Median class) 07-10	40 = *f*	36 + 40 = 76
10-13	16	76 + 16 = 92
13-16	4	92 + 4 = 96
16-19	4	96 + 4 = 100
Total	*N* = 100	

[1]

(i) $\because \quad \frac{N}{2} = \frac{100}{2} = 50$

Since, cumulative frequency 50 lies in the interval 7-10.

Here, (Lower median class) $l = 7$, $f = 40$, $cf = 36$, (Class width) $h = 3$, (Total observations) $N = 100$

$$\text{Median} = l + \left\{\frac{\frac{N}{2} - cf}{f}\right\} \times h \qquad [1]$$
$$= 7 + \left\{\frac{50 - 36}{40}\right\} \times 3$$
$$= 7 + \frac{14 \times 3}{40}$$
$$= 7 + \frac{21}{20} = 7 + 1.05 = 8.05$$

Hence, the median number of letters in the surnames is 8.05.

(ii) Modal class is (7-10) (because it has maximum frequency $f_m = 40$).

(Lower modal class) $l = 7$, $f_m = 40$, $f_1 = 30$, $f_2 = 16$, (Class width) $h = 3$ **[1]**

Mode

$$= l + \left\{\frac{f_m - f_1}{2f_m - f_1 - f_2}\right\} \times h = 7 + \left\{\frac{40 - 30}{80 - 30 - 16}\right\} \times 3$$
$$= 7 + \frac{30}{34} = 7 + 0.88 = 7.88$$

Hence, the modal size of the surnames is 7.88. **[1]**