Stage I

SAMPLE Question Paper

Fully Solved (Question-Solution)

MATHEMATICS

A Highly Simulated Practice Question Paper
for CBSE Class X Term II Examination (SA II)

Time : 3 hrs **Max. Marks : 90**

General Instructions

1. All questions are compulsory.
2. Draw neat labelled diagram whenever necessary to explain your answer.
3. Q. Nos. 1-8 are multiple choice questions, carrying 1 mark each.
4. Q. Nos. 9-14 are short answer type questions, carrying 2 marks each.
5. Q. Nos. 15-24 are short answer type questions, carrying 3 marks each.
6. Q. Nos. 25-34 are long answer type questions, carrying 4 marks each.

Section A

Que 1. If the roots of $x^2 + Kx + 12 = 0$ are in the ratio $1:3$, then the value of K is equal to

(a) ± 6 (b) ± 7 (c) ± 8 (d) ± 9

Sol. (c) Let the roots be α and 3α.

$\therefore$ Sum of roots, $\alpha + 3\alpha = -K$

$$\Rightarrow \quad 4\alpha = -K$$

$$\Rightarrow \quad \alpha = \frac{-K}{4}$$

Also, product of roots,

$$(\alpha)(3\alpha) = 12$$

$$\Rightarrow \quad 3\alpha^2 = 12 \quad \Rightarrow \quad \alpha^2 = 4$$

$$\Rightarrow \quad \left(\frac{-K}{4}\right)^2 = 4 \quad \Rightarrow \quad \frac{K^2}{16} = 4$$

$$\Rightarrow \quad K^2 = 64 \quad \Rightarrow \quad K = \pm 8$$

Que 2. A metallic cube of edge 1 cm is drawn into a wire of diameter 4 mm, then the length of the wire is

(a) $\frac{100}{\pi}$ cm (b) 100π cm

(c) $\frac{25}{\pi}$ cm (d) 10000 cm

Sol. (c) Let h be length of the wire and radius = 2 mm

$\because$ A metallic cube is drawn into a wire.

$\therefore$ Volume of wire = Volume of cube

$$\Rightarrow \quad \pi \times \frac{2}{10} \times \frac{2}{10} \times h = 1 \times 1 \times 1 \quad (\because 1 \text{ mm} = 10 \text{ cm})$$

$$\Rightarrow \quad \frac{\pi}{25} h = 1$$

$$\Rightarrow \quad h = \frac{25}{\pi} \text{ cm}$$

Que 3. For an AP, if $a_{25} - a_{20} = 35$, then d equals

(a) 9 (b) -9
(c) 7 (d) 23

Sol. (c) Given, $a_{25} - a_{20} = 35$

$$\Rightarrow (a + 24d) - (a + 19d) = 35$$
$$\Rightarrow a + 24d - a - 19d = 35$$
$$\Rightarrow 5d = 35$$
$$\therefore d = 7$$

Que 4. The perimeter of a quadrant of a circle of radius r is

(a) $\frac{\pi r}{2}$ (b) $2\pi r$
(c) $\frac{r}{2}(\pi + 4)$ (d) $2\pi r + \frac{r}{2}$

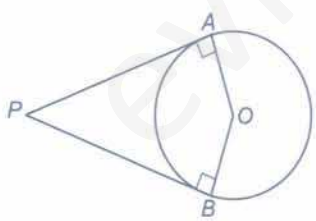
Sol. (c) Perimeter of a quadrant $= r + r + \frac{1}{4} \times 2\pi r$

$$= 2r + \frac{1}{2}\pi r$$
$$= \frac{r}{2}(\pi + 4)$$

Que 5. If the angle between two radii of a circle is 100°, the angle between the tangents at the ends of these radii is

(a) 90° (b) 80°
(c) 70° (d) 60°

Sol. (b) Let PA and PB are tangents to the circle with centre O.



$\therefore$ $OA \perp AP$ and $OB \perp PB$

and $\angle AOB = 100°$

$$\angle APB = 180° - \angle AOB$$
$$= 180° - 100° = 80°$$

Que 6. The area of a triangle with vertices $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$ is

(a) $(a + b + c)^2$ (b) 0
(c) $a + b + c$ (d) abc

Sol. (b) Let $(x_1, y_1) = (a, b + c), (x_2, y_2) = (b, c + a)$

and $(x_3, y_3) = (c, a + b)$

$\therefore$ Area of triangle

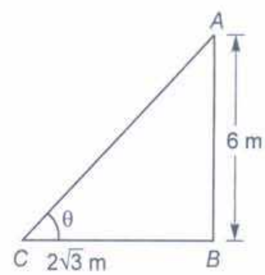
$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$= \frac{1}{2}[a\{(c + a - (a + b)\} + b\{(a + b - (b + c)\} + c\{(b + c - (c + a)\}]$$
$$= \frac{1}{2}[a(c - b) + b(a - c) + c(b - a)]$$
$$= \frac{1}{2}(ac - ab + ba - bc + cb - ca)$$
$$= \frac{1}{2}[0] = 0$$

Que 7. A pole 6 m high casts a shadow $2\sqrt{3}$ m on the ground, then the angle of elevation of the sun's is

(a) 30° (b) 45°
(c) 60° (d) 90°

Sol. (c) Let AB be the height of pole and BC be its shadow.
In right angled ΔABC, $\angle B = 90°$

$\therefore$ $AB = 6$ m, $BC = 2\sqrt{3}$ m



Let $\angle ACB = \theta$

$$\therefore \tan\theta = \frac{AB}{BC}$$
$$\Rightarrow \tan\theta = \frac{6}{2\sqrt{3}}$$
$$\Rightarrow \tan\theta = \sqrt{3}$$
$$\Rightarrow \theta = 60°$$

Que 8. For a race of 1980 m, number of rounds one have to take on a circular track of radius 35 m is

(a) 5 (b) 6 (c) 8 (d) 9

Sol. (d) $\because n\cdot(2\pi r) = 1980$, where n = number of rounds taken

$$n = \frac{1980}{2\pi r} = \frac{1980}{2 \times \frac{22}{7} \times 35}$$
$$n = \frac{1980}{220} = 9$$
$$\therefore n = 9$$

Section B

Que 9. The sum of two numbers is 9 and the sum of their reciprocals is $\frac{1}{2}$. Find the numbers.

