Design of Question Paper

Mathematics - Class XII

Time : 3 hours **Max. Marks : 100**

Weightage of marks over different dimensions of the question paper shall be as follows :

A. Weightage to different topics/content units

S.No.	Topics	Marks
1.	Relations and functions	10
2.	Algebra	13
3.	Calculus	44
4.	Vectors & three-dimensional Geometry	17
5.	Linear programming	06
6.	Probability	10
	Total	**100**

B. Weightage to different forms of questions

S.No.	Forms of Questions	Marks for each question	No. of Questions	Total Marks
1.	Very Short Answer questions (VSA)	01	10	10
2.	Short answer questions (SA)	04	12	48
3.	Long answer questions (LA)	06	07	42
	Total		**29**	**100**

C. Scheme of Options

There will be no overall choice. However, an internal choice in any four questions of four marks each and any two questions of six marks each has been provided.

D. Difficulty level of questions

S.No.	Estimated difficulty level	Percentage of marks
1.	Easy	15
2.	Average	70
3.	Difficult	15

Based on the above design, separate sample papers along with their blue prints and Marking schemes have been included in this document. About 20% weightage has been assigned to questions testing higher order thinking skills of learners.

Class XII

MATHEMATICS

Blue-Print I

S.No.	TOPIC	VSA (1)Mark	SA (4) Marks	LA (6) Marks	Total	
1. (a)	Relations & Functions	1 (1)	4 (1)	-	5 (2)	10 (4)
(b)	Inverse Trigonometric functions	1 (1)	4 (1)	-	5 (2)	
2. (a)	Matrices	2 (2)	-	6 (1)	8 (3)	13 (5)
(b)	Determinants	1 (1)	4 (1)	-	5 (2)	
3. (a)	Continuity & Differentiability	-	8 (2)	-	8 (2)	19 (5)
(b)	Applications of Derivatives	1 (1)	4 (1)	6 (1)	11 (3)	
(c)	Integrals	1 (1)	12 (3)	-	13 (4)	19 (5)
(d)	Applications of Integrals	-	-	6 (1)	6 (1)	
(e)	Differential Equations	-	-	6 (1)	6 (1)	6 (1)
4. (a)	Vectors	2 (2)	4 (1)	-	6 (3)	17 (6)
(b)	Three Dimensional Geometry	1 (1)	4 (1)	6 (1)	11 (3)	
5.	Linear Programming	-	-	6 (1)	6 (1)	6 (1)
6.	Probability	-	4 (1)	6 (1)	10 (2)	10 (2)
	Total	10 (10)	48 (12)	42 (7)	**100 (29)**	

Sample Question Paper - I

MATHEMATICS

Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions

1. All questions are compulsory.

2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.

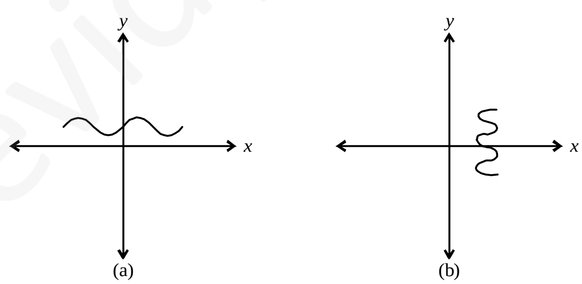
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

1. Which one of the following graphs represent the function of x ? Why ?



2. What is the principal value of

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right) ?$$

3. A matrix A of order 3×3 has determinant 5. What is the value of $|3A|$?

4. For what value of x, the following matrix is singular ?

$$\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$$

5. Find the point on the curve $y = x^2 - 2x + 3$, where the tangent is parallel to x-axis.

6. What is the angle between vectors $\vec{a}$ & $\vec{b}$ with magnitude √3 and 2 respectively ? Given $\vec{a} . \vec{b} = 3$.

7. Cartesian equations of a line AB are.

$$\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$$

Write the direction ratios of a line parallel to AB.

8. Write a value of $\int e^{3\log x}\left(x^4\right)dx$

9. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ & $\vec{b}$ externally in the ratio

1 : 4, where $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$

10. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

Write the order of AB and BA.

SECTION - B

11. Show that the function f : **R** → **R** defined by $f(x) = \frac{2x-1}{3}$, $x \in$ **R** is one-one and onto function. Also find the inverse of the function f.

OR

Examine which of the following is a binary operation

(i) $a * b = \frac{a+b}{2}$, $a, b \in N$

(ii) $a * b = \frac{a+b}{2}$, $a, b \in Q$

for binary operation check the commutative and associative property.

12. Prove that

$$\tan^{-1}\left(\frac{63}{16}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$$

13. Using elementary transformations, find the inverse of

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

14. Find all the points of discontinuity of the function **f** defined by

$$f(x) = \begin{matrix} x+2, & x \le 1 \\ x-2, & 1 < x < 2 \\ 0, & x \ge 2 \end{matrix}$$

15. If $x^p y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

OR

Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right], 0 < |x| < 1$

16. Evaluate $\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}\,dx$

17. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10m.

18. Evaluate the following integral as limit of sum $\int_1^2 (3x^2-1)\,dx$

19. Evaluate $\int_0^{\pi/2} \log \sin x \, dx$

20. Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through (3, 0, –4). Also

find the distance between these two lines.

21. In a regular hexagon ABC DEF, if $\vec{AB} = \vec{a}$ and $\vec{BC} = \vec{b}$, then express $\vec{CD}, \vec{DE}, \vec{EF}, \vec{FA}, \vec{AC}, \vec{AD}, \vec{AE}$ and $\vec{CE}$ in terms of $\vec{a}$ and $\vec{b}$.

22. A football match may be either won, drawn or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches.

OR

A candidate has to reach the examination centre in time. Probability of him going by bus or scooter or by other means of transport are $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively, if he travels by bus or scooter. But he reaches in time if he uses any other mode of transport. He reached late at the centre. Find the probability that he travelled by bus.

SECTION - C

23. Find the matrix P satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

24. Find all the local maximum values and local minimum values of the function

$$f(x) = \sin 2x - x, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

OR

A given quantity of metal is to be cast into a solid half circular cylinder (i.e., with rectangular base and semicircular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is $\pi : (\pi + 2)$.

25. Sketch the graph of

$$f(x) = \begin{cases} |x-2|+2, & x \le 2 \\ x^2 - 2, & x > 2 \end{cases}$$

Evaluate $\int_0^4 f(x)\,dx$. What does the value of this integral represent on the graph ?

26. Solve the following differential equation $(1-x^2)\frac{dy}{dx} - xy = x^2$, given $y = 2$ when $x = 0$

27. Find the foot of the perpendicular from P(1, 2, 3) on the line

$$\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$$

Also obtain the equation of the plane containing the line and the point (1, 2, 3)

28. Let X denote the number of colleges where you will apply after your results and P(X = x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x)=\begin{cases} kx & \text{if } x=0 \text{ or } 1 \\ 2kx & \text{if } x=2 \\ k(5-x) & \text{if } x=3 \text{ or } 4 \end{cases}, \quad \text{k is +ve constant}$$

(a) Find the value of k.

