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CBSE ANNUAL EXAMINATION
[Solutions With Detailed Explanations]
SECTION – A (Each question carries 1 Mark)

Q01. Find the distance of the plane $3x-4y+12z=3$ from the origin.

Sol. As the distance of plane $ax+by+cz+d=0$ from a point (x_1, y_1, z_1) is $\dfrac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}\, units$.

So, distance of origin (0, 0, 0) from the given plane $3x-4y+12z=3$ is

$$=\frac{|3(0)-4(0)+12(0)-3|}{\sqrt{(3)^2+(-4)^2+(12)^2}}=\frac{3}{\sqrt{169}}\,units=\frac{3}{13}\,units.$$

Q02. Find the scalar components of the vector $\overrightarrow{AB}$ with initial point A(2, 1) and terminal point B(–5, 7).

Sol. As $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(-5\hat{i}+7\hat{j})-(2\hat{i}+\hat{j})$

$\Rightarrow \overrightarrow{AB}=-7\hat{i}+6\hat{j}$.

So, the scalar components of $\overrightarrow{AB}$ are –7, 6.

Q03. Find the principal value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$.

Sol. As range of $\tan^{-1}x$ is $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ and that of $\sec^{-1}x$ is $[0,\pi]-\left\{\dfrac{\pi}{2}\right\}$.

So, $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)=\tan^{-1}\left(\tan\dfrac{\pi}{3}\right)-\sec^{-1}\left(\sec\dfrac{2\pi}{3}\right)$

$$\therefore \quad =\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}.$$

Q04. Given $\int e^x(\tan x+1)\sec x\,dx=e^x f(x)+c$. Write $f(x)$ satisfying this.

Sol. Let $I=\int e^x(\tan x+1)\sec x\,dx=e^x f(x)+c$

$\Rightarrow e^x f(x)+c=\int e^x(\tan x+1)\sec x\,dx$

$\Rightarrow \quad =\int e^x(\sec x+\sec x\tan x)\,dx \qquad \Rightarrow \quad =\int e^x\sec x\,dx+\int e^x\sec x\tan x\,dx$

$\Rightarrow \quad =\sec x\int e^x dx-\int\left[\dfrac{d}{dx}\sec x\int e^x dx\right]dx+\int e^x\sec x\tan x\,dx$

(On applying **By parts** in first integral)

$\Rightarrow \quad =e^x\sec x-\int e^x\sec x\tan x\,dx+\int e^x\sec x\tan x\,dx$

$\Rightarrow e^x f(x)+c=e^x\sec x+c$

On comparing both the sides, we get

$$\therefore \quad f(x)=\sec x.$$

Q05. Evaluate: $\displaystyle\int_0^2\sqrt{4-x^2}\,dx$.

Sol. Let $I=\displaystyle\int_0^2\sqrt{4-x^2}\,dx \qquad \Rightarrow I=\int_0^2\sqrt{(2)^2-x^2}\,dx=\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$$\Rightarrow I=\left[\frac{2}{2}\times 0+2\sin^{-1}\left(\frac{2}{2}\right)\right]-\left[0+2\sin^{-1}(0)\right]=2\sin^{-1}\left(\sin\frac{\pi}{2}\right)=2\times\frac{\pi}{2}$$

$\therefore \quad I=\pi$.

Q06. Let A be a square matrix of order 3×3. Write the value of |2A|, where $|A|=4$.

Sol. As $|kA|=k^n|A|$, where n is the order of matrix A and k is any non-zero scalar.

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So, $|2A| = 2^3|A| = (8)(4) = 32$.

Q07. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$.

Sol. As $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

So, $A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$.

Q08. The binary operation $*: R \times R \to R$ is defined as $a*b = 2a + b$. Find $(2*3)*4$.

Sol. Since $*: R \times R \to R$ is defined as $a*b = 2a + b$.

So, $2*3 = 2(2) + 3 = 7$.

Then, $(2*3)*4 = 7*4 = 2(7) + 4 = 18$.

Q09. Write the value of $(\hat{k} \times \hat{i}).\hat{j} + \hat{i}.\hat{k}$.

Sol. Since $\hat{k} \times \hat{i} = \hat{j}$ and $i.\hat{k} = 0$

So, $(\hat{k} \times \hat{i}).\hat{j} + \hat{i}.\hat{k} = \hat{j}.\hat{j} + 0$

$\therefore \quad = 1 + 0 = 1.$

Q10. Find the value of $x + y$ from the following equation:

$$2\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}.$$

Sol. $2\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 6 \\ 0 & 2x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2+y & 6 \\ 0+1 & 2x+2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix} \Rightarrow \begin{pmatrix} 2+y & 6 \\ 1 & 2x+2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$

By equality of matrices, we have $2 + y = 5,\ 2x + 2 = 8 \quad \Rightarrow y = 3,\ x = 3$.

So, $x + y = 3 + 3 = 6$.

SECTION – B (Each question carries 4 Marks)

Q11. Using properties of determinants, show that: $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$.

Sol. Consider **LHS** and, let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$= \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ [Applying $R_1 \to R_1 - (R_2 + R_3)$

$= 2\begin{vmatrix} 0 & -c & -b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ [Taking 2 common from R_1

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$$= 2\begin{vmatrix} 0 & -c & -b \\ b & a & 0 \\ c & 0 & a \end{vmatrix}$$ [Applying $R_3 \to R_3 + R_1$ and $R_2 \to R_2 + R_1$

$$= 2\left\{0\begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} - (-c)\begin{vmatrix} b & 0 \\ c & a \end{vmatrix} - b\begin{vmatrix} b & a \\ c & 0 \end{vmatrix}\right\}$$ [Expanding along R_1

$$= 2\{c(ab-0) - b(0-ac)\} = 2(2abc)$$

$$= 4abc = \textbf{RHS}.$$ [Hence Proved.