Sol. Let the smaller number be x, then the other number is $9-x$.

According to question,

$$\frac{1}{x}+\frac{1}{9-x}=\frac{1}{2}$$

$$\Rightarrow \quad \frac{9-x+x}{x(9-x)}=\frac{1}{2} \qquad [1]$$

$$\Rightarrow \quad 18=9x-x^2$$

$$\Rightarrow \quad x^2-9x+18=0$$

$$\Rightarrow \quad x^2-6x-3x+18=0$$

$$\Rightarrow \quad x(x-6)-3(x-6)=0$$

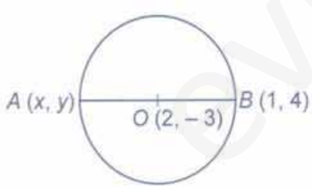
$$\Rightarrow \quad (x-6)(x-3)=0$$

$$\therefore \quad x=6, 3$$

Hence, the smaller number is 3 and other is 6. **[1]**

Que 10. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is $(2, -3)$ and B is $(1, 4)$.

Sol. Let the coordinates of a point A be (x, y). As AB is diameter and O is centre of circle. Then, O will be mid-point of AB.



$\therefore$ Coordinates of O = Coordinates of mid-point of AB

$$(2, -3)=\left(\frac{x+1}{2}, \frac{y+4}{2}\right) \qquad [1]$$

$$\Rightarrow \quad 2=\frac{x+1}{2}$$

$$\Rightarrow \quad x+1=4 \quad \Rightarrow \quad x=3$$

$$\text{and} \quad -3=\frac{y+4}{2} \quad \Rightarrow \quad y+4=-6$$

$$y=-10$$

Hence, coordinates of point A are $(3, -10)$. **[1]**

Que 11. The 8th term of an AP is 37 and its 12th term is 57. Find the AP.

OR

If 6th term of an AP is – 10 and its 10th term is – 26, then find the 17th term of the AP.

Sol. Let a be the first term and d be the common difference of given AP.

$$\text{Given,} \quad a_8=37$$

$$\Rightarrow \quad a+(8-1)d=37$$

$$\Rightarrow \quad a+7d=37 \qquad \text{...(i)}$$

$$\text{and} \quad a_{12}=57$$

$$\Rightarrow \quad a+(12-1)d=57$$

$$\Rightarrow \quad a+11d=57 \qquad \text{...(ii)} \; [1]$$

Subtracting Eq. (i) from Eq. (ii), we get

$$4d=20$$

$$\Rightarrow \quad d=5$$

Now, from Eq. (i), we get

$$a+7(5)=37$$

$$a=2$$

$\therefore$ Required AP is 2, 7, 12, 17, ... **[1]**

OR

Let a be the first term and d be the common difference of given AP.

$$\text{Given,} \quad a_6=-10 \quad \Rightarrow \quad a+5d=-10 \qquad \text{...(i)}$$

$$\text{and} \quad a_{10}=-26 \quad \Rightarrow \quad a+9d=-26 \qquad \text{...(ii)}$$

Subtracting Eq. (i) from Eq. (ii), we get

$$4d=-16$$

$$d=-4 \qquad [1]$$

Now, from Eq. (i), we get

$$a+5(-4)=-10$$

$$\Rightarrow \quad a-20=-10$$

$$\Rightarrow \quad a=10$$

$$\therefore \quad a_{17}=a+16d=10+16\times(-4)$$

$$=-54 \qquad [1]$$

Que 12. Two coins are tossed together. Find the probability of getting atleast one tail.

Sol. When two coins are tossed together, then the number of possible outcomes = 4

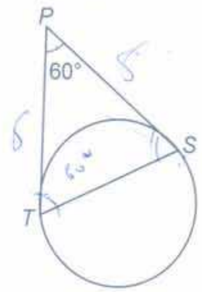
i.e., (HH, HT, TH, TT) **[1/2]**

$\therefore$ Number of favourable outcomes (atleast one tail)

= 3, as (HT, TH, TT) **[1/2]**

$\therefore$ Required probability $=\frac{3}{4}$ **[1]**

Que 13. In figure, PT and PS are tangents to a circle from a point P such that $PT = 5$ cm and $\angle TPS = 60°$. Find the length of chord TS.



Sol. Since, tangents drawn from external point to the circle are equal.

$$\therefore \quad PS = PT = 5 \text{ cm}$$

$$\angle PTS = \angle PST$$

[$\because$ angle opposite to equal sides are equal] **[1/2]**

In ΔPTS, we have

$$\angle PTS + \angle PST + \angle TPS = 180°$$

$$\Rightarrow \quad \angle PTS + \angle PTS + 60° = 180°$$

[$\because \angle PST = \angle PTS$]

$$\Rightarrow \quad 2\angle PTS = 180° - 60°$$

$$\Rightarrow \quad 2\angle PTS = 120°$$

$$\therefore \quad \angle PTS = \frac{120}{2} = 60°$$

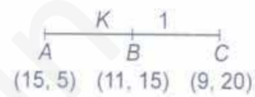
$\therefore$ ΔPTS is an equilateral triangle.

$$\therefore \quad TS = 5 \text{ cm}$$ **[1½]**

Que 14. Find the ratio in which the point (11, 15) divides the line segment joining the points (15, 5) and (9, 20).