(b) What is the probability that you will get admission in exactly two colleges?

(c) Find the mean and variance of the probability distribution.

OR

Two bags A and B contain 4 white 3 black balls and 2 white and 2 black balls respectively. From bag A two balls are transferred to bag B. Find the probability of drawing

(a) 2 white balls from bag B ?

(b) 2 black balls from bag B ?

(c) 1 white & 1 black ball from bag B ?

29. A catering agency has two kitchens to prepare food at two places A and B. From these places 'Mid-day Meal' is to be supplied to three different schools situated at P, Q, R. The monthly requirements of the schools are respectively 40, 40 and 50 food packets. A packet contains lunch for 1000 students. Preparing capacity of kitchens A and B are 60 and 70 packets per month respectively. The transportation cost per packet from the kitchens to schools is given below :

Transportation cost per packet (in rupees)		
To	**From**	
	A	**B**
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported to school so that the cost of transportation is minimum ? Also find the minimum cost.

MARKING SCHEME
SAMPLE PAPER - I
Mathematics - XII

Q. No.	Value Points	Marks
	SECTION A	
1.	(a)	$\frac{1}{2}$
$\therefore$	for every value of x there is unique y	$\frac{1}{2}$
2.	π	1
3.	135	1
4.	3	1
5.	(1, 2)	1
6.	$\pi/_6$	1
7.	(1, - 7, 2) or their any multiple	1
8.	$\frac{x^8}{8}+c$	1
9.	$3\hat{i}+\frac{11}{3}\hat{j}+5\hat{k}$	1
10.	order of AB is 2 x 2	$\frac{1}{2}$
	order of BA is 3 x 3	$\frac{1}{2}$

Q. No. **Value Points** **Marks**

SECTION B

11. $f(x) = \dfrac{2x-1}{3}, x \in \mathrm{R}$

To show *f* is one-one

Let $x_1, x_2 \in \mathrm{R}$ s.t. $x_1 \neq x_2$

$\Rightarrow$ $2x_1 \neq 2x_2$

$\Rightarrow$ $2x_1 - 1 \neq 2x_2 - 1$

$\Rightarrow$ $\dfrac{2x_1 - 1}{3} \neq \dfrac{2x_2 - 1}{3}$ ½

$\Rightarrow$ $f(x_1) \neq f(x_2)$ ½

$\Rightarrow$ *f* is one-one

To show *f* is onto

Let $y = \dfrac{2x-1}{3}, y \in \mathrm{R}$ (codomain of f)

or $3y = 2x - 1$

or $x = \dfrac{3y+1}{2} \in \mathrm{R}$ 1

$\therefore$ for all $y \in$ R (codomain of *f*), there exist

$x = \dfrac{3y+1}{2} \in \mathrm{R}$ (codomain of *f*), such that

$$f(x) = f\left(\frac{3y+1}{2}\right) = \frac{2\left(\frac{3y+1}{2}\right) - 1}{3} = y$$

$\Rightarrow$ every element in codomain of *f* has its pre-image in the domain of *f*. ½

$\Rightarrow$ *f* is onto.

To find f^{-1}

Let $f(x) = y, \quad x = \dfrac{3y+1}{2}$

$\Rightarrow$ $f^{-1}(y) = x$

Q. No.	Value Points	Marks

$\Rightarrow$ $f^{-1}(y) = \dfrac{3y+1}{2}$

$\therefore$ f^{-1} : R $\to$ R given by

$f^{-1}(y) = \dfrac{3y+1}{2}$ 1½

OR

(i) $a * b = \dfrac{a+b}{2}, \quad a, b \in N$

$\forall$ $a, b \in N$ $\dfrac{a+b}{2}$ may or may not belong to N. ½

$\therefore$ a * b is not always natural no.

$\therefore$ '*' is not a binary operation on N ½

(ii) $a * b = \dfrac{a+b}{2}, \quad a, b \in Q$

$\forall$ $a, b \in Q$

$\dfrac{a+b}{2} \in Q$ ½

$\Rightarrow$ $a * b \in Q$.

$\Rightarrow$ '*' is a binary operation on Q ½

(iii) For $a * b = \dfrac{a+b}{2}, \quad a, b \in Q$

$a * b = \dfrac{a+b}{2} = \dfrac{b+a}{2} = b * a$ ½

$\Rightarrow$ * is commutative ½

(iv) $(a * b) * c \quad = \left(\dfrac{a+b}{2}\right) * c$

$\forall$ a, b, c, $\in$ Q.

$= \dfrac{\dfrac{a+b}{2} + c}{2}$

$= \dfrac{a+b+2c}{4}$

Q. No.	Value Points	Marks

$$a * (b * c) = a * \left(\frac{b+c}{2}\right)$$

$$= \frac{a + \left(\frac{b+c}{2}\right)}{2}$$

$$= \frac{2a+b+c}{4}$$

$(a * b) * c \neq a * (b * c) \ \forall\ a, b, c, \in Q$ ½

∴ '$*$' is not associative, ½

12. Let $\sin^{-1}\left(\frac{5}{13}\right) = x$ & $\cos^{-1}\left(\frac{3}{5}\right) = y$

$\Rightarrow$ $\sin x = \frac{5}{13}$ & $\cos y = \frac{3}{5}$

& $\cos x = \frac{12}{13}$ & $\sin y = \frac{4}{5}$

$\Rightarrow$ $\tan x = \frac{5}{13}$ & $\tan y = \frac{4}{3}$ (1 + 1)

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$ (½)

$$\tan(x+y) = \frac{63}{16}$$ (1)

$\Rightarrow$ $x + y = \tan^{-1}\left(\frac{63}{16}\right)$ (½)

$\Rightarrow$ $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right)$

Q. No. | **Value Points** | **Marks**

Sol.13. Let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

$A = I A$ (½)

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$\Rightarrow$ $$\begin{bmatrix} 1 & -2 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A$$ (½)

$R_2 \to R_2 - 2R_1$ (1)

$\Rightarrow$ $$\begin{bmatrix} 1 & -2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A$$

$R_1 \to R_1 - R_2(1)$

$\Rightarrow$ $$\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix} A$$

$R_2 \to -\frac{1}{2} R_2$ (½)

$\Rightarrow$ $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

$\therefore$ $$A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$ (½)

OR

Operate $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+bac \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$$ 1

Q. No.	**Value Points**	**Marks**

Take a, b, c common from C_1, C_2, C_3 respectively 1

$$= \begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ba \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$= \begin{vmatrix} ab+bc+ac & ab+bc+ac & ab+bc+ac \\ ab+bc & -ac & bc+ba \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ca)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ba \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$C_1 \rightarrow C_1 - C_3$, $C_2 \rightarrow C_2 - C_3$

$$= (ab+bc+ca)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -(ab+bc+ca) & bc+ba \\ ac+bc+ab & bc+ac+ab & -ab \end{vmatrix}$$ 1½

On expanding by R_1 we get ½

$= (ab+bc+ca)^3$

Sol.14. Being a polynomial function $f(x)$ is continuous at all point for $x < 1$, $1 < x < 2$ and $x \geq 2$. 1

Thus the possible points of discontinuity are $x = 1$ and $x = 2$.