Q12. Evaluate: $\int_{-1}^{2} |x^3 - x|\,dx$. **OR** Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$.

Sol. Let $I = \int_{-1}^{2} |x^3 - x|\,dx$

$$\Rightarrow \quad = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx, \text{ where } f(x) = |x^3 - x|$$

Now, $f(x) = \begin{cases} (x^3 - x), \ if \ -1 < x < 0 \\ -(x^3 - x), \ if \ 0 < x < 1 \\ (x^3 - x), \ if \ 1 < x < 2 \end{cases}$.

So, $I = \int_{-1}^{0} (x^3 - x)\,dx + \int_{0}^{1} (x - x^3)\,dx + \int_{1}^{2} (x^3 - x)\,dx$

$$\Rightarrow \quad = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$

$$\Rightarrow \quad = \left\{[0-0] - \left[\frac{1}{4} - \frac{1}{2}\right]\right\} + \left\{\left[\frac{1}{2} - \frac{1}{4}\right] - [0-0]\right\} + \left\{\left[\frac{16}{4} - \frac{4}{2}\right] - \left[\frac{1}{4} - \frac{1}{2}\right]\right\}$$

$$\Rightarrow \quad = \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}.$$

OR

Let $I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$ ...(*i*)

$$= \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx$$ [By using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx$$ ...(*ii*)

On adding equations (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{(x + \pi - x)\sin x}{1+\cos^2 x}dx \quad \Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

Let $f(x) = \frac{\sin x}{1+\cos^2 x} \Rightarrow f(\pi - x) = \frac{\sin(\pi - x)}{1+\cos^2(\pi - x)} = \frac{\sin x}{1+\cos^2 x}$

i.e., $f(\pi - x) = f(x)$. So by using, $\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx,\ if\ f(2a - x) = f(x)$, we get

$$I = \left(\frac{\pi}{2}\right) \times 2\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx = \pi\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$$

Put $\cos x = t \Rightarrow \sin x\,dx = -dt$. Also when $x = 0 \Rightarrow t = 1$ and when $x = \pi/2 \Rightarrow t = 0$.

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So, $I = \pi\int_1^0 \frac{-dt}{1+t^2} = \pi\int_0^1 \frac{dt}{1+t^2}$ $\Rightarrow I = \pi\left[\tan^{-1} t\right]_0^1 = \pi\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$

$\therefore\ I = \pi\left(\frac{\pi}{4} - 0\right) = \frac{\pi^2}{4}$ or, $\left(\frac{\pi}{2}\right)^2$.

Q13. A ladder $5m$ long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of $2cm/s$. How fast is its height on the wall decreasing when the foot of the ladder is $4m$ away from the wall?

Sol. Let $y\ m$ be the height of the wall at which the ladder touches. Also, let the foot of the ladder be $x\ m$ away from the wall. Then by Pythagoras theorem, we have

$$x^2 + y^2 = 5^2 \Rightarrow y = \sqrt{25 - x^2}$$

Then, the rate of change of height (i.e., y) with respect to time t is given by,

$\frac{dy}{dt} = -\frac{x}{\sqrt{25-x^2}} \times \frac{dx}{dt} = -\frac{2x}{\sqrt{25-x^2}}$ [As it is given that $\frac{dx}{dt} = 2cm/s$

Now, when $x = 4\,m$, we have : $\frac{dy}{dt} = -\frac{2\times 4}{\sqrt{25-4^2}} = -\frac{8}{3}\ cm/s.$

Hence, the height of the ladder on the wall is decreasing at the rate of $\frac{8}{3}\ cm/s$.

Q14. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}.\vec{c} = 18$.

Sol. Let $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$. Since $\vec{p}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, so $\vec{p}.\vec{a} = 0$ and $\vec{p}.\vec{b} = 0$.

That means, $\vec{p}.\vec{a} = (x\hat{i} + y\hat{j} + z\hat{k}).(\hat{i} + 4\hat{j} + 2\hat{k}) = 0 \Rightarrow x + 4y + 2z = 0$ ...(i)

and, $\vec{p}.\vec{b} = (x\hat{i} + y\hat{j} + z\hat{k}).(3\hat{i} - 2\hat{j} + 7\hat{k}) = 0 \Rightarrow 3x - 2y + 7z = 0$ ...(ii)

Also, we have, $\vec{p}.\vec{c} = 18 \Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}).(2\hat{i} - \hat{j} + 4\hat{k}) = 18 = 0 \Rightarrow 2x - y + 4z = 18$ ...(iii)

Solving (i) and (ii) by using Cross-multiplication, we have

$\frac{x}{28+4} = \frac{y}{6-7} = \frac{z}{-2-12} = \lambda$ $\Rightarrow x = 32\lambda, y = -\lambda, z = -14\lambda$.

Substituting these values in (iii), we get $2(32\lambda) - (-\lambda) + 4(-14\lambda) = 18 \Rightarrow \lambda = 2$.

So, $x = 64, y = -2, z = -28$.

Hence, the required vector $\vec{p}$ is, $\vec{p} = 64\hat{i} - 2\hat{j} - 28\hat{k}$.

Q15. If $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

OR Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ with respect to x.

