Series : PTS/20

Code No. 13/2/3

Roll No. | | | | | | | |

Candidates must write the Code on the title page of the answer-book.

Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION – A

Q01. If A is a square matrix of order n, then write the value of adj.(adj.A).

Q02. Let $A = \{1, 2, 3\}$. Write the number of equivalence relations containing (1, 2).

Q03. Let the unit vector in the direction of $(5\hat{i}+3\hat{j}-4\hat{k})$ is given as $\vec{a}$. Write the value of $50\vec{a}$.

Q04. Find the value of "p" if $(2\hat{i}+3\hat{j}-4\hat{k})$ and $(6\hat{i}+p\,\hat{j}+9\,\hat{k})$ are orthogonal vectors.

Q05. Find the distance between the planes 3x + 4y = 8 and 3x + 4y = 3.

Q06. Find the value of $\int \frac{\sin 2x}{\sin^2 x}dx$.

Q07. Evaluate : $\frac{3\pi}{4}-\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

Q08. Write the intercepts cut off by the plane 3x – 2y + 9z + 21 = 0.

Q09. Evaluate : $\sin\left(\int_0^1 \frac{dx}{\sqrt{16-(1-x)^2}}\right)$.

Q10. If $\begin{vmatrix} 22 & -1 \\ -3 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ -7 & x \end{vmatrix}$, find the value of x.

SECTION – B

Q11. Simplify : $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}-\cos^{-1}\frac{36}{85}$.

OR $3\sin^{-1}\left(\frac{2x}{1+x^2}\right)-4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+2\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$

Q12. Write the shortest distance between the lines :

$\frac{x+4}{3}=-\frac{y}{2}=-\frac{z+1}{2}$ and $\vec{r}=(6\hat{i}+2\hat{j}+2\hat{k})+\lambda(\hat{i}-2\hat{j}+2\hat{k})$.

Q13. Find the value of 'a' so that $f(x)=\begin{cases}\frac{3x^2-5x-2}{x-2}, \text{if } x\neq 2\\ a\,x-33, \text{ if } x=2\end{cases}$ is continuous at x = 2.

If "5a + 7" denotes the number of students who participated in an inter school competition, then how many students participated in the competition? What is the importance of participation in these kinds of competitions held in schools?

Q14. Let $A=R-\{2\}$, $B=R-\{3\}$. If $f: A\to B$ is a mapping defined by $f(x)=\frac{6x-1}{2x-2}$ then, show that f is invertible. Also find $f^{-1}(x)$.

Q15. Show that $|\vec{a}\times\vec{b}|^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b}\\ \vec{a}.\vec{b} & \vec{b}.\vec{b}\end{vmatrix}$.

Q16. Fifteen cards numbered from one to fifteen are placed in a box and a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, find the probability that it is an even number. If 50 times this probability be the number of students who suffered in a fire accident in a school building then, find the number of those students. What measures should be taken to avoid such mishap in the futures?

Q17. If $y=\sin^{-1}[\sqrt{x^4-x^6}+\sqrt{x^2-x^6}]$ then, prove that $\frac{dy}{dx}=\frac{2x}{\sqrt{1-x^4}}+\frac{1}{\sqrt{1-x^2}}$.

OR If $x^{\cos x}+(\sin x)^x$, then find $\frac{dy}{dx}$.

Q18. Find the particular solution of the differential equation, $(1+e^{2x})\,dy+(1+y^2)\,e^x\,dx=0$, given that $y=1$ when $x=0$.

Q19. Evaluate : $\int\frac{x^4}{x^4+81}dx$. **OR** Evaluate : $\int x^2\sin^{-1}x\,dx$.

Q20. Using properties of definite integrals, evaluate the integral : $\int_1^4[|x-1|+|x-2|+|x-4|]dx$.

Q21. Using properties of determinants, prove the followings :

$$\begin{vmatrix} a & b-c & b+c \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix}=(a+b+c)(a^2+b^2+c^2).$$

Q22. Find the intervals of increasing & decreasing for the function $2x^2-\log|x|$.

OR If curves $y=3e^{2x}$ and $y=be^{-2x}$ cut each other orthogonally, then determine the value of b.

SECTION – C

Q23. In a bolt factory machines, A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from production and is found to be defective. What is the probability that it manufactured by the machine B? What is the importance of machines?

OR A factory has two machines A and B. Past record shows that machines A produced 60% of the items of output and machine b produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B? What is the importance of machines?

Q24. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inserted in a given right circular cone is half that of the cone.

OR If $f(x)=x^3+ax^2+bx+5c$ has a maxima at $x=-1$ and minima at $x=3$, then find the value of a, b and c.

Q25. Find the image of the line l : $\frac{x-1}{3}=\frac{y-3}{1}=\frac{4-z}{5}$ in the plane $2x-y+z+3=0$. What is the distance between the line l from its image?

Q26. Using elementary operations, find the inverse of the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$.

Q27. Using integration, find the area of the region bounded by $4y=x^2$ and $4y=x+2$.

Q28. Prove that : $\int_0^{\pi/2}\log(\sin x)dx=\int_0^{\pi/2}\log(\cos x)dx=\frac{\pi}{2}\log\left(\frac{1}{2}\right)$.

Q29. A dealer deals in two items A and B. he has Rs.15000 to invest and a space to store at the most 80 pieces. Item A costs him Rs.300 and item B costs him Rs.150. He can sell items A and B at a profit of Rs.40 and Rs.25 respectively. Assuming that he can sell all that he buys, formulate the above as an LPP for maximum profit and solve it graphically.