To check continuity at $x = 1$

$\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1} x+2 =$

$\lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1} x-2 = -$ 1

$f(1) = 3$.

since, $\lim\limits_{x\to1^-} f(x) = f(1) \neq \lim\limits_{x\to1^+} f(x$

$\therefore$ $f(x)$ is not continuous at $x =$

Q. No. **Value Points** **Marks**

To check continuity at $x = 2$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x - 2 = 0$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} 0 = 0$$

$$f(2) = 0$$ (1)

since $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2) = 0$

∴ $f(x)$ is continuous at $x = 2$.

∴ The only point of discontinuity is $x = 1$. (1)

Sol.15. $x^p y^q = (x+y)^{p+q}$

Take log on both sides

$$p\log x + q\log y = (p+q)\log(x+y)$$

$$\frac{p}{x} + \frac{q}{y}\cdot\frac{dy}{dx} = \frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right)$$ (2)

or $$\frac{p}{x} - \frac{p+q}{x+y} = \frac{dy}{dx}\left(\frac{p+q}{x+y} - \frac{q}{y}\right)$$ (1)

or $$\frac{px+py-px-qx}{x(x+y)} = \frac{dy}{dx}\left(\frac{py+qy-qx-qy}{y(x+by)}\right)$$

or $$\frac{py-qx}{x} = \frac{dy}{dx}\left(\frac{py-qx}{y}\right)$$

or $$\frac{y}{x} = \frac{dy}{dx}$$ (1)

OR

$$y = \tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$$

Put $x^2 = \cos\theta$

$$y = \tan^{-1}\left[\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right]$$ (½)

$$= \tan^{-1}\left[\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right]$$ (1)

$$= \tan^{-1}\left[\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]$$

$$y = \frac{\pi}{4}+\frac{\theta}{2}$$ (1)

or $$y = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(x^2\right)$$ (½)

$$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{2x}{\sqrt{1-x^4}}\right)$$

$$= \frac{-x}{\sqrt{1-x^4}}$$ (1)

Sol. 16. $$\int \frac{\left(x^2+1\right)\left(x^2+4\right)}{\left(x^2+3\right)\left(x^2-5\right)}\,dx$$

Consider

$$\frac{\left(x^2+1\right)\left(x^2+4\right)}{\left(x^2+3\right)\left(x^2-5\right)} = \frac{(t+1)(t+4)}{(t+3)(t-5)} \quad \text{where } t = x^2$$

$$= 1+\frac{7t+19}{(t+3)(t-5)}$$ (1)

Q. No.	Value Points	Marks

Consider

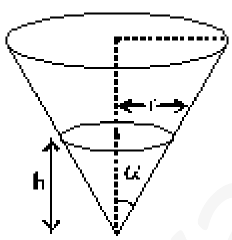
$$\frac{7t+19}{(t+3)(t-5)} = \frac{A}{t+3} + \frac{B}{t-5}$$

$$A = \frac{1}{4}, \quad B = \frac{27}{4}$$ (1)

$$\therefore \quad \int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}\,dx = \int dx + \frac{1}{4}\int \frac{dx}{x^2+3} + \frac{27}{4}\int \frac{dx}{x^2-5}$$

$$= x + \frac{1}{4\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + \frac{27}{8\sqrt{5}}\log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + c$$ (2)

Sol.17.



Let r = radius of cone formed by water at any time

h = height of cone formed by water at any time

Given $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

$\therefore \quad \tan\alpha = \frac{1}{2}$

Also $\tan\alpha = \frac{r}{h}$

$\Rightarrow \quad h = 2r$ (1)

Volume of this cone

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h$$

Q. No.		Value Points	Marks
		$v = \frac{\pi}{12} h^3$	(1)
		$\frac{dv}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$	(1)
		$= \frac{\pi}{4} h^2 \frac{dh}{dt}$	
	But	$\frac{dv}{dt} = 5\ m^3/\text{minute}$	
	∴	$5 = \frac{\pi}{4} h^2 \frac{dh}{dt}$	
	or	$\frac{dh}{dt} = \frac{20}{\pi(10)^2}$ when h = 10m	
	or	$\frac{dh}{dt} = \frac{1}{5\pi}$ m/minute	(1)
18.	For	$\int_1^2 (3x^2 - 1)\,dx$	
		a = 1, b = 2, $h = \frac{1}{n}$ as $n \to \infty$, $h \to 0$	(½)
		$f(x) = 3x^2 - 1$	
		$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + \ldots + f(a+(n-1)h)\right]$	(½)
		$\int_1^2 (3x^2 - 1)dx = \lim_{h\to 0} h\left[3n + 3h^2(1^2 + 2^2 + \ldots + (n-1)^2) + 6h(1 + 2 + \ldots + (n-1)) - n\right]$	(1)
		$= \lim_{h\to 0} h\left[2n + 3h^2\frac{(n)(n-1)(2n-1)}{6} + 3h(n)(n-1)\right]$	(1)
		$= \lim_{h\to 0} h\left[\frac{2}{h} + \frac{3h^2(1-h)(2-h)}{6h^3} + 3h\left(\frac{1}{h}\right)\left(\frac{1-h}{h}\right)\right]$	

Q. No.		Value Points	Marks
		$= 2+\frac{1}{2}(2)+3$	(1)
		$= 6$	
19.	Given	$I = \int_0^{\pi/2} \log \sin x \, dx$	
		$I = \int_0^{\pi/2} \log \cos x \, dx \qquad \left(\because \int_0^a f(a-x)dx = \int_0^a f(x)dx \right)$	(1)
	∴	$2I = \int_0^{\pi/2} (\log \sin 2x \, dx - \log 2) dx$	
		$I = \frac{1}{2}\int_0^{\pi/2} \log \sin 2x \, dx - \frac{\pi}{4}\log 2 \qquad \text{..............} (1)$	(1)
	Consider		
		$I_1 = \int_0^{\pi/2} \log \sin 2x \, dx$	
		$= \frac{1}{2}\int_0^{\pi} \log \sin t \, dt \qquad$ Put $2x = t$, $dx = \frac{dt}{2}$, $x = 0 \Rightarrow t = 0$, $x = \frac{\pi}{2} \Rightarrow t = \pi$	
		$= \frac{1}{2}.2\int_0^{\pi/2} \log \sin t \, dt \qquad \left(\because \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx \text{ if } f(2a-x) = f(x) \right)$	
		$= \int_0^{\pi/2} \log \sin t \, dt$	
		$I_1 = \int_0^{\pi/2} \log \sin x \, dx \qquad \text{.............} (2)$	(1)

Q. No.	**Value Points**	**Marks**

From (1) and (2)

$$I = \frac{1}{2}\int_0^{\pi/2} \log\sin dx - \frac{\pi}{4}\log 2$$

$$I - \frac{1}{2}I = -\frac{\pi}{4}\log 2$$

$$I = -\frac{\pi}{2}\log 2.$$ (1)

20. Given line

$$\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$$ (½).

or, $$\frac{x-1}{5} = \frac{y-3}{-2} = \frac{z-(-1)}{4}$$ .... (i)

is passing through (1, 3, –1) and has D.R. 5, –2, 4 .