Sol. We have $x = \sqrt{a^{\sin^{-1} t}}$

Taking logarithm on both the sides, we get

$\log x = \log\sqrt{a^{\sin^{-1} t}}$

$\Rightarrow \log x = \left(\frac{\log a}{2}\right)\sin^{-1} t$

On differentiating w.r.t. t both the sides,

$\frac{1}{x}\frac{dx}{dt} = \left(\frac{\log a}{2}\right)\frac{1}{\sqrt{1-t^2}}$

$\Rightarrow \frac{dx}{dt} = \left(\frac{\log a}{2}\right)\frac{x}{\sqrt{1-t^2}}$

And, $y = \sqrt{a^{\cos^{-1} t}}$

Taking logarithm on both the sides, we get

$\log y = \log\sqrt{a^{\cos^{-1} t}}$

$\Rightarrow \log y = \left(\frac{\log a}{2}\right)\cos^{-1} t$

On differentiating w.r.t. t both the sides,

$\frac{1}{y}\frac{dy}{dt} = \left(\frac{\log a}{2}\right)\left(-\frac{1}{\sqrt{1-t^2}}\right)$

$\Rightarrow \frac{dy}{dt} = \left(\frac{\log a}{2}\right)\left(-\frac{y}{\sqrt{1-t^2}}\right)$.

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So, $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \left(\frac{\log a}{2}\right)\left(-\frac{y}{\sqrt{1-t^2}}\right)\left(\frac{2}{\log a}\right)\frac{\sqrt{1-t^2}}{x} = -\frac{y}{x}$

$\therefore \quad \frac{dy}{dx} = -\frac{y}{x}$. [Hence Proved.

OR

Let $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$ ...(*i*)

So, $y = \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right] = \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right]$ $\Rightarrow y = \tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right]$

$\Rightarrow y = \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$ $\Rightarrow y = \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right] = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$

$\Rightarrow y = \frac{1}{2}(\tan^{-1} x)$ [By (i)

On differentiating with respect to x, we have: $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{1+x^2}$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2(1+x^2)}.$$

Q16. Show that $f : \mathrm{N} \to \mathrm{N}$, given by $f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x-1, \text{ if } x \text{ is even} \end{cases}$ is both one-one and onto.

OR Consider the binary operations *: R × R→R and o: R × R→R defined as $a * b = |a - b|$ and $a \; o \; b = a$ for all $a, b \in \mathrm{R}$. Show that '*' is commutative but not associative, '*o*' is associative but not commutative.

Sol. Suppose $f(x_1) = f(x_2)$. If x_1 is odd and x_2 is even, then we will have $x_1 + 1 = x_2 - 1$, i.e., $x_2 - x_1 = 2$ which is impossible. Similarly, the possibility of x_1 being even and x_2 being odd is ruled out, using the same argument. Therefore, both x_1 and x_2 must be either odd or even. Suppose both x_1 and x_2 are odd. Then,

$$f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2.$$

Similarly if both x_1 and x_2 are even. Then,

$$f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1 \Rightarrow x_1 = x_2.$$

Thus, f is one-one.

Also, any odd number $2r+1$ in the co-domain N is the image of $2r+2$ in the domain N and any even number $2r$ in the co-domain N is the image of $2r-1$ in the domain N. Thus, f is onto.

OR

It is given that *: R ×R → R and *o*: R × R → R is defined as $a * b = |a - b|$ and $a \; o \; b = a$ for all $a, b \in \mathrm{R}$. For $a, b \in \mathrm{R}$, we have: $a * b = |a - b| \Rightarrow b * a = |b - a| = |-(a-b)| = |a - b|$.

So, $a * b = b * a$. Thus, the operation * is commutative.

It can be observed that,
(1*2)*3 = (|1–2|)*3 = 1*3 = |1–3| = 2. Also, 1*(2*3) = 1*(|2–3|) = 1*1 = |1–1| = 0.
$\therefore$ (1*2)*3 ≠ 1*(2*3) where 1, 2, 3 ∈ R. Thus, the operation * is not associative.

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Now, consider the operation o:
It can be observed that $1\ o\ 2 = 1$ and $2\ o\ 1 = 2$.
$\therefore\ 1\ o\ 2 \neq 2\ o\ 1$ where $1, 2 \in \mathrm{R}$.
$\therefore$ The operation o is not commutative.

Let $a, b, c \in \mathrm{R}$. Then, we have: $(a\ o\ b)\ o\ c = a\ o\ c = a$. Also, $a\ o\ (b\ o\ c) = a\ o\ b = a$.
$\therefore\ (a\ o\ b)\ o\ c = a\ o\ (b\ o\ c)$
Thus, the operation o is associative.

Q17. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

Sol. Let the number of red cards drawn be denoted by X which is a random variable. Clearly, X can take the values 0, 1 or 2.

$$\therefore\ \mathrm{P}(\mathrm{X}=0) = \mathrm{P}(\text{two non-red cards}) = \frac{^{26}\mathrm{C}_2}{^{52}\mathrm{C}_2} = \frac{25}{102}.$$

$$\mathrm{P}(\mathrm{X}=1) = \mathrm{P}(\text{one red card and one non-red cards}) = \frac{^{26}\mathrm{C}_1 \times {}^{26}\mathrm{C}_1}{^{52}\mathrm{C}_2} = \frac{52}{102}.$$

$$\therefore\ \mathrm{P}(\mathrm{X}=2) = \mathrm{P}(\text{two red cards}) = \frac{^{26}\mathrm{C}_2}{^{52}\mathrm{C}_2} = \frac{25}{102}.$$

$$\text{Therefore, mean of X} = \mathrm{E(X)} = \sum_{i=1}^{n} \mathrm{X}_i \mathrm{P}(\mathrm{X}_i) == 0 \times \frac{25}{102} + 1 \times \frac{52}{102} + 2 \times \frac{25}{102} = 1$$

$$\text{Also, } \mathrm{Var(X)} = \sum_{i=1}^{n} \mathrm{X}_i^2 \mathrm{P}(\mathrm{X}_i) - \left[\mathrm{E(X)}\right]^2$$

$$\Rightarrow \quad = \left[0^2 \times \frac{25}{102} + 1^2 \times \frac{52}{102} + 2^2 \times \frac{25}{102}\right] - (1)^2 = \frac{76}{51} - 1$$

$$\therefore\ \mathrm{Var(X)} = \frac{25}{51}.$$

Q18. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

OR Find the particular solution of the differential equation: $x(x^2-1)\dfrac{dy}{dx} = 1;\ y = 0$ when $x = 2$.