Sol. Let the point $C(11, 15)$ divides the line segment AB joining $A(15, 5)$ and $B(9, 20)$ in the ratio $K:1$, then $x_1 = 15$, $y_1 = 5$, $x_2 = 9$, $y_2 = 20$, $x = 11$, $y = 15$

$$\therefore \quad x = \frac{Kx_2 + x_1}{K+1}, \; y = \frac{Ky_2 + y_1}{K+1}$$ **[1]**



$$\Rightarrow \quad 11 = \frac{K \times 9 + 15}{K+1}, \; 15 = \frac{K \times 20 + 5}{K+1}$$

$$\Rightarrow \quad 11K + 11 = 9K + 15, \; 15K + 15 = 20K + 5$$

$$\Rightarrow \quad 2K = 4 \qquad -5K = -10$$

$$\Rightarrow \quad K = 2 \qquad K = 2$$

Hence, the required ratio is 2 : 1. **[1]**

Section C

Que 15. Solve for $x : 9^{x+2} - 6 \cdot 3^{x+1} + 1 = 0$

Sol. The given equation is

$$9^{x+2} - 6 \cdot 3^{x+1} + 1 = 0$$

$$\Rightarrow \quad 9^x \cdot 9^2 - 6 \cdot 3^x \cdot 3 + 1 = 0$$

$$\Rightarrow \quad 81(3^2)^x - 18 \cdot 3^x + 1 = 0 \qquad \text{...(i)}$$ **[1]**

Now, putting $3^x = y$ in Eq. (i), we get

$$81y^2 - 18y + 1 = 0$$

$$\Rightarrow \quad 81y^2 - 9y - 9y + 1 = 0$$

$$\Rightarrow \quad 9y(9y-1) - 1(9y-1) = 0$$

$$\Rightarrow \quad (9y-1)(9y-1) = 0$$

$$\Rightarrow \quad (9y-1)^2 = 0$$

$$\Rightarrow \quad 9y - 1 = 0$$

$$\Rightarrow \quad 9y = 1$$

$$\Rightarrow \quad y = \frac{1}{9}$$ **[1]**

$$\Rightarrow \quad 3^x = \frac{1}{9} = \frac{1}{3^2} = 3^{-2} \qquad [\because y = 3^x]$$

$$\Rightarrow \quad 3^x = 3^{-2}$$

$$\therefore \quad x = -2$$

Hence, the required solution is $x = -2$. **[1]**

Que 16. If $a \neq b \neq c$, prove that the points (a, a^2), (b, b^2) and (c, c^2) can never be collinear.

Sol. If the area of the triangle formed by joining the given points is zero, then the points are collinear.

$$\text{Area of triangle} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \quad \text{...(i)}$$

Here, $x_1 = a$, $y_1 = a^2$; $x_2 = b$, $y_2 = b^2$;

$x_3 = c$, $y_3 = c^2$ **[1]**

Substituting these values in Eq. (i), we get

Area of triangle

$$=\frac{1}{2}[a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)]$$

$$=\frac{1}{2}[ab^2-ac^2+bc^2-a^2b+a^2c-cb^2]$$

$$=\frac{1}{2}[-a^2(b-c)+a(b^2-c^2)-bc(b-c)]$$ [1]

$$=\frac{1}{2}[(b-c)\{-a^2+a(b+c)-bc\}]$$

$$=\frac{1}{2}[(b-c)(-a^2+ab+ac-bc)]$$

$$=\frac{1}{2}[(b-c)\{-a(a-b)+c(a-b)\}]$$

$$=\frac{1}{2}[(b-c)(a-b)(c-a)]$$

It is given that $a \neq b \neq c$

$\therefore$ Area of the triangle can never be 0.

Hence, the points (a, a^2), (b, b^2) and (c, c^2) never be collinear. **[1]**

Que 17. Which terms of AP, 121, 117, 113, ... is its first negative term?

OR

Find the sum of all natural numbers between 200 and 1000, exactly divisible by 6.

Sol. Here, $a=121, d=117-121=-4$

$\because$ $a_n=a+(n-1)d$

$=121+(n-1)(-4)$

$=121-4n+4$

$=125-4n$ **[1]**

For first term to be negative,

$\therefore$ $a_n<0$

$\Rightarrow$ $125-4n<0$

$\Rightarrow$ $125<4n$

$\Rightarrow$ $4n>125$

$\Rightarrow$ $n>\frac{125}{4}=31\frac{1}{4}$

Hence, 32nd term will be the first negative term. **[2]**

OR

Natural numbers between 200 and 1000 exactly divisible by 6 are

204, 210, 216, ..., 996

Here, $a=204$, $d=6$ and $a_n=996$

$\Rightarrow$ $a+(n-1)d=996$

$\Rightarrow$ $204+(n-1)6=996$

$\Rightarrow$ $(n-1)6=996-204=792$

$\Rightarrow$ $(n-1)=\frac{792}{6}=132$

$\Rightarrow$ $n=132+1=133$ **[1]**

$\therefore$ $S_n=\frac{n}{2}[a+a_n]$

$=\frac{133}{2}[204+996]$

$=\frac{133}{2}\times 1200=133\times 600$

$=79800$ **[2]**

Que 18. A lot consists of 144 ball pens of which 20 are defective and the others are good. Neha will buy a pen if it is good, but will not buy it, if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that (i) she will buy it? (ii) she will not buy it?

Sol. Total number of ball pens $=144$

Number of defective pens $=20$

Number of good pens $=144-20=124$

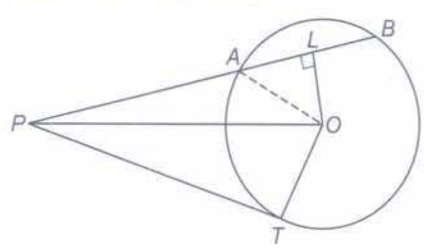
(i) $P(\text{she will buy it})=\frac{n(\text{good pens})}{n(S)}=\frac{124}{144}=\frac{31}{36}$ **[1]**

(ii) $P(\text{she will not buy it})=\frac{n(\text{defective pens})}{n(S)}$

$=\frac{20}{144}=\frac{5}{36}$ **[2]**

Que 19. If PAB is a secant to a circle intersecting the circle at A and B and PT is a tangent segment, then prove that $PA\times PB=PT^2$.