Equations of line passing through (3, 0, –4) and parallel to given line is

$$\frac{x-3}{5} = \frac{y-0}{-2} = \frac{z+4}{4}$$ .... (ii)

Vector equations of line (i) & (ii)

$$\vec{r} = \hat{i} + 3\hat{j} - \hat{k} + \lambda\left(5\hat{i} - 2\hat{j} + 4\hat{k}\right)$$

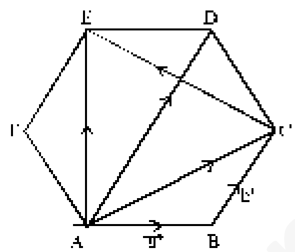
$$\vec{r} = 3\hat{i} - 4\hat{k} + \mu\left(5\hat{i} - 2\hat{j} + 4\hat{k}\right)$$ (1)

∴ $$\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 3\hat{j} - 3\hat{k}$$ (½)

$$\left|\vec{b}\right| = \sqrt{(5)^2 + (-2)^2 + (4)^2} = \sqrt{45} = 3\sqrt{5}.$$

Also $$\vec{b} \times \left(\vec{a}_2 - \vec{a}_1\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -2 & 4 \\ 2 & -3 & -3 \end{vmatrix}$$

$$= 18\hat{i} + 23\hat{j} - 11\hat{k}$$

Q. No.	**Value Points**	**Marks**
	$\therefore \quad \left|\vec{b}\times\left(\vec{a_2}-\vec{a_1}\right)\right| = \sqrt{(18)^2+(23)^2+(11)^2} = \sqrt{974}$	(1)
	$\therefore$ Distance between two parallel lines.	
	$= \dfrac{\left|\vec{b}\times\left(\vec{a_2}-\vec{a_1}\right)\right|}{\left|\vec{b}\right|}$	(½)
	$= \dfrac{\sqrt{974}}{\sqrt{45}} = \sqrt{\dfrac{974}{45}}$ units	(½)
21.		
	From fig. $\vec{DE} = -\vec{a}$	(½)
	$\vec{EF} = -\vec{b}$	(½)
	$\vec{AC} = \vec{AB} + \vec{BC}$	
	$= \vec{a} + \vec{b}$	(½)
	$\vec{AD} = 2\vec{BC} = 2\vec{b}$	(½)
	$\vec{AD} = \vec{AC} + \vec{CD}$	
	$\Rightarrow \quad \vec{CD} = \vec{AD} - \vec{AC}$	
	$= \vec{b} - \vec{a}$	(½)
	$\vec{FA} = -\vec{CD} = \vec{a} - \vec{b}$	(½)
	$\vec{CE} = \vec{CD} + \vec{DE} = \vec{b} - 2\vec{a}$	(½)

Q. No.	Value Points	Marks

$\vec{AE} = \vec{AD} + \vec{DE} = 2\vec{b} - \vec{a}$ (½)

22. P (Correct forecast) $= \frac{1}{3}$ (1)

P (Incorrect forecast) $= \frac{2}{3}$

P (At least three correct forecasts for four matches)

= P(3 correct) + P(4 correct) (½)

$$= {}^4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1 + {}^4C_4 \left(\frac{1}{3}\right)^4$$ (1+1)

$$= \frac{8}{81} + \frac{1}{81}$$

$$= \frac{9}{81} = \frac{1}{9} \text{ Ans.}$$ (½)

OR

Let E : Candidate Reaches late

A_1 : Candidate travels by bus

A_2 : Candidate travels by scooter

A_3 : Candidate travels by other modes of transport

$$P(A_1) = \frac{3}{10}, \quad P(A_2) = \frac{1}{10}, \quad P(A_3) = \frac{3}{5}$$ (½)

$$P(E/A_1) = \frac{1}{4}, \quad P(E/A_2) = \frac{1}{3}, \quad P(E/A_3) = 0$$ (1½)

∴ By Baye's Theorem

$$P(A_1/E) = \frac{P(A_1)P(E/A_1)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2) + P(A_3)P(E/A_3)}$$ (1)

$$= \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{3}{40} + \frac{1}{30} + 0}$$

$$= \frac{9}{13}$$ (1)

Q. No.	Value Points	Marks

SECTION C

23. Given $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Let $R = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ then $|R| = 1$ (½)

$S = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$ then $|S| = -1$ (½)

$Q = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Since R and S are non-singular matrices

$\therefore$ R^{-1} and S^{-1} exist.

$$R^{-1} = \frac{\text{Adj } R}{|R|} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$ (1)

$$S^{-1} = \frac{\text{Adj } S}{|S|} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$ (1)

Now given

$$\left.\begin{aligned} RPS &= Q \\ R^{-1}(RPS) &= R^{-1}Q \\ (R^{-1}R)PS &= R^{-1}Q \\ PS &= R^{-1}Q \\ PSS^{-1} &= R^{-1}QS^{-1} \\ P &= R^{-1}QS^{-1} \end{aligned}\right\} (\because R^{-1}R = I \quad I.P = P)$$ (1)

$$\therefore \quad P = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$$ (2)

Q. No. **Value Points** **Marks**

24. $f(x) = \sin 2x - x \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$f'(x) = 2\cos 2x - 1$ (½)

$f'(x) = 0 \Rightarrow \cos 2x = \frac{1}{2}$

or $2x = -\frac{\pi}{3}, \frac{\pi}{3}$ (1)

or $x = -\frac{\pi}{6}, \frac{\pi}{6}$

$f''(x) = -4\sin 2x$ (½)

$f''(x) = 2\sqrt{3} > 0 \text{ at } x = -\frac{\pi}{6}$ (1)

$\Rightarrow$ $x = -\frac{\pi}{6}$ is point of local minima

$f''(x) = -2\sqrt{3} < 0 \text{ at } x = \frac{\pi}{6}$ (1)

$\Rightarrow$ $x = \frac{\pi}{6}$ is point of local maxima

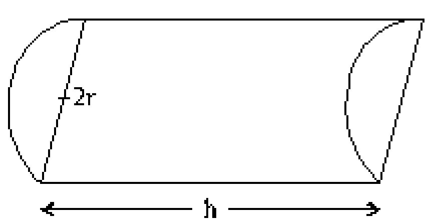
$\therefore$ Local minimum value is

$f\left(-\frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2} + \frac{\pi}{6}$ (1)

Local maximum value is

$f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ (1)