Sol. Let C denote the family of circles in the second quadrant and touching the coordinate axes. Let $(-h, h)$ be the coordinate of the centre of any member of this family. It is clear that the radius will be h.
So, equation representing this family is: $(x+h)^2 + (y-h)^2 = h^2$ …(i)
i.e., $x^2 + y^2 + 2hx - 2hy + h^2 = 0$ …(ii)

On differentiating (ii) w.r.t. x, we get $2x + 2y\dfrac{dy}{dx} + 2h - 2h\dfrac{dy}{dx} = 0$

$$\Rightarrow x + y\frac{dy}{dx} = h\left(\frac{dy}{dx} - 1\right) \qquad \Rightarrow h = \frac{x + yy'}{y' - 1}.$$

Substituting the value of h in equation (i), we get

$$\left(x + \frac{x+yy'}{y'-1}\right)^2 + \left(y - \frac{x+yy'}{y'-1}\right)^2 = \left(\frac{x+yy'}{y'-1}\right)^2$$

$$\Rightarrow \left[xy' - x + x + yy'\right]^2 + \left[yy' - y - x - yy'\right]^2 = \left[x + yy'\right]^2 \qquad \Rightarrow (x+y)^2(y')^2 + (x+y)^2 = (x+yy')^2$$

$$\therefore\ (x+y)^2\left[(y')^2 + 1\right] = \left[x + yy'\right]^2$$

$$\text{i.e., } \therefore\ (x+y)^2\left[\left(\frac{dy}{dx}\right)^2 + 1\right] = \left[x + y\frac{dy}{dx}\right]^2$$

This is the required differential equation representing the given family of circles.

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OR We have, $x(x^2-1)\dfrac{dy}{dx}=1$

$$\Rightarrow dy=\frac{dx}{x(x^2-1)} \qquad \Rightarrow \int dy=\int\frac{dx}{x(x-1)(x+1)} \qquad \ldots\text{(i)}$$

Consider, $\dfrac{1}{x(x-1)(x+1)}=\dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{x+1}$

$$\Rightarrow \quad 1=(A+B+C)x^2+(B-C)x-A.$$

On equating the coefficients of like terms, we get: $A+B+C=0,\ B-C=0,\ -A=1$.

On solving these equations, we have: $A=-1,\ B=\dfrac{1}{2},\ C=\dfrac{1}{2}$.

So by (i) we have: $\displaystyle\int dy=-\int\frac{1}{x}dx+\frac{1}{2}\int\frac{1}{x-1}dx+\frac{1}{2}\int\frac{1}{x+1}dx$

$$\Rightarrow \quad y=-\log|x|+\frac{1}{2}\log|x-1|+\frac{1}{2}\log|x+1|+k$$

$$\Rightarrow y=\frac{1}{2}\log\left|\frac{x^2-1}{x^2}\right|+k \qquad \ldots\text{(ii)}$$

Now since $y=0$, when $x=2$. So, $0=\dfrac{1}{2}\log\left|\dfrac{4-1}{4}\right|+k \Rightarrow k=\dfrac{1}{2}\log\left(\dfrac{4}{3}\right)$

Substituting the value of k in equation (ii), we get

$$y=\frac{1}{2}\log\left|\frac{x^2-1}{x^2}\right|+\frac{1}{2}\log\left(\frac{4}{3}\right)$$

$$\therefore \quad y=\frac{1}{2}\log\left|\frac{4(x^2-1)}{3x^2}\right|.$$

This is the required particular solution of the given differential equation.

Q19. If $x=a\left(\cos t+\log\tan\dfrac{t}{2}\right)$, $y=a\sin t$, find $\dfrac{d^2y}{dt^2}$ and $\dfrac{d^2y}{dx^2}$.

Sol. Given $x=a\left(\cos t+\log\tan\dfrac{t}{2}\right)$ [On differentiating with respect to t both the sides

$$\frac{dx}{dt}=a\left(-\sin t+\frac{1}{\tan\frac{t}{2}}\times\sec^2\left(\frac{t}{2}\right)\times\frac{1}{2}\right)=a\left(-\sin t+\frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}\times\frac{1}{\cos^2\frac{t}{2}}\times\frac{1}{2}\right)=a\left(-\sin t+\frac{1}{t\ \sin}\right)$$

$$\Rightarrow \frac{dx}{dt}=a\cos t\cot t \qquad \ldots\text{(i)}$$

Also, $y=a\sin t$ [On differentiating with respect to t both the sides

$$\Rightarrow \frac{dy}{dt}=a\cos t \qquad \ldots\text{(ii)}$$

Again differentiating with respect to t both the sides, we have

$$\frac{d^2y}{dt^2}=\frac{d}{dt}\left(a\cos t\right)=a(-\sin t)$$

$$\therefore \quad \frac{d^2y}{dt^2}=-a\sin t.$$

Now, by (i) and (ii), we have

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=(a\cos t)\times\frac{1}{a\cos t\cot t} \qquad \Rightarrow \frac{dy}{dx}=\tan t$$

On differentiating with respect to x both the sides

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$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\tan t) = (\sec^2 t)\frac{dt}{dx} \quad \Rightarrow \frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{a\cos t \cot t}$$

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{\sec^3 t \ \tan t}{a}.$$

Q20. Find the coordinates of the point where the line through the points (3,–4,–5) and (2,–3, 1) crosses the plane $3x+2y+z+14=0$.