Sol. Given, A secant PAB to a circle $C(O, r)$ intersecting it in A and B and PT is a tangent segment.



[1]

To prove $PA\times PB=PT^2$

Construction $OL\perp AB$, join OP, OT and OA

Proof Since, $OL\perp$ chord AB

$\Rightarrow$ $AL=BL$

$\therefore$ $PA\times PB=(PL-AL)\times(PL+BL)$

$$= (PL - AL) \times (PL + AL)$$

$$PA \times PB = PL^2 - AL^2 \quad \text{[1]}$$

$$PA \times PB = (OP^2 - OL^2) - AL^2$$

$$[\because OP^2 = PL^2 + OL^2]$$

$$= OP^2 - (OL^2 + AL^2)$$

$$[\because OA^2 = OL^2 + AL^2]$$

$$= OP^2 - OA^2$$

$$= OP^2 - OT^2 \qquad [OA = OT = r]$$

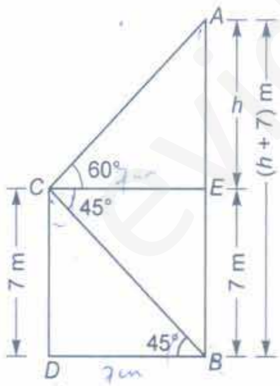
$$\therefore \quad PA \times PB = PT^2 \quad [\because PT^2 = OP^2 - OT^2] \quad \text{[1]}$$

Que 20. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

OR

A tree breaks due to the storm and the broken part bends, so that the top of the tree touches the ground making an angle of 30° with the ground. The distance from the foot of the tree to the point, where the top touches the ground is 8 m. Find the height of the tree.

Sol. Let AB is the tower of height $(h+7)$ m and CD is the building 7 m high, such that $\angle ACE = 60°$, $\angle ECB = 45°$



[1]

$$\Rightarrow \quad \angle CBD = 45°$$

In ΔCDB, $\dfrac{CD}{DB} = \tan 45°$

$$\Rightarrow \quad \frac{7}{DB} = 1$$

$$\Rightarrow \quad DB = 7$$

$$\Rightarrow \quad CE = 7 \qquad [\because DB = CE = 7] \quad \text{[1]}$$

In ΔAEC, $\dfrac{AE}{CE} = \tan 60°$

$$\Rightarrow \quad \frac{h}{7} = \sqrt{3}$$

$$\Rightarrow \quad h = 7\sqrt{3} \text{ m}$$

Hence, height of the tower $= (h+7)$ m

$$= (7\sqrt{3} + 7) \text{ m}$$

$$= 7(\sqrt{3} + 1) \text{ m}$$

$$= 7(1.73 + 1)$$

$$= 7(2.73) \quad \text{[1]}$$

$$= 19.11 \text{ m}$$

OR

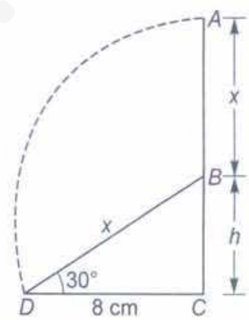
Let CBA be the tree of height $(h+x)$ m.

Let the height of the tree after broken part be h m.

Let $AB = BD = x$ and $CD = 8$ m [given]

In ΔBCD, $\dfrac{CD}{x} = \cos 30°$

$$\Rightarrow \quad \frac{8}{x} = \frac{\sqrt{3}}{2} \quad \text{[1]}$$



$$\Rightarrow \quad x = \frac{16}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{16\sqrt{3}}{3}$$

Again, in ΔBCD, $\dfrac{h}{8} = \tan 30° = \dfrac{1}{\sqrt{3}}$ [1]

$$\Rightarrow \quad h = \frac{8}{\sqrt{3}} = \frac{8}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$$

$$\therefore \text{ Height of the tree} = h + x = \frac{8\sqrt{3}}{3} + \frac{16\sqrt{3}}{3}$$

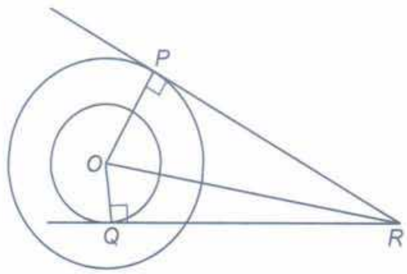
$$= \frac{24\sqrt{3}}{3} = 8\sqrt{3}$$

$$= 8 \times 1.73 = 13.84 \text{ m} \quad \text{[1]}$$

Que 21. Two concentric circles are of radii 10 cm 8 cm. RP and RQ are tangents to the two circles from R. If the length of RP is 24 cm, find the length of RQ.

Sol. Given that $OP = 10$ cm and $OQ = 8$ cm

and $RP = 24$ cm, join OR



[1]

In ΔOPR, we have $OP \perp PR$

$$\therefore \quad OR = \sqrt{PR^2 + OP^2}$$
$$= \sqrt{24^2 + 10^2}$$
$$= \sqrt{576 + 100}$$
$$= \sqrt{676} = 26 \text{ cm} \quad [1]$$

In ΔOQR, we have $OQ \perp QR$

$$\therefore \quad OR^2 = RQ^2 + OQ^2$$
$$\Rightarrow \quad RQ^2 = OR^2 - OQ^2$$
$$= 26^2 - 8^2$$
$$RQ^2 = 676 - 64$$
$$RQ^2 = 612$$
$$RQ = \sqrt{612}$$
$$RQ = \sqrt{6 \times 6 \times 17} = 6\sqrt{17} \text{ cm} \quad [1]$$

Que 22. Do the points (3, 2), (−2, −3) and (2, 3) form a triangle? If so, name the type of the triangle formed.