OR



Q. No. **Value Points** **Marks**

Let $h =$ length of cylinder

$r =$ radius of semi-circular ends of cylinder (½)

$$v = \frac{1}{2}\pi r^2 h$$

S = Total surface area of half circular cylinder

= 2 (Area of semi circular ends) + Curved surface area of half circular cylinder + Area of rectangular base. (1)

$$= 2\left(\frac{1}{2}\pi r^2\right) + \frac{1}{2}(2\pi rh) + 2rh$$ (½)

$$= \pi r^2 + (\pi + 2)rh$$

$$= \pi r^2 + (\pi + 2)r.\frac{2v}{\pi r^2}$$ (1)

$$\frac{ds}{dr} = 2\pi r - \frac{2v(\pi + 2)}{\pi}\left(\frac{1}{r^2}\right)$$

$$\frac{ds}{dr} = 0 \Rightarrow r^3 = \frac{(\pi + 2)v}{\pi^2}$$ (1)

$$\frac{d^2s}{dr^2} = 2\pi + \frac{2v(\pi + 2)}{\pi}.\frac{2}{r^3} > 0$$ (1)

$\therefore$ S is minimum when $r^3 = \frac{(\pi + 2)v}{\pi^2}$

$$= \frac{(\pi + 2)}{\pi^2}\left(\frac{1}{2}\pi r^2 h\right)$$

$$\Rightarrow r = \frac{\pi + 2}{2\pi}.h$$

$$\therefore \frac{h}{2r} = \frac{\pi}{\pi + 2}$$ (1)

Which is required result.

eVidyarthi
FREE Education

Q. No. **Value Points** **Marks**

25. $f(x)\begin{cases} -(x-2)+2 & x \le 2 \\ x^2-2, & x > 2 \end{cases}$

or $f(x)=\begin{cases} 4-x & x \le 2 \\ x^2-2, & x > 2 \end{cases}$ (1)

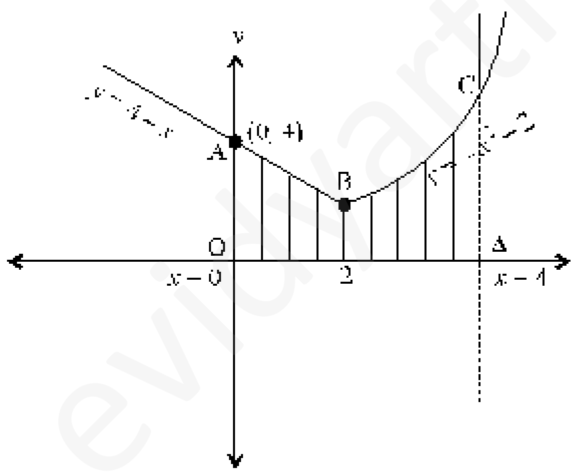
To sketch the graph of above function following tables are required.

For $f(x)=4-x, \quad x\le 2$ & for $f(x)=x^2-2,\ x\ge 2$

x	-1	0	1	2
y	5	4	3	2

Also $f(x) = x^2 - 2$ represent parabolic curve.

x	2	3	4	5	6
y	2	7	14	23	34



(2)

$$\text{Area} = \int_0^4 f(x)\,dx = \int_0^2 (4-x)\,dx + \int_2^4 (x^2-2)\,dx$$

$$= 4x - \frac{x^2}{2}\Big|_0^2 + \frac{x^3}{3} - 2x\Big|_2^4$$ (2)

$$= 6 + \frac{44}{3} = \frac{62}{3} \text{ sq. units}$$

Q. No.	Value Points	Marks

On the graph $\int_0^4 f(x)\,dx$ represents the area bounded by x-axis the lines $x = 0$; $x = 4$ and the curve $y = f(x)$.

i.e. area of shaded region shown in fig. (1)

26. $$(1-x^2)\frac{dy}{dx} - xy = x^2$$

or $$\frac{dy}{dx} - \frac{x}{1-x^2}.y = \frac{x^2}{1-x^2}$$

$$P = -\frac{x}{1-x^2}\ ,\quad Q = \frac{x^2}{1-x^2}$$ (½)

I.F. $$= e^{\int P dx} = e^{\int -\frac{-x}{1-x^2}dx}$$

$$= e^{\frac{1}{2}\log(1-x^2)}$$

$$= \sqrt{1-x^2}$$ (1)

∴ Solution of diff. equation is

$$y\sqrt{1-x^2} = \int \frac{x^2}{1-x^2}.\sqrt{1-x^2}\ dx$$ (1)

$$= \int \left(\frac{1}{\sqrt{1-x^2}} - \sqrt{1-x^2}\right) dx$$

$$= \sin^{-1}x - \left(x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right) + c$$ (1)

$$y\sqrt{1-x^2} = \frac{1}{2}\sin^{-1}x - x\sqrt{1-x^2} + c$$ (1)

When $x = 0$, $y = 2$ (1)

$2 = c$

∴ Solution is

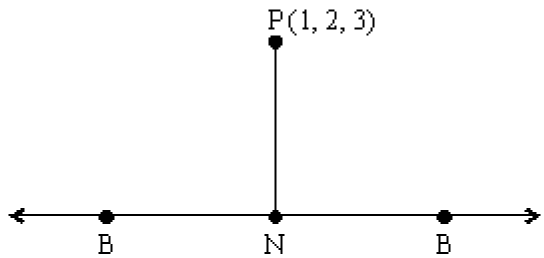
$$y\sqrt{1-x^2} = \frac{1}{2}\sin^{-1}x - x\sqrt{1-x^2} + 2$$ (½)

Q. No.	**Value Points**	**Marks**

27. The given line is

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda \text{ (say)} \qquad \text{.................. (i)}$$

Let N be the foot of the perpendicular from P(1, 2, 3) to the given line



Coordinates of N = (3λ + 6, 2λ + 7 –2λ + 7) (½)

D.R. of NP 3λ + 5 , 2 λ + 5 , –2λ + 4 (1)

D.R. of AB 3 , 2 , –2

Since NP ⊥ AB

∴ 3(3λ + 5) + 2(2λ + 5) – 2(–2λ + 4) = 0

or λ = –1 (1)

∴ Coordinates of foot of perpendicular N are (3, 5, 9) (½)

Equation of plane containing line (i) and point (1, 2, 3) is (½)

Equation of plane containing point (6, 7, 7) & (1, 2, 3) and parallel to line with D.R. 3 , 2 , –2 is

$$\begin{vmatrix} x-6 & y-7 & z-7 \\ -5 & -5 & -4 \\ 3 & 2 & -2 \end{vmatrix} = 0$$ (1½)

or, $18x - 22y + 5z + 11 = 0.$ (1)

28. Given

x	$P(x)$
0	0
1	k
2	4k
3	2k
4	k
	$\Sigma p_i = 8k$

(1)

But $\Sigma\ p_i = 1$ (1)

Q. No. **Value Points** **Marks**

$\Rightarrow$ $\boxed{k = \frac{1}{8}}$

$\therefore$ Probability distribution is

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	0	0	0
1	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{2}$	1	2
3	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{9}{4}$
4	$\frac{1}{8}$	$\frac{1}{2}$	2

(1)

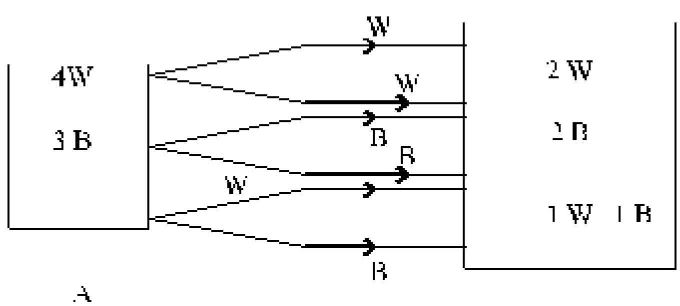
Probability of getting admission in two colleges = $\frac{1}{2}$ (1)

$$\text{Mean} = \mu = \Sigma p_i x_i = \frac{19}{8}$$ (1)

$$\text{Variance} = \sigma^2 = \Sigma p_i x_i^2 - \mu^2 = \frac{51}{8} - \left(\frac{19}{8}\right)^2$$

$$= \frac{47}{64}$$ (1)

OR



Three cases arise, when 2 balls from bag A are shifted to bag B.