Sol. The equation of the straight line passing through the points (3,–4,–5) and (2,–3, 1) is:

$$\frac{x-3}{2-3} = \frac{y-(-4)}{-3-(-4)} = \frac{z-(-5)}{1-(-5)} \quad \Rightarrow \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda \ (say)$$

The coordinates of any random point on this line is $\text{P}(3-\lambda,\ \lambda-4,\ 6\lambda-5)$.

Consider that the line intersects the given plane $3x+2y+z+14=0$ at $\text{P}(3-\lambda,\ \lambda-4,\ 6\lambda-5)$.

So, $3(3-\lambda)+2(\lambda-4)+(6\lambda-5)+14=0 \quad \Rightarrow \lambda=-2$.

Thus, the required **point of intersection** is $\text{P}\big(3-(-2),\ (-2)-4,\ 6(-2)-5\big)$ i.e., $\text{P}(5,-6,-17)$.

Q21. Find the particular solution of the following differential equation:

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0, \text{ given that when } x=2,\ y=\pi.$$

Sol. We have $x\dfrac{dy}{dx} - y + x\sin\left(\dfrac{y}{x}\right) = 0 \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x} - \sin\left(\dfrac{y}{x}\right)$ ...(i)

It is evident that the given differential equation is homogeneous.

So, put $y = vx \ \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ (On differentiating w.r.t. x both sides)

Substituting these in equation (i), we have

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \sin\left(\frac{vx}{x}\right) \quad \Rightarrow v + x\frac{dv}{dx} = v - \sin v \quad \Rightarrow x\frac{dv}{dx} = -\sin v$$

$$\Rightarrow x\frac{dv}{dx} = -\sin v \quad \Rightarrow \int \operatorname{cosec} v\, dv = -\int \frac{dx}{x}$$

$$\Rightarrow \log|\operatorname{cosec} v - \cot v| = -\log|x| + \log|k| \quad \Rightarrow \log|\operatorname{cosec} v - \cot v| = \log\left|\frac{k}{x}\right|$$

$$\Rightarrow \operatorname{cosec}\left(\frac{y}{x}\right) - \cot\left(\frac{y}{x}\right) = \frac{k}{x} \quad \Rightarrow k\sin\left(\frac{y}{x}\right) = x\left[1-\cos\left(\frac{y}{x}\right)\right]$$

It is given that when $x=2,\ y=\pi$.

$$\text{So, } k\sin\left(\frac{\pi}{2}\right) = 2\left[1-\cos\left(\frac{\pi}{2}\right)\right] \quad \Rightarrow k(1) = 2\ [1-0] \quad \Rightarrow k=2.$$

$$\text{Thus, } x\left[1-\cos\left(\frac{y}{x}\right)\right] = 2\sin\left(\frac{y}{x}\right).$$

This is the required particular solution of the given differential equation.

Q22. Prove that: $\cos^{-1}\left(\dfrac{12}{13}\right) + \sin^{-1}\left(\dfrac{3}{5}\right) = \sin^{-1}\left(\dfrac{56}{65}\right)$.

Sol. Let $\sin^{-1}\left(\dfrac{3}{5}\right) = x \Rightarrow \sin x = \dfrac{3}{5} \Rightarrow \tan x = \dfrac{3}{4} \Rightarrow x = \tan^{-1}\left(\dfrac{3}{4}\right)$.

$$\therefore \ \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right) \quad \ldots(i)$$

Also, let $\cos^{-1}\left(\dfrac{12}{13}\right) = y \Rightarrow \cos y = \dfrac{12}{13} \Rightarrow \tan y = \dfrac{5}{12} \Rightarrow y = \tan^{-1}\left(\dfrac{5}{12}\right)$

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$\therefore \cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$ ...(ii)

And, let $\sin^{-1}\left(\frac{56}{65}\right) = z \Rightarrow \sin z = \frac{56}{65} \Rightarrow \tan z = \frac{56}{33} \Rightarrow z = \tan^{-1}\left(\frac{56}{33}\right)$

$\therefore \sin^{-1}\left(\frac{56}{65}\right) = \tan^{-1}\left(\frac{56}{33}\right)$ ...(iii)

Now, we take **LHS**:

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{3}{4}\right)$$

$$= \tan^{-1}\left[\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}.\frac{3}{4}}\right] = \tan^{-1}\left[\frac{20+36}{48-15}\right]$$

$$= \tan^{-1}\left(\frac{56}{33}\right)$$ [Using equation (iii)

$$= \sin^{-1}\left(\frac{56}{65}\right) = \textbf{RHS}.$$ [Hence Proved.

SECTION – C (Each question carries 6 Marks)

Q23. Evaluate: $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$. **OR** Evaluate: $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$.

Sol. Let $I = \int \sin^{-1}x\left(\frac{x}{\sqrt{1-x^2}}\right)dx$

Integrating by parts taking $\sin^{-1}x$ as first function and $\frac{x}{\sqrt{1-x^2}}$ as second function.

$$I = \sin^{-1}x\int\left(\frac{x}{\sqrt{1-x^2}}\right)dx - \int\left[\frac{d}{dx}(\sin^{-1}x)\int\left(\frac{x}{\sqrt{1-x^2}}\right)dx\right]dx$$

$$\Rightarrow \;= -\frac{1}{2}\sin^{-1}x\int\left(\frac{-2x}{\sqrt{1-x^2}}\right)dx + \frac{1}{2}\int\left[\frac{1}{\sqrt{1-x^2}}\int\left(\frac{-2x}{\sqrt{1-x^2}}\right)dx\right]dx$$

$$\Rightarrow \;= -\frac{1}{2}\sin^{-1}x\left[2\sqrt{1-x^2}\right] + \frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}\left[2\sqrt{1-x^2}\right]dx$$

$$\Rightarrow \;= -\sqrt{1-x^2}\sin^{-1}x + \int 1dx \qquad \Rightarrow I = -\sqrt{1-x^2}\sin^{-1}x + x + k.$$

OR Let $I = \int \frac{x^2+1}{(x-1)^2(x+3)}dx$ ...(i)

Consider $\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+3}$

$$\Rightarrow x^2+1 = A(x-1)(x+3)+B(x+3)+C(x-1)^2.$$

On equating the coefficients of like terms, we obtain $A=\frac{3}{8}, B=\frac{1}{2}, C=\frac{5}{8}$.