OR

Show that the points $P(a, a), Q(-a, -a)$ and $R(-a\sqrt{3}, a\sqrt{3})$ form an equilateral triangle.

Sol. Let the given three points be $A(3, 2), B(-2, -3)$ and $C(2, 3)$.

$\therefore$ By using distance formula, we have

$$AB = \sqrt{(-2-3)^2 + (-3-2)^2}$$
$$= \sqrt{(-5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \text{ units}$$
$$= 7.07 \text{ units}$$
$$BC = \sqrt{(2+2)^2 + (3+3)^2}$$
$$= \sqrt{(4)^2 + (6)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13} \text{ units}$$
$$= 7.21 \text{ units}$$
$$CA = \sqrt{(3-2)^2 + (2-3)^2} \quad [1]$$
$$= \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2} \text{ units} = 1.41 \text{ units}$$

Now, $AB + BC = 7.07 + 7.21 = 14.28 \text{ units} > CA$

$$BC + CA = 7.21 + 1.41 = 8.62 \text{ units} > AB$$

and $AB + CA = 7.07 + 1.41 = 8.48 \text{ units} > BC$ [1]

Thus, the given points, A, B and C form a triangle.

Also, $(AB)^2 = (\sqrt{50})^2 = 50$ units

$$(BC)^2 = (\sqrt{52})^2 = 52 \text{ units}$$
$$(CA)^2 = (\sqrt{2})^2 = 2 \text{ units}$$

i.e., $(AB)^2 + (CA)^2 = (BC)^2$

[since, the sum of the squares of two sides is equal to square of the third side.]

$\therefore$ By converse of Pythagoras theorem, we have $\angle A = 90°$. Hence, ΔABC is a right triangle. [1]

OR

Let coordinates of the vertices of a triangle be $P(a, a), Q(-a, -a)$ and $R(-a\sqrt{3}, a\sqrt{3})$.

$$\therefore \quad PQ = \sqrt{(-a-a)^2 + (-a-a)^2}$$
$$= \sqrt{(-2a)^2 + (-2a)^2}$$
$$= \sqrt{4a^2 + 4a^2} = \sqrt{8a^2} = 2\sqrt{2}\, a \text{ units} \quad [1]$$
$$QR = \sqrt{(-a\sqrt{3} + a)^2 + (a\sqrt{3} + a)^2}$$
$$= \sqrt{a^2 + 3a^2 - 2\sqrt{3}\, a^2 + a^2 + 3a^2 + 2\sqrt{3}\, a^2}$$
$$= \sqrt{8a^2} = 2\sqrt{2}\, a \text{ units}$$
$$RP = \sqrt{(a + a\sqrt{3})^2 + (a - a\sqrt{3})^2} \quad [1]$$
$$= \sqrt{a^2 + 3a^2 + 2\sqrt{3}\, a^2 + a^2 + 3a^2 - 2\sqrt{3}\, a^2}$$
$$= \sqrt{8a^2} = 2\sqrt{2}\, a \text{ units}$$

Here, $|PQ| = |QR| = |RP|$

Hence, the given triangle is an equilateral triangle. [1]

Que 23. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

Sol. Total number of balls = 12

Number of black balls = x

$$P(\text{a black ball}) = \frac{x}{12} \quad \text{...(i)} \quad [1]$$

If 6 more black balls are added to the box, then total number of balls = 12 + 6 = 18

According to question,

P(black balls) = 2 × Previous probability of balck ball is drawn A [1]

$$\Rightarrow \quad \frac{6+x}{18} = 2 \times \frac{x}{12}$$
$$\Rightarrow \quad 6(6+x) = 18x$$
$$\Rightarrow \quad 6 + x = 3x$$
$$\Rightarrow \quad 2x = 6$$
$$\Rightarrow \quad x = 3$$
[1]

Que 24. If the roots of the equation $(a-b)x^2 + (b-c)\ x + (c-a) = 0$ are equal, then prove that $2a = b + c$.

Sol. The given equation is

$$(a-b)x^2 + (b-c)x + (c-a) = 0$$

Here, $A = (a-b), B = (b-c), C = (c-a)$ **[1/2]**

For equal roots, we have

Discriminant $(D) = 0$

$$\Rightarrow \quad b^2 - 4ac = 0$$
$$\Rightarrow \quad (b-c)^2 - 4 \times (a-b)(c-a) = 0$$
[1]
$$\Rightarrow \quad b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$$
$$\Rightarrow \quad b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$$
$$\Rightarrow \quad 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ca = 0$$
$$\Rightarrow \quad (2a - b - c)^2 = 0$$
$$[\because A^2 + B^2 + C^2 + 2AB + 2BC + 2CA = (A+B+C)^2]$$
$$\Rightarrow \quad 2a - b - c = 0$$
$$\Rightarrow \quad 2a = b + c$$

Hence proved. **[1½]**

Section D

Que 25. Out of a number of saras birds, one fourth of the number are moving about in lotus plants; 1/9th coupled (along) with 1/4th as well as 7 times the square root of the number move on a hill. 56 birds remain in vakula trees. What is the total number of birds?

OR

If (-5) is a root of the equation $2x^2 + Px - 15 = 0$ and the quadratic equation $P(x^2 + x) + K = 0$ has equal roots, then find the value of P and K.

