First Mock Board Exam

Duration: $2\frac{1}{2}$ **Hours**   **Class XII**   **Max. Mark 100.**

General Instructions:

1. All questions are compulsory.

2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six mark each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section-A
(*1 mark each*)

1. Write the antiderivative of $\dfrac{2^x}{1+4^x}$ w.r.t. 'x'.

2. Write the principal value branch of $arc \sin x$.

3. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ and $|3A| = \lambda|A|$ then find λ.

4. Find the angle between line $\dfrac{x+1}{2} = \dfrac{y}{3} = \dfrac{z-1}{6}$ and the plane $10x + 2y - 11z = 3$.

5. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on vector $7\hat{i} - \hat{j} + 8\hat{k}$.

6. Write the number of all binary operation on set A = {a, b, c} to itself.

7. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric matrix then find the value of *a, b* and *c*.

8. Find the value of $\}$ such that the line $\dfrac{x-2}{9} = \dfrac{y-1}{\}} = \dfrac{z+3}{-6}$ is perpendicular to the plane $3x - y - 2z = 7$.

9. Find the value of $\cos^{-1}\left(-\dfrac{1}{2}\right)+\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

10. Differentiate $\dfrac{\sin\lfloor x}{y^{\circ}}\,w.r.t.\ 'y'.$

Section-B
(*4 marks each*)

11. Evaluate $\displaystyle\int x^{x}.(1+\log x)\,dx$

OR

Evaluate: $\displaystyle\int \dfrac{x^{2}}{(1+x^{2})\left(1+\sqrt{1+x^{2}}\right)}\,dx$

12. Let $A=N\times N$. Let * be binary operation on A defined by $(a,b)*(c,d)=(ad+bc,bd)$. Then

find the identity element of $(A,*)$. Is $(A,*)$ commutative?

13. Solve for x, $2\tan^{-1}(\cos x)=\tan^{-1}(2\cos ec\ x)$.

OR

Prove that: $\tan\left(\dfrac{f}{4}+\dfrac{1}{2}\cos^{-1}\dfrac{a}{b}\right)+\tan\left(\dfrac{f}{4}-\dfrac{1}{2}\cos^{-1}\dfrac{a}{b}\right)=\dfrac{2b}{a}.$

14. Prove by using properties of determinants:

$$\begin{vmatrix} -a(b^{2}+c^{2}-a^{2}) & 2b^{3} & 2c^{3} \\ 2a^{3} & -b(c^{2}+a^{2}-b^{2}) & 2c^{3} \\ 2a^{3} & 2b^{3} & -c(a^{2}+b^{2}-c^{2}) \end{vmatrix}=abc(a^{2}+b^{2}+c^{2})^{3}.$$

15. For what value of a and b, the function defined as:

$$f(x)=\begin{cases} 3ax+b & ;\ if\ x<1 \\ 11 & ;\ if\ x=1 \\ 5ax-2b & ;\ if\ x>1 \end{cases}\qquad \text{is continuous at } x=1.$$

16. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\dfrac{\left[1+(y')^2\right]^{3/2}}{y''}$ is a constant and free from a and b.

OR

If $\sqrt{1-x^{2n}} + \sqrt{1-y^{2n}} = a(x^n - y^n)$, prove that $\dfrac{dy}{dx} = \dfrac{x^n}{y^n}\sqrt{\dfrac{1-y^{2n}}{1-x^{2n}}}$.

17. Find the interval in which the function f given by $f(x) = \sin^4 x + \cos^4 x$ is strictly increasing and strictly decreasing $0 < x < \dfrac{f}{2}$.

18. Solve the differential equation: $\dfrac{dy}{dx} - \dfrac{1}{x}\cdot y = 2x^2$. *Social websites are consuming precious time and money of the students, do you agree/disagree, give two lines in support of your answer.*

19. Solve: $(x^3 + x^2 + x + 1)\dfrac{dy}{dx} = 2x^2 + x$.

20. There are three bags each bag contains 4red & 5black, 7red & 5black and 4red & 6black. One bag is selected at random and two black balls are drawn from the bag and distributed one-one into other bags then a ball is drawn, and that is black find the probability that the two black balls were drawn from the bag which now contained 7 balls.

21. $\vec{a}, \vec{b}, \vec{c}$ are the unit vectors. Suppose $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\dfrac{f}{6}$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

OR

If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

22. If the straight lines $\dfrac{x+1}{2} = \dfrac{y+1}{5} = \dfrac{z}{2}$ and $\dfrac{x+1}{5} = \dfrac{y+1}{2} = \dfrac{z}{5}$ are coplanar, find the equation of plane containing these two lines.

Section-C
(6 marks each)

23. Evaluate $\displaystyle\int_1^4 (x^2 - x)dx$ as the limit of a sum.

24. A bag contains 4 balls. Two balls are drawn at random, and are found to be blue. What is the probability that 50% balls were blue in colour, in that bag?

25. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$ find A^{-1}, and hence solve the following system of equations:

$$2x + y + 3z = 3$$
$$4x - y \qquad = 3$$
$$-7x + 2y + z = 2.$$

26. Using integration find the area of the region included between the curves

$y = x^2 + 1, y = x, x = 0$ *and* $y = 2.$

OR

Using definite integration, find the area of the region: $\left\{ (x, y) : |x-1| \le y \le \sqrt{5-x^2} \right\}.$

27. Find the distance of the point (1, -2, 3) from the plane x – y + z = 5 measured parallel to the line

$\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{-6}.$

OR

Find foot, length and equation of the perpendicular drawn from the point (1, 1, 1) to the plane

$\vec{r}.(2\hat{i} - 2\hat{j} + 4\hat{k}) + 5 = 0$

28. A window of fixed perimeter (including the base of the triangle) is in the form of a rectangle surmounted by an equilateral triangle. The triangular portion is filled with colored glass while the rectangular part is filled with clear glass. The coloured glass stops 30% light fall on it while clear glass only 1%. What is the ratio of the sides of the rectangle so that the window transmits the maximum light? *Use sunlight to save electricity and serve the nation directly comments on it.*

29. There are two factories located one a place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirement of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

To From	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum? What will be the minimum transportation cost? *How we can help our parents/nation to save money by using easy/public transportation.*