So by (i), we have: $I = \frac{3}{8}\int\frac{1}{(x-1)}dx + \frac{1}{2}\int\frac{1}{(x-1)^2}dx + \frac{5}{8}\int\frac{1}{(x+3)}dx$

$$\Rightarrow I = \frac{3}{8}\log|x-1| + \frac{1}{2}\left(-\frac{1}{x-1}\right) + \frac{5}{8}\log|x+3| + k$$

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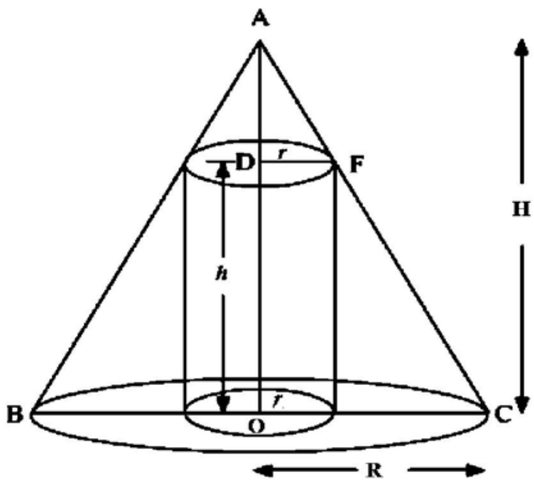
$\therefore\ \mathrm{I}=\frac{3}{8}\log|x-1|+\frac{5}{8}\log|x+3|-\frac{1}{2(x-1)}+k$.

Q24. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

OR An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

Sol.



Let H and R be the height and radius of the base of the cone ABC respectively. Suppose the radius and height of the cylinder inscribed in the cone be r and h respectively

Now, DF = r. AD = H – h.

As $\Delta\mathrm{ADF}\sim\Delta\mathrm{AOC}$

So, $\frac{\mathrm{AD}}{\mathrm{AO}}=\frac{\mathrm{DF}}{\mathrm{OC}}\Rightarrow\frac{\mathrm{H}-h}{\mathrm{H}}=\frac{r}{\mathrm{R}}$

i.e., $h=\left(1-\frac{r}{\mathrm{R}}\right)\mathrm{H}$.

Let S be the curved surface area of the cylinder.

So, $\mathrm{S}=2\pi rh$ $\Rightarrow\mathrm{S}=2\pi r\left(1-\frac{r}{\mathrm{R}}\right)\mathrm{H}$ $\Rightarrow\mathrm{S}=2\pi\mathrm{H}\left(r-\frac{r^2}{\mathrm{R}}\right)$

Now, differentiating with respect to r, we get: $\frac{d\mathrm{S}}{dr}=2\pi\mathrm{H}\left(1-\frac{2r}{\mathrm{R}}\right)$

Again differentiating with respect to r, we get: $\frac{d^2\mathrm{S}}{dr^2}=2\pi\mathrm{H}\left(-\frac{2}{\mathrm{R}}\right)=-\frac{4\pi\mathrm{H}}{\mathrm{R}}$.

For the points of local maxima or minima, $\frac{d\mathrm{S}}{dr}=0$

i.e., $2\pi\mathrm{H}\left(1-\frac{2r}{\mathrm{R}}\right)=0\Rightarrow r=\frac{\mathrm{R}}{2}$

Now, $\left.\frac{d^2\mathrm{S}}{dr^2}\right]_{at\ r=\frac{\mathrm{R}}{2}}=-\frac{4\pi\mathrm{H}}{\mathrm{R}}<0$.

So, curved surface area S, of the cylinder is maximum at $r=\mathrm{R}/2$.

Hence, the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

OR

Let the length, breadth and height of the open box be x, x and h units respectively.

$\therefore$ Area of cardboard used in the open box $=x^2+4xh$ $\Rightarrow c^2=x^2+4xh$ $\Rightarrow h=\frac{c^2-x^2}{4x}$.

Suppose V be the volume of the open box.

$\therefore\ \mathrm{V}=x^2h=x^2\left(\frac{c^2-x^2}{4x}\right)$ $\Rightarrow\mathrm{V}=\frac{1}{4}(c^2x-x^3)$

On differentiating w.r.t. x, we have: $\frac{d\mathrm{V}}{dx}=\frac{1}{4}(c^2-3x^2)$

Again differentiating w.r.t. x, we have: $\frac{d^2\mathrm{V}}{dx^2}=-\frac{3}{2}x$

For the points of local maxima or minima, $\frac{d\mathrm{V}}{dx}=0$

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i.e., $\frac{1}{4}(c^2-3x^2)=0$ $\quad\Rightarrow x=\frac{c}{\sqrt{3}}$ $\quad$ [Rejecting $x=-\frac{c}{\sqrt{3}}$ as, x can't be negative.

Now, $\left.\frac{d^2V}{dx^2}\right]_{at\ x=\frac{c}{\sqrt{3}}}=-\left(\frac{3}{2}\right)\left(\frac{c}{\sqrt{3}}\right)<0$

Hence, volume of the open box is maximum when $x=\frac{c}{\sqrt{3}}$ units.

