MODEL PRACTICE TEST PAPER - III
MATHEMATICS
CLASS 12 - CBSE 2011

Time : 3 hrs

Max. Marks: 100

General Instructions:

1. All questions are compulsory
2. The question paper consists of 29 questions divided into three sections A,B and C. Section A contains 10 questions of 1 mark each, Section B contains 12 questions of 4 marks each and section C contains 07 questions of 6 marks each.

Section – A
(Questions 1 – 10 carry one mark each)

1. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^5 xdx$
2. Write the number of all one-one function form the set A = {a,b,c} to itself.
3. Write the range of the principal branch of $\sec^{-1} x$ defined on the domain R-(-1,1)
4. Find the distance of the point (a,b,c) from x-axis
5. If $\vec{a}$ and $\vec{b}$ represent the two adjacent sides of a parallelogram, then write the area of the parallelogram in terms of $\vec{a}$ and $\vec{b}$
6. Find the direction cosines of the line passing through origin and lying in the first octant, making equal angles with the three coordinate axes.
7. Write $\sin^{-1}(2x\sqrt{1-x^2})$ in the simplest form
8. If f(x) = $sinx^0$, find $\frac{dy}{dx}$
9. If the vectors a$\hat{\imath}$ + a$\hat{\jmath}$ + c$\hat{k}$, $\hat{\imath}$ + $\hat{k}$ and c$\hat{\imath}$ + c$\hat{\jmath}$ + b$\hat{k}$ be coplanar, show that c^2 = ab
10. If $\vec{a}$ = 2$\hat{\imath}$ - $\hat{\jmath}$ + 3$\hat{k}$ and $\vec{b}$ = 6$\hat{\imath}$ + $\mu\hat{\jmath}$ + 9$\hat{k}$ and $\vec{a}$ | | $\vec{b}$, find the value of μ

Section – B
(Questions 11 – 22 carry four marks each)

11. The length x of a reactangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rate of change of (a) the perimeter (b) the area of the rectangle
12. If y = $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that (1-x²)$\frac{d^2y}{dx^2}$ - 3x$\frac{dy}{dx}$ – y = 0
13. Form a differential equation of the family of circles touching the x-axis at origin.
14. Solve the following for x : 2 $\tan^{-1} cosx = \tan^{-1}(2cosecx)$ or,
Prove the following : $\cot^{-1}\{\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\} = \frac{x}{2}$, x∈ $(0,\frac{\pi}{4})$
15. Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the equation $\vec{a}+\vec{b}+\vec{c}$. Find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$, if $|\vec{a}|$= 1, $|\vec{b}|$ =4 and $|\vec{c}| = 2$
16. If $x\sqrt{1+y} + y\sqrt{1+x}$ = 0, for -1< $x, y < 1$, prove the following : $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ or
If x = a(cost + logtan$\frac{t}{2}$) and y = a sint, find $\frac{dy}{dx}$
17. Using properties of determinants, show that $\begin{bmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{bmatrix}$ = (1+a²+b²)³
If f(x) = 3x²+15x+5, then find the approximate value of f(3.02), using differentials
18. Find the shortest distance between the following two lines :
$\vec{r}$ = (1+μ)$\hat{\imath}$ + (2-μ)$\hat{\jmath}$ + (μ + 1)$\hat{k}$; (2$\hat{\imath}$ − $\hat{\jmath}$ - $\hat{k}$) + μ(2$\hat{\imath}$ + $\hat{\jmath}$ + 2$\hat{k}$)
19. Evaluate $\int x\log(x+1)\,dx$ or,
using properties of definite integrals, evaluate the following : $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$
20. A family has two children. What is the probability that both the children are boys, given that atleast one of them is a boy?

21. Solve the following differential equation : $x^2\frac{dy}{dx} = y^2 + 2xy$ given that y=1, when x=1
22. If f(x), defined by the following, is continuous at x=0, find the values of a,b and c

$$F(x) = \begin{cases} \frac{\sin(a+1)x+\sin x}{x} & if\ x < 0 \\ c & if\ x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}} & if\ x > 0 \end{cases}$$

Section – C
(Questions 23 – 29 carry Six marks each)

23. Find the distance of the point (2,3,4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane 3x+2y+2z-5=0
24. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$

Or,

Using the method of integration, find the area of the region bounded by the lines 2x+y=4, 3x-2y=6 and x-3y+5=0
25. There are three coins. One is a two headed coin(having head on both faces), another is a biased coin that comes up tails 25% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin ?
26. Show that the semi-vertical angle of a right circular cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$
27. Evaluate $\int_0^a \sin^{-1}\sqrt{(\frac{x}{x+a})}\, dx$
28. Find the coordinate of the image of the point (1,3,4) in the plane 2 x - y + z + 3 =0
29. Two tailors A and B are paid Rs.150 and Rs.200 per day respectively. A can stitich 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimise the labour cost to produce atleast 60 shirts and 32 pants. Solve the problem graphically.