Q. No. **Value Points** **Marks**

Case 1 : If 2 white balls are transferred from bag A.

$$P(W_A W_A) = \frac{4}{7} \cdot \frac{2}{6} = \frac{2}{7}$$ (1)

Case 2 : If 2 black balls are transferred from bag A

$$P(B_A B_A) = \frac{3}{7} \cdot \frac{2}{6} = \frac{1}{7}$$ (1)

Case 3 : If 1 white and 1 black ball is transferred from bag A

$$P(W_A B_A) = 2\left(\frac{4}{7} \cdot \frac{3}{6}\right) = \frac{4}{7}$$ (1)

(a) Probability of drawing 2 white balls from bag B

$= P(W_A W_A). P(W_B W_B) + P(B_A B_A). P(W_B W_B) + P(W_A B_A).P(W_B.W_B)$

$$= \frac{2}{7}\left(\frac{4}{6} \cdot \frac{3}{5}\right) + \frac{1}{7}\left(\frac{2}{6} \cdot \frac{1}{5}\right) + \frac{4}{7}\left(\frac{3}{6} \cdot \frac{2}{5}\right) = \frac{5}{21}$$ (1)

(b) Probability of drawing 2 black balls from bag B

$= P(W_A W_A). P(B_B B_B) + P(B_A.B_A). P(B_B B_B) + P(W_A B_A).P(B_B.B_B)$

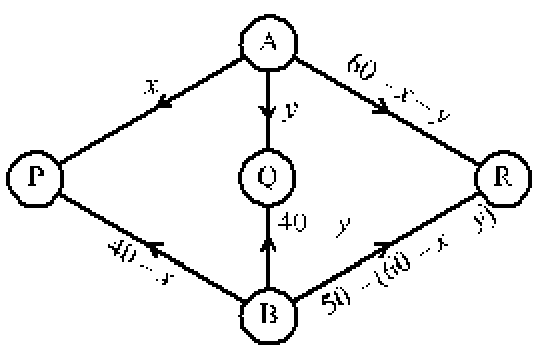
$$= \frac{2}{7}\left(\frac{2}{6} \cdot \frac{1}{5}\right) + \frac{1}{7}\left(\frac{4}{6} \cdot \frac{3}{4}\right) + \frac{4}{7}\left(\frac{3}{6} \cdot \frac{2}{5}\right)$$ (1)

$$= \frac{4}{21}$$

(c) Probability of drawing 1 white and 1 black ball from bag B

$$= \frac{2}{7}\left(\frac{4}{6} \cdot \frac{2.2}{5}\right) + \frac{1}{7}\left(\frac{2.2}{6} \cdot \frac{4}{5}\right) + \frac{4}{7}\left(\frac{2.3}{6} \cdot \frac{3}{5}\right) = \frac{4}{7}$$ (1)

29.



Q. No. **Value Points** **Marks**

Let x no. of packets from kitchen A are transported to P and y of packets from kitchen A to Q. Then only $60 - x - y$ packets can be transported to R from A.

Similarily from B $40 - x$ packets can be transported to P and $40 - y$ to Q. Remaining requirement of R i.e. $50 - (60 - x - y)$ can be transported from B to Q.

∴ Constraints are (1)

$$\left.\begin{array}{l} 40 - x \geq 0 \\ 40 - y \geq 0 \\ 60 - x - y \geq 0 \\ -10 + x + y \geq 0 \\ x \geq 0, y \geq 0 \end{array}\right\}$$ (1)

Objective function is:

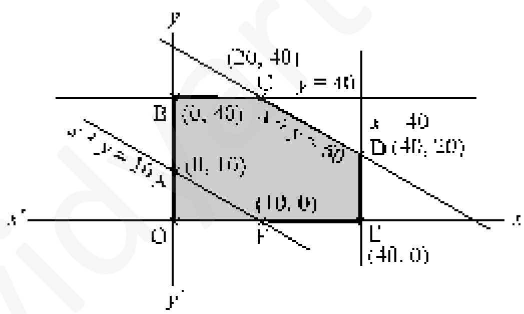
Minimise. $z = 5x + 4y + 3(60 - x - y) + 4(40 - x) + 2(40 - y) + 5(x + y - 10)$

∴ L.P.P. is

To Minimise. $z = 3x + 4y + 370$ (1)

subject to constraints

$$\left.\begin{array}{l} x \leq 40 \\ y \leq 40 \\ x + y \leq 60 \\ x + y \geq 10 \\ x \geq 0,\ y \geq 0 \end{array}\right\}$$



(1)

Feasible Region is A B C D E F A with corner points

A (0, 10) z = 3(0) + 4(10) + 370 = 410
B (0, 40) z = 3(10) + 4(40) + 370 =530
C (20, 40) z = 3(20) + 4(40) + 370 = 590
D (40, 20) z = 3(40) + 4(20) + 370 = 570
E (40, 0) z = 3(40) + 4(0) + 370 = 490
F (10, 0) z = 3(10) + 4(0) + 370 = 400 (1)

∴ $x = 10, y = 0$ gives minimum cost of transportion.

Thus No. of packets can be transported as follows (1)

	A	B
P	10	30
Q	0	40
R	50	0

Minimum cost of transportation is Rs. 400.

BLUE PRINT - II
MATHEMATICS
CLASS - XII

S.No.	Topic	VSA (1 Mark)	SA (4 Marks)	LA (6 Marks)	TOTAL
1. (a)	Relations and Functions	———	4 (1)	6 (1)	10 (2)
(b)	Inverse trigonometric Functions				
2. (a)	Matrices Determinants	2 (2)		6 (1)	8 (3) } 13 (5)
(b)		1 (1)	4 (1)		5 (2)
3. (a)	Continuity and Differentiability	1 (1)	8 (2)	-	} 20 (6)
(b)	Applications of Derivatives	1 (1)	4 (1)	6 (1)	
(c)	Integrals	1 (1)	12 (3)		} 19 (5) 44 (13)
(d)	Applications of Integrals			6 (1)	
(e)	Differential equations	1 (1)	4 (1)		5 (2)
4. (a)	Vectors	1 (1)	4 (1)	-	
(b)	3 - dimensional geometry	-	-	12 (2)	17 (4)
5.	Linear - Programming	-	-	6 (1)	6 (1)
6.	Probability	2 (2)	8 (2)	-	10 (4)
	Totals	10 (10)	48 (12)	42 (7)	100 (29)

Sample Question Paper - II
Mathematics - Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

Q.1. If A is a square matrix of order 3 such that |adj A| = 64, find |A|.