Now, maximum volume of the open box is, $V=\frac{1}{4}\left(c^2\left(\frac{c}{\sqrt{3}}\right)-\left(\frac{c}{\sqrt{3}}\right)^3\right)=\frac{1}{4}\left(\frac{c^3}{\sqrt{3}}-\frac{c^3}{3\sqrt{3}}\right)$

i.e., $V=\frac{c^3}{4\sqrt{3}}\left(1-\frac{1}{3}\right)$ $\quad\Rightarrow V=\frac{c^3}{6\sqrt{3}}$ cubic units. $\quad$ [Hence Proved.

Q25. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3, or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3, or 4 with the die?

Sol. Let E_1 be the event that the outcome on the die is 5 or 6 and E_2 be the event that the outcome on the die is 1, 2, 3, or 4. So, $P(E_1) = 2/6 = 1/3$, $P(E_2) = 4/6 = 2/3$.
Let E be the event of getting exactly one head.
$P(E|E_1)$ = Probability of getting exactly one head by tossing the coin three times if she gets 5 or 6 = 3/8.
$P(E|E_2)$ = Probability of getting exactly one head in a single throw of coin if she gets 1, 2, 3, or 4 = 1/2.

Observe that the probability that the girl threw 1, 2, 3, or 4 with the die, if she obtained exactly one head, is given by $P(E_2|E)$.
Using Bayes' theorem, we have

$$P(E_2|E)=\frac{P(E_2)P(E|E_2)}{P(E_1)P(E|E_1)+P(E_2)P(E|E_2)}$$

$$\Rightarrow \quad =\frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}}=\frac{8}{11}.$$

Q26. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at last 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹5 per kg to purchase Food I and ₹7 per kg to purchase Food II. Determine the minimum cost of such a mixture. Formulate the above as a LPP and solve it graphically.

Sol. Let the dietician mix x kg of food I and y kg of food II to make the mixture.
To minimize, Z = ₹$(5x + 7y)$
Subject to the constraints:
$2x + y \geq 8$ …(i)
$x + 2y \geq 10$ …(ii)
and $x, y \geq 0$

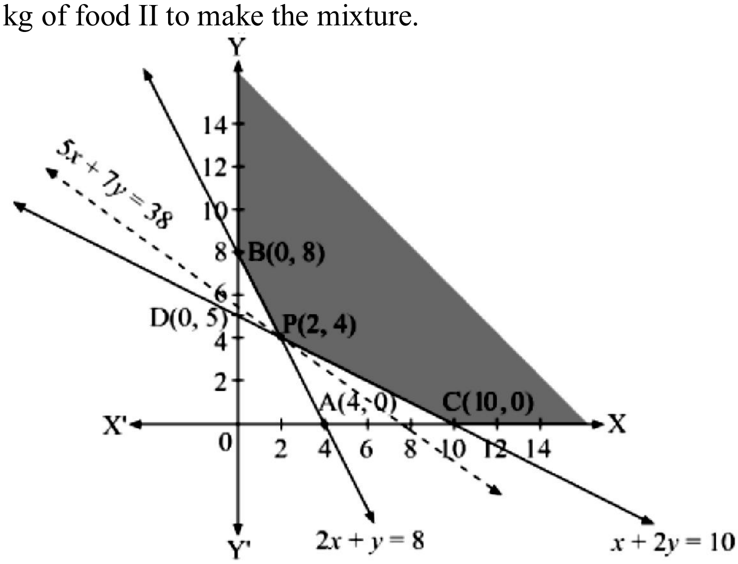
Considering the equations corresponding to the inequations (i) and (ii),

$2x + y = 8$

x	0	4
Y	8	0

$x + 2y = 10$

x	0	10
Y	5	0



Take the testing points as (0, 0) for (i), we have: $2(0)+(0)\geq 8 \Rightarrow 0\geq 8$, which is false.
Take the testing points as (0, 0) for (ii), we have: $(0)+2(0)\geq 10 \Rightarrow 0\geq 10$, which is false.
The shaded region as shown in the given figure is the feasible region, which is **unbounded**.
The coordinates of the corner points of the feasible region are B(0, 8), P(2, 4) and C(10, 0).

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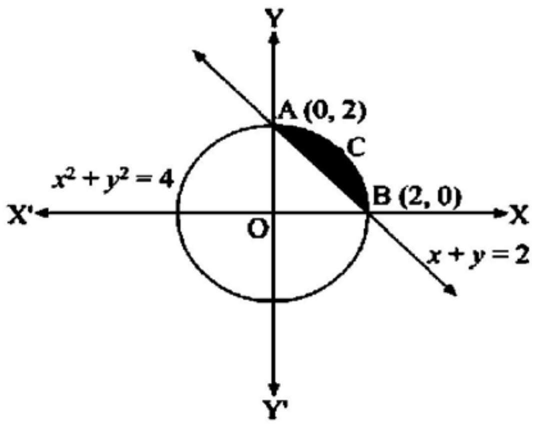
So, Value of Z at B(0, 8) = ₹(5× 0 + 7× 8) = ₹56
Value of Z at P(2, 4) = ₹(5× 2 + 7× 4) = ₹38
Value of Z at C(10, 0) = ₹(5× 10 + 7× 0) = ₹50
Thus, the minimum value of Z is ₹38.

Since the feasible region is unbounded, we need to verify whether Z = ₹38 is minimum value of given objective function or not.
For this, draw a graph of $5x + 7y < 38$.
We observe that the open half plane determined by $5x + 7y < 38$ has no points in common with the feasible region. So, Z is minimum for $x = 2$ and $y = 4$ and the minimum value of Z is ₹38.

Thus, the minimum cost of the mixture is ₹38 for 2kg of food I and 4kg of food II.