Sol. Let the total number of birds $= x$

Birds moving in lotus plants $= \frac{1}{4}x$

Birds moving on a hill $= \frac{1}{9}x + \frac{1}{4}x + 7\sqrt{x}$ **[1]**

According to the question,

$$\frac{1}{4}x + \frac{1}{9}x + \frac{1}{4}x + 7\sqrt{x} + 56 = x$$

Multiplying each term by 36, we get

$$9x + 4x + 9x - 36x + 7 \times 36\sqrt{x} + 56 \times 36 = 0$$
$$\Rightarrow \quad -14x + 7 \times 36\sqrt{x} + 56 \times 36 = 0$$

Dividing both sides by (-14), we get

$$x - 18\sqrt{x} - 144 = 0$$
[1]

It is quadratic equation in $\sqrt{x}$

$$\Rightarrow \quad (\sqrt{x})^2 - 24\sqrt{x} + 6\sqrt{x} - 144 = 0$$
$$\Rightarrow \quad \sqrt{x}(\sqrt{x} - 24) + 6(\sqrt{x} - 24) = 0$$
$$\Rightarrow \quad (\sqrt{x} + 6)(\sqrt{x} - 24) = 0$$
$$\Rightarrow \quad \sqrt{x} - 24 = 0 \quad \text{or} \quad \sqrt{x} + 6 = 0$$
$$\Rightarrow \quad \sqrt{x} = 24 \quad \text{or} \quad \sqrt{x} = -6 \ \text{[rejected]}$$
$$\Rightarrow \quad \sqrt{x} = 24$$
$$\Rightarrow \quad x = (24)^2 = 576$$

Hence, the total number of birds $= 576$. **[2]**

OR

Since, (-5) is a root of the quadratic equation $2x^2 + Px - 15 = 0$

$$\therefore \quad 2(-5)^2 + P(-5) - 15 = 0$$
$$\Rightarrow \quad 50 - 5P - 15 = 0$$
$$\Rightarrow \quad -5P = -35$$
$$\Rightarrow \quad P = \frac{-35}{-5} = 7 \qquad ...(i)$$
[1]

Also, quadratic equation $P(x^2 + x) + K = 0$ or $Px^2 + Px + K = 0$ has equal roots. **[1]**

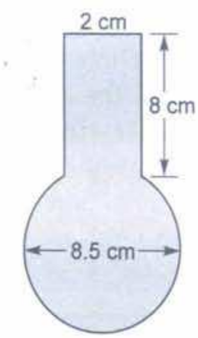
$$\therefore \quad P^2 - 4(P)(K) = 0 \quad [\because D = b^2 - 4ac = 0]$$
$$\Rightarrow \quad (7)^2 - 4(7)(K) = 0 \quad \text{[using Eq. (i)]}$$
$$\Rightarrow \quad 49 - 28K = 0$$
$$\Rightarrow \quad -28K = -49$$
$$\Rightarrow \quad K = \frac{-49}{-28} = \frac{7}{4}$$
$$\Rightarrow \quad K = \frac{7}{4}$$

Hence, the required values of P and K are respectively 7 and $\frac{7}{4}$. **[2]**

Que 26. A spherical glass has a cylindrical neck 8 cm long, 2 cm in diameter, the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds a child. Find its volume to be 345 cm^3. Check whether she is correct taking the above as the inside measurements. [Use $\pi = 3.14$]

Sol. For cylindrical part : $r = \frac{2}{2}$ cm $= 1$ cm, $h = 8$ cm

For spherical part : Radius $(R) = \frac{85}{20} = \frac{17}{4}$ cm



[1½]

Volume of glass solid = Volume of cylindrical part + Volume of the spherical part

$$= \pi r^2 h + \frac{4}{3}\pi R^3 = \pi\left[r^2 h + \frac{4}{3}R^3\right] \quad \textbf{[1]}$$

$$= \frac{314}{100}\left[1 \times 1 \times 8 + \frac{4}{3} \times \frac{17}{4} \times \frac{17}{4} \times \frac{17}{4}\right] \text{cm}^3$$

$$= \frac{314}{100}\left[8 + \frac{4913}{48}\right] \text{cm}^3 = \frac{314}{100}\left[\frac{384 + 4913}{48}\right] \text{cm}^3$$

$$= \frac{314}{100} \times \frac{5297}{48} \text{cm}^3 = \frac{1663258}{4800} \text{cm}^3$$

$$= 346.51 \text{ cm}^3$$

Hence, it is not correct. **[1½]**

Que 27. Find the common difference of an AP whose first term is 1 and the sum of the first four term is one-third to the sum of the next four terms.

Sol. Let a be the first term and d be the common difference of an AP. Given that $a = 1$ and

$$3(t_1 + t_2 + t_3 + t_4) = (t_5 + t_6 + t_7 + t_8) \quad \ldots\text{(i)}$$

Adding $(t_1 + t_2 + t_3 + t_4)$ on both sides of Eq. (i), we get **[1]**

$$(t_1 + t_2 + t_3 + t_4) + 3(t_1 + t_2 + t_3 + t_4) = (t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 + t_8)$$

$$\Rightarrow 4(t_1 + t_2 + t_3 + t_4) = (t_1 + t_2 + t_3 + t_4 + \ldots + t_8)$$

[1]

$$\Rightarrow \quad 4S_4 = S_8 \quad \ldots\text{(ii)}$$

Now $S_4 = \frac{4}{2}[2 \times 1 + (4-1)d] = 4 + 6d$

[$\because$ first term $= a = 1$]

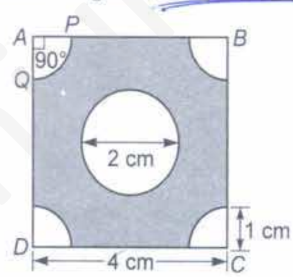
and $S_8 = \frac{8}{2}[2 \times 1 + (8-1)d] = 8 + 28d$ **[1]**

According to the question,

$$4(4 + 6d) = 8 + 28d \quad \text{[from Eq. (ii)]}$$

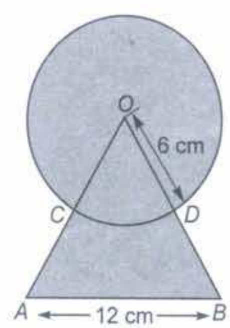
$$\Rightarrow \quad 4d = 8 \Rightarrow d = 2 \quad \textbf{[1]}$$

Que 28. $ABCD$ is a square of side 4 cm. At each corner of radius 1 cm and the centre of a circle of radius 1 cm are drawn as shown in figure. Find the area of the shaded region [Use $\pi = 3.14$]



OR

Find the area of the shaded region in figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral ΔOAB of side 12 cm as centre.