Q.2. If A, B, C are three non zero square matrices of same order, find the condition on A such that AB = AC $\Rightarrow$ B = C

Q.3. Give an example of two non zero 2 × 2 matrices A, B such that AB = 0.

Q.4. If $f(1) = 4$; $f'(1) = 2$, find the value of the derivative of log $f(e^x)$ w.r.t x at the point $x = 0$.

Q.5. Find a, for which $f(x) = a(x + \sin x) + a$ is increasing.

Q.6. Evaluate, $\int_0^{1.5} [x]\,dx$ (where [x] is greatest integer function)

Q.7. Write the order and degree of the differential equation, $y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

Q.8. If $\vec{a} = \hat{i} + \hat{j}$; $\vec{b} = \hat{j} + \hat{k}$; $\vec{c} = \hat{k} + \hat{i}$, find a unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$

Q.9. A four digit number is formed using the digits 1, 2, 3, 5 with no repetitions. Find the probability that the number is divisible by 5.

Q.10. The probability that an event happens in one trial of an experiment is 0.4. Three independent trials of the experiment are performed. Find the probability that the event happens at least once.

SECTION - B

Q.11. Find the value of $2\tan^{-1}\left(\frac{1}{5}\right) + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2\tan^{-1}\frac{1}{8}$

Q.12. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(aI + bA)^n = a^n.I + na^{n-1}bA$ where I is a unit matrix of order 2 and n is a positive integer

OR

Using properties of determinants, prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

Q.13. If $x = a \sin pt$ and $y = b \cos pt$, find the value of $\frac{d^2y}{dx^2}$ at t = 0.

Q.14. Find the equations of tangent lines to the curve $y = 4x^3 - 3x + 5$ which are perpendicular to the line $9y + x + 3 = 0$.

Q.15. Show that the function $f(x) = |x + 2|$ is continuous at every $x \in R$ but fails to be differentiable at $x = -2$.

Q.16. Evaluate $\int \frac{x^2 + 4}{x^4 + x^2 + 16} dx$

Q.17. Evaluate $\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$

OR

Evaluate $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx$

Q.18. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of a ΔABC respectivily. Find an expression for the area of ΔABC and hence deduce the condition for the points A, B, C to be collinear.

OR

Show that the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively, are the vertices of a right triangle. Also find the remaining angles of the triangle.

Q.19. Evaluate, $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x}, dx \qquad a, b > 0$

Q.20. Solve the differential equation, $\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x \,;\, y(0) = 1$

OR

$$2xy + y^2 - 2x^2\frac{dy}{dx} = 0\,;\, y(1) = 2$$

Q.21. In a bolt factory machines, A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product.

(i) What is the probability that the bolt drawn is defective ?
(ii) If the bolt is found to be defective find the probability that it is a product of machine B.

Q.22. Two dice are thrown simultaneously. Let X denote the number of sixes, find the probability distribution of X. Also find the mean and variance of X, using the probability distribution table.

SECTION - C

Q.23. Let X be a non-empty set. P(x) be its power set. Let '*' be an operation defined on elements of P(x) by,

$$A * B = A \cap B \ \forall \ A, B \in P(X)$$

Then,

(i) Prove that * is a binary operation in P(X).
(ii) Is * commutative ?
(iii) Is * associative ?
(iv) Find the identity element in P(X) w.r.t. *
(v) Find all the invertible elements of P(X)
(vi) If o is another binary operation defined on P(X) as A o B = , $A \cup B$ then verify that o distributes itself over *.

OR

Consider f : $R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible. Find the inverse of f.

Q.24. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.

Q.25. Make a rough sketch of the region given below and find it's area using integration

Q.26. Every gram of wheat provides 0.1 gm of proteins and 0.25 gm of carbohydrates. The corresponding values for rice are 0.05 gm and 0.5 gm respectively. Wheat costs Rs. 4 per kg and rice Rs. 6 per kg. The minimum daily requirements of proteins and carbohydrates for an average child are 50 gms and 200 gms respectively. In what quantities should wheat and rice be mixed in the daily diet to provide minimum daily requirements of proteins and carbohydrates at minimum cost. Frame an L.P.P. and solve it graphically.

Q.27. font $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations

$$x + 2y + z = 4$$

$$-x + y + z = 0$$

$$x - 3y + z = 2$$

$$\{(x, y);\ y^2 \le 4x,\ 4x^2 + 4y^2 \le 9\}$$

Q.28. Find the equation of the plane containing the lines,

$$\vec{r} = i + j + \lambda(\hat{i} + 2\hat{j} - \hat{k}) \quad \text{and} \quad \vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$$

Find the distance of this plane from origin and also from the point (1, 1, 1)

OR

Find the equation of the plane passing through the intersection of the planes, $2x + 3y - z + 1 = 0$; $x + y - 2z + 3 = 0$ and perpendicular the plane $3x - y - 2z - 4 = 0$. Also find the inclination of this plane with the xy plane.

Q.29. Prove that the image of the point (3, –2, 1) in the plane $3x - y + 4z = 2$ lies on the plane, $x + y + z + 4 = 0$.

MARKING SCHEME
SAMPLE PAPER - II
Mathematics - XII

Q. No.	Value Points	Marks
	SECTION A	
1.	± 8	(1)
2.	$\|A\| \neq 0$ / A is non singular A^{-1} exists / A is invertible	(1)
3.	$A = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$	(1)
4.	$\frac{1}{2}$	(1)
5.	$a > 0$	(1)
6.	0.5	(1)
7.	order 1, degree 2	(½, ½)
8.	$\pm \frac{1}{\sqrt{3}} \left(\hat{i} + \hat{j} + \hat{k} \right)$	(1)
.9.	$\frac{1}{4}$	(1)
10.	$1 - (0.6)^3$	(1)
	SECTION - B	
11.	$2\left[\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right)\right] + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)$	

Q. No.	Value Points	Marks
	$= 2\tan^{-1}\left(\dfrac{\left(\frac{1}{5}+\frac{1}{8}\right)}{1-\frac{1}{5}\times\frac{1}{8}}\right)+\sec^{-1}\left(\dfrac{5\sqrt{2}}{7}\right)$	(1)
	$= 2\tan^{-1}\left(\dfrac{\frac{13}{40}}{1-\frac{1}{40}}\right)+\tan^{-1}\left(\dfrac{1}{7}\right)$	(1)
	$= 2\tan^{-1}\left(\dfrac{13}{39}\right)+\tan^{-1}\left(\dfrac{1}{7}\right)$	
	$= 2\tan^{-1}\left(\dfrac{1}{13}\right)+\tan^{-1}\left(\dfrac{1}{7}\right)$	
	$= \tan^{-1}\left(\dfrac{\frac{2}{3}}{1-\frac{1}{9}}\right)+\tan^{-1}\left(\dfrac{1}{7}\right)$	(1)
	$= \tan^{-1}\left(\dfrac{2}{3}\times\dfrac{9}{8}\right)+\tan^{-1}\left(\dfrac{1}{7}\right)$	
	$= \tan^{-1}\left(\dfrac{3}{4}\right)+\tan^{-1}\left(\dfrac{1}{7}\right)$	
	$= \tan^{-1}\left(\dfrac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$	
	$= \tan^{-1}\left(\dfrac{\frac{21+4}{28}}{1-\frac{3}{28}}\right)=\tan^{-1}(1)=\dfrac{\pi}{4}$	(1)
12.	$A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$	
	for n = 1	