Q27. Find area of the region $\{(x,y): x^2+y^2 \le 4,\ x+y \ge 2\}$.

Sol.



We have $\{(x,y): x^2+y^2 \le 4,\ x+y \ge 2\}$.

Consider $x^2+y^2=4$ ...(i), $x+y=2$ ...(ii).

On solving curves (i) and (ii) simultaneously to get the point of intersection, we have

$$x^2+(2-x)^2=4 \Rightarrow 2x^2-4x=0 \Rightarrow 2x(x-2)=0$$
$$\therefore \quad x=0,\ x=2 \Rightarrow y=2,\ y=0.$$

So, we have (0, 2) and (2, 0) as the point of intersections of given two curves.

$\therefore$ Required area = Area (OACBO) – Area (OABO)

$$\Rightarrow \quad = \int_0^2 y_c\,dx - \int_0^2 y_l\,dx$$

$$\Rightarrow \quad = \int_0^2 \sqrt{2^2-x^2}\,dx - \int_0^2 (2-x)\,dx \quad \Rightarrow \quad = \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2 - \left[\frac{(2-x)^2}{2\times(-1)}\right]_0^2$$

$$\Rightarrow \quad = \left[\left(0+2\sin^{-1}\left(\frac{2}{2}\right)\right)-\left(0+2\sin^{-1}(0)\right)\right]+\frac{1}{2}[0-4]$$

$$\Rightarrow \quad = (\pi-2) \text{ sq.units}.$$

Q28. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r} = -\hat{i}+3\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k})$. Also find the image of P in this line.

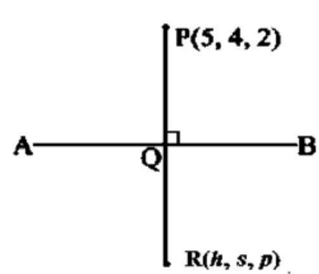
Sol. The given point is P(5, 4, 2) and the given line AB (*say*) is $\vec{r} = -\hat{i}+3\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k})$.

Cartesian equation of line is: $\dfrac{x+1}{2}=\dfrac{y-3}{3}=\dfrac{z-1}{-1}=\lambda$.

So, the coordinates of any random point Q on this line is $Q(2\lambda-1,\ 3\lambda+3,\ 1-\lambda)$.

Let Q be the foot of the perpendicular on the given line for some value of λ.
Direction Ratios of PQ are $2\lambda-1-5,\ 3\lambda+3-4,\ 1-\lambda-2$ i.e., $2\lambda-6, 3\lambda-1, -1-\lambda$.

As PQ is perpendicular to given line.



So, $2(2\lambda-6)+3(3\lambda-1)-1(-1-\lambda)=0 \Rightarrow \lambda=1$.

$\therefore$ Coordinates of the **Foot of perpendicular** on the line is $Q(1,6,0)$.

And, **Length of perpendicular** is $PQ=\sqrt{(1-5)^2+(6-4)^2+(0-2)^2}$

$$\Rightarrow PQ = 2\sqrt{6} \text{ units}.$$

Also, let the image of P in the line be R(h, s, p). Then Q will be the mid-point of PQ.

So, $Q(1,6,0)=Q\left(\dfrac{5+h}{2},\dfrac{4+s}{2},\dfrac{2+p}{2}\right) \quad \Rightarrow h=-3,\ s=8,\ p=-2$.

Hence, the **Image of point P** in the given line is R(−3, 8, −2).

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Q29. Using matrices, solve the following system of linear equations:

$3x+4y+7z=4;\ 2x-y+3z=-3;\ x+2y-3z=8$.

Sol. The given system of equations is: $3x+4y+7z=4;$

$$2x-y+3z=-3;$$

$$x+2y-3z=8$$

By using matrix method: let $A=\begin{bmatrix}3 & 4 & 7\\ 2 & -1 & 3\\ 1 & 2 & -3\end{bmatrix}$, $B=\begin{bmatrix}4\\ -3\\ 8\end{bmatrix}$ and, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$.

Since $AX=B \qquad \Rightarrow X=A^{-1}B \qquad ...(i)$

Now, $|A|=\begin{vmatrix}3 & 4 & 7\\ 2 & -1 & 3\\ 1 & 2 & -3\end{vmatrix}=3(3-6)-4(-6-3)+7(4+1)$

$\therefore\ |A|=62\neq 0 \qquad \Rightarrow$ A is **non-singular** and hence, it is invertible i.e., A^{-1} exists.

Consider C_{ij} be the cofactors of element a_{ij} in matrix A, we have

$$C_{11}=-3,\quad C_{12}=9,\quad C_{13}=5$$
$$C_{21}=26,\quad C_{22}=-16,\quad C_{23}=-2$$
$$C_{31}=19,\quad C_{32}=5,\quad C_{33}=-11$$

So, $adjA=\begin{bmatrix}-3 & 9 & 5\\ 26 & -16 & -2\\ 19 & 5 & -11\end{bmatrix}^T=\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}$

$$\therefore\ A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{62}\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}$$

Now by (i), we have $X=A^{-1}B$

So, $$X=\frac{1}{62}\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}\begin{bmatrix}4\\ -3\\ 8\end{bmatrix}$$

$$\Rightarrow \quad =\frac{1}{62}\begin{bmatrix}-12-78+152\\ 36+48+40\\ 20+6-88\end{bmatrix}$$

$$\Rightarrow \quad =\frac{1}{62}\begin{bmatrix}62\\ 124\\ -62\end{bmatrix}$$

i.e., $$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 2\\ -1\end{bmatrix}$$

By equality of matrices, we get

$$x=1,\ y=2,\ z=-1$$

Hence, $x=1,\ y=2,\ z=-1$ is the required solution.

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