Sol. Area of a square with side 4 cm $= 4 \times 4 = 16$ cm^2 **[1]**

Area of smaller circle of radius $\left[\frac{2}{2} = 1 \text{ cm}\right]$

$$= \pi \times 1 \times 1 = 3.14 \text{ cm}^2 \quad \textbf{[1]}$$

Area of sector $APQ = \frac{\pi \times AP^2 \times \theta}{360°}$

$$= 3.14 \times 1 \times 1 \times \frac{90°}{360°} = \frac{1}{4} \times 3.14 \text{ cm}^2 \quad \textbf{[1]}$$

Area of such 4 sectors at each corner of a square

$$=4 \times \frac{1}{4} \times 3.14 = 3.14 \text{ cm}^2$$

$\therefore$ Area of the shaded portion = Area of a square − (Area of smaller circle + Area of 4 such sectors)

$$=16-(3.14+3.14)$$
$$=16-6.28=9.72 \text{ cm}^2 \quad [1]$$

OR

Area of a an circle with radius (6 cm)

$$=\pi r^2 = \frac{22}{7} \times 6 \times 6 = \frac{729}{7} \text{ cm}^2$$

Area of an equilateral ΔOAB

$$=\frac{\sqrt{3}}{4} \times a^2 = \frac{\sqrt{3}}{4} \times 12 \times 12 \text{ cm}^2$$
$$=\sqrt{3} \times 3 \times 12$$
$$=36\sqrt{3} \text{ cm}^2 \quad [1]$$

Area of sector OCD with angle 60°

$$=\frac{\pi \times r^2 \ \theta}{360°} = \frac{22}{7} \times \frac{6 \times 6 \times 60°}{360°} = \frac{132}{7} \text{ cm}^2$$

$\therefore$ Required area (shaded portion) [1]

$$= \text{Area of a circle} + \text{Area of } \Delta OAB - \text{Area of sector } OCD$$
$$=\left(\frac{792}{7} + \frac{36\sqrt{3}}{1} - \frac{132}{7}\right) \text{cm}^2$$
$$=\left(\frac{660}{7} + 36\sqrt{3}\right) \text{cm}^2$$

Hence, the area of the shaded region

$$=\left(\frac{660}{7} + 36\sqrt{3}\right) \text{cm}^2 \quad [1]$$

Que 29. A chord AB of the larger of the two concentric circles is tangent to the smaller circle at the point C. Show that C is the mid-point of the chord AB.

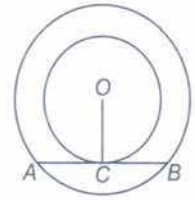
Sol. Given, two concentric circles with centre O. A chord AB is the larger circle of a tangent to the smaller circle at the point C.

To prove $AC = CB$ [1]

Proof Since, OC is the radius of the smaller circle and ACB is the tangent to the smaller circle.

$\therefore$ $OC \perp AB$

[$\because$ radius of a circle is perpendicular to the tangent at the point of contact.] [1]



Since, AB is a chord of the bigger circle, and the line OC from the centre of the circle is perpendicular to it. [1]

$\therefore$ OC bisects AB

$\Rightarrow$ C is the mid-point of the chord AB

$\Rightarrow$ $AC = CB$

Hence proved. [1]

Que 30. Suppose there are two windows in a house. A window of the house is at a height of 1.5 m above the ground and the other window is 3 m vertically above the lower window. Anil and Sanjeev are sitting inside the two windows. At an instant, the angles of elevation of a balloon from these windows are observed as 45° and 30°, respectively.

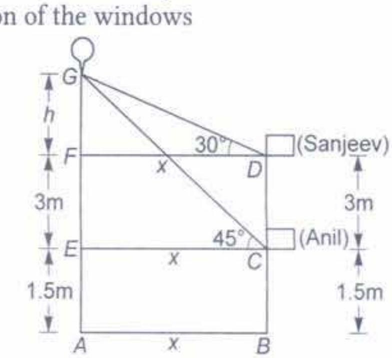
(a) Find the height of the balloon from the ground.

(b) Among Anil and Sanjeev, who is more closer to the balloon?

(c) Why windows are essential in any construction commercial or residential?

(d) If the balloon is moving towards the building, then both the angles of elevation will remains same?

Sol. (a) Let a be the height of the balloon and C and D be the position of the windows



At points C and D, angles of elevation are

$$\angle ECG = 45° \quad \text{and} \quad \angle FDG = 30°$$

Draw a perpendicular line EC and FD on AG, [1]

Let, $CE = DF = x$ m

and $FG = h$ m

In right angled ΔECG,

$$\tan 45° = \frac{EG}{EC} = \frac{3+h}{x}$$

$$\Rightarrow \quad 1 = \frac{3+h}{x}$$

$$\Rightarrow \quad x = 3+h \qquad \text{...(i)} \quad \textbf{[1/2]}$$

In right angled ΔFDG,

$$\tan 30° = \frac{GF}{DF} = \frac{h}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h \qquad \text{...(ii)}$$

On putting $x = \sqrt{3}h$ in Eq. (i), we get

$$3 + h = \sqrt{3}h$$

$$\Rightarrow \quad h(\sqrt{3}-1) = 3$$

$$\Rightarrow \quad h = \frac{3}{(\sqrt{3}-1)}$$

$$= \frac{3}{1.732-1} = \frac{3}{0.732}$$

$$= 4.098 \text{ m} \qquad \textbf{[1]}$$

Hence, the height of the balloon is 4.098 m.

(b) The person who makes small angle of elevation is more closer to the balloon.

Hence, Sanjeev is more close to the balloon. **[1/2]**

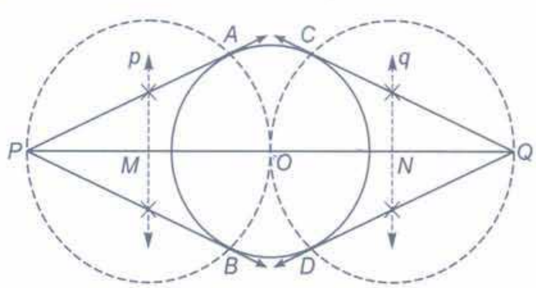
(c) Windows are the most important part of any building they add value to it.

They are useful for the proper vertilation, which is very much required as natural air, keeps the building fresh and suffocation free.

(d) No, when the balloon is moving towards the building, then the angle of elevation will automatically increase. **[1]**

Que 31. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangent to the circle from these two points P and Q.

Sol.



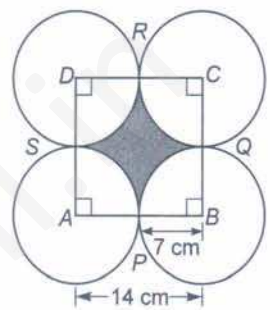
[2]

Steps of construction

1. Draw a circle with centre O and radius = 3 cm.
2. Mark two points P and Q on extended diameter, such that $OP = OQ = 7$ cm.
3. Draw the perpendicular bisectors (P and Q) of OP and OQ, let they intersect OP in M and OQ in N. **[1]**
4. With M as centre and radius MP, draw a circle which intersects the given circle in A and B.
5. With N as centre and radius NQ, draw a circle which intersects the given circle in C and D.
6. Join PA, PB, QC and QD.

   Thus, PA, PB, QC, and QD are the required tangents. **[1]**

Que 32. In figure, $ABCD$ is a square of side 14 cm. With centres A, B, C and D, four circles are drawn, such that each circles touches externally two of the remaining three circles. Find the area of the shaded region.



Sol. $AB = BC = CD = DA = 14$ cm

$$\Rightarrow \text{Radius of each sector} = \frac{1}{2} \times AB = \frac{1}{2} \times 14 = 7 \text{ cm} \quad \textbf{[1]}$$

Area of such 4 equal sectors

$$= 4 \times \frac{\pi r^2 \theta}{360°} = 4 \times \frac{22}{7} \times 7 \times 7 \times \frac{90°}{360°} \quad \textbf{[1]}$$

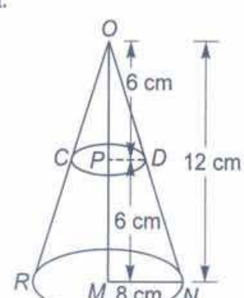
$$= 154 \text{ cm}^2$$

Area of a square $ABCD = 14 \times 14 = 196 \text{ cm}^2$ **[1]**

Area of the shaded portion $= (196 - 154) = 42 \text{ cm}^2$ **[1]**

Que 33. A cone of radius 8 cm and height 12 cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of two parts.

Sol. Let ORN be the cone, then given radius of the base of the cone $r_1 = 8$ cm.



[1]

and height of the cone, $h = OM = 12$ cm

Let P be the mid-point of OM, then

$$OP = PM = \frac{12}{2} = 6 \text{ cm}$$

Now, $\Delta OPD \sim \Delta OMN$

$$\therefore \quad \frac{OP}{OM} = \frac{PD}{MN}$$

$$\Rightarrow \quad \frac{6}{12} = \frac{PD}{8} \Rightarrow \frac{1}{2} = \frac{PD}{8}$$

$$\Rightarrow \quad PD = 4 \text{ cm}$$ [1]

The plane CD divides the cone into two parts, namey

(i) a smaller cone of radius 4 cm and height 6 cm

and (ii) frustum of a cone for which radius of the top of the frustum, $r_1 = 8$ cm

Radius of the bottom, $r_2 = 4$ cm

and height of the frustum, $h = 6$ cm

$\therefore$ Volume of smaller cone

$$= \left(\frac{1}{3}\pi \times 4 \times 4 \times 6\right) \text{cm}^3 = 32\pi \text{ cm}^3$$

and volume of the frustum of cone [1]

$$= \frac{1}{3} \times \pi \times 6 [(8)^2 + (4)^2 + 8 \times 4] \text{ cm}^3$$

$$= 2\pi (64 + 16 + 32) = 224\pi \text{ cm}^3$$

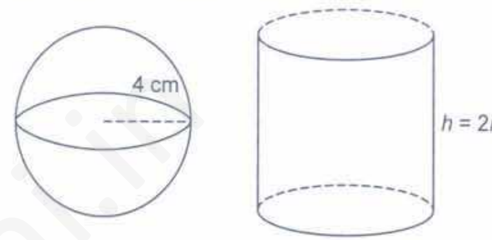
$\therefore$ Required ratio = Volume of cone : Volume of frustum

$$= 32\pi : 224\pi$$

$$= 1 : 7$$ [1]

Que 34. A cylinder whose height is equal to its diameter has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder correct to one decimal place.

Sol. The volume of the sphere



[1]

$$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 4^3 \text{ cm}^3$$

The volume of the cylinder

$$= \pi r^2 h = \pi r^2 \cdot 2r \qquad (\because h = 2r)$$ [1]

$$= \frac{22}{7} \times 2 \times r^3$$

According to question,

$$\frac{22}{7} \times 2 \times r^3 = \frac{4}{3} \times \frac{22}{7} \times 4^3$$ [1]

$$\Rightarrow \quad 2r^3 = \frac{4}{3} \cdot 4^3$$

$$\Rightarrow \quad r^3 = \frac{2}{3} \cdot 4^3$$

$$\Rightarrow \quad r = 4 \times \sqrt[3]{\frac{2}{3}} \text{ cm}$$ [1]