Q. No. **Value Points** **Marks**

LHS = aI + bA, RHS = aI + bA

$\therefore$ The result is true for n = 1 (1)

Let $(aI + bA)^k = a^k I + k\, a^{k-1} b A$ be true

Now, $(aI + bA)^{k-1} = (aI + bA)^k . (aI + bA)$ (½)

$= (ka^{k-1} bA + a^k I)(aI + bA)$

$= k\, a^k\, bA + k\, a^{k-1} b^2 A^2 + a^{k+1} I + a^k bA$

$(aI + bA)^{k+1} = k\, a^k bA + a^{k+1} I + a^k bA$

$(a + bA)^{k+1} = a^k b(k+1) A + a^{k+1} I \quad \left(\text{as } A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right)$ (2)

$\Rightarrow (aI + bA)^n = a^n I + na^{n-1} bA$

is true $\forall\, n \in N$ by principle of mathematical induction. (½)

OR

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

$c_1 \to c_1 + c_2 + c_3$

$$= \begin{vmatrix} 2a+2b+2c & a & b \\ 2a+2b+2c & b+c+2a & b \\ 2a+2b+2c & a & c+a+2b \end{vmatrix}$$ (1)

$$2(a+b+c) \begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$ (1)

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$= 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 0 & b+c+a & 0 \\ 0 & 0 & b+c+a \end{vmatrix}$$ (1)

$$= 2(a+b+c)\left[(b+c+a)\right]^2 = 2(a+b+c)^3$$ (1)

13. $\dfrac{dx}{dt} = ap\cos pt \ ; \ \dfrac{dy}{dt} = -bp\sin pt$

Q. No. **Value Points** **Marks**

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-b}{a}\tan pt \qquad (1)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\times\frac{dt}{dx} \qquad (1½)$$

$$= \frac{d}{dt}\left(\frac{-b}{a}\tan pt\right)\times\frac{1}{ap\cos pt}$$

$$= \frac{-b}{a}p\ .\ \sec^2 pt\times\frac{1}{ap}\sec pt \qquad (½)$$

$$\left(\frac{d^2y}{dx^2}\right)_{t=0} = \left(\frac{-b}{a^2}\sec^3 pt\right)_{t=0} = \frac{-b}{a^2} \qquad (1)$$

14. Let P (x_1, y_1) be the point on the curve $y = 4x^3 - 3x + 5$ where the tangent is perpendicular to the line $9y + x + 3 = 0$

$\Rightarrow \quad y_1 = 4x_1^3 - 3x_1 + 5$ (½)

$\Rightarrow \quad \frac{dy}{dx} = 12x^2 - 3 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 12x_1^2 - 3$ (½)

Slope of a line perpendicular to $9y + x + 3 = 0$ is 9

$\therefore \quad 12x_1^2 - 3 = 9$

$\Rightarrow \quad x_1 = \pm 1$ (1)

$\Rightarrow$ Two coresponding points on the curve $y = 4x^3 - 3x + 5$ are (1, 6) and (–1, 4). (½)

Therefore, equations of tangents are

$y - 4 = 9(x+1) \Rightarrow 9x - y + 13 = 0$ and $y - 6 = 9(x-1) \Rightarrow 9x - y - 3 = 0$ (1½)

15. Let $f(x) = |x + 2|$

$$\Rightarrow \quad f(x) = \begin{cases} x+2, & x \geq -2 \\ -(x+2), & x < -2 \end{cases} \qquad (½)$$

When $x > -2$ or $x < -2$, $f(x)$ being a polynomial function is continuous. (½)

We check continuity at $x = -2$

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2} -(x+2) = 0$$

Q. No.	Value Points	Marks

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2}(x+2) = 0$$

Also $f(-2) = 0$

$\Rightarrow$ $f(x)$ is continuous at $x = -2$ (1)

$\Rightarrow$ $f(x)$ is continuous $\forall\, x \in R$ (½)

Now, We check differentiability at $x = -2$

LHD at $x = -2$

$$= \lim_{x\to -2} \frac{f(x) - f(-2)}{x-(-2)} = \lim_{x\to -2} -\frac{x-2-0}{x+2} = -1$$ (½)

RHD at $x = -2$

$$= \lim_{x\to -2^+} \frac{f(x)-f(-2)}{x-(-2)} = \lim_{x\to -2} \frac{x+2-0}{x+2} = 1$$ (½)

$\Rightarrow$ $f(x)$ is not differentiable at $x = -2$ (½)

16. $$\int \frac{x^2+4}{x^4+x^2+16}\,dx = \int \frac{1+4/x^2}{x^2+1+16/x^2}\,dx$$ (½)

$$\left.\begin{aligned} &\text{Let } x - \frac{4}{x} = t \\ &\Rightarrow \left(1+\frac{4}{x^2}\right)dx = dt \\ &\text{and } x^2 + \frac{16}{x^2} - 8 = t^2 \end{aligned}\right\}$$ (1)

$$= \int \frac{1}{t^2+9}\,dt = \int \frac{1}{t^2+3^2}\,dt$$ (1)

$$= \frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right) + c$$

$$= \frac{1}{3}\tan^{-1}\left[\left(x - \frac{4}{x}\right)\times\frac{1}{3}\right] + c$$ (1)

$$= \frac{1}{3}\tan^{-1}\left(\frac{x^2-4}{3x}\right) + c$$ (½)

Q. No.	Value Points	Marks
17.	$\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}\,dx = \int_0^{\pi/2} \frac{x+2\sin{}^{x}/_{2}\ \text{Cos}{}^{x}/_{2}}{2\cos^2\frac{x}{2}}\,dx$	(½)
	$= \frac{1}{2}\int_0^{\pi/2} x\sec^2{}^{x}/_{2}\ dx + \int_0^{\pi/2} \tan\frac{x}{2}\,dx$	(1)
	$= \frac{1}{2}\left[\left[x\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi/2} - \int_0^{\pi/2}\frac{\tan\frac{x}{2}\,dx}{\frac{1}{2}}\,dx\right] + \int_0^{\pi/2}\tan\frac{x}{2}\,dx$	(1½)
	$= \left\| x\tan\frac{x}{2}\right\|_0^{\pi/2}$	(½)
	$= \frac{\pi}{2}$	(½)

OR

$$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx$$

Let $\left.\begin{aligned} e^x &= t \\ e^x\,dx &= dt \end{aligned}\right\}$ (1)

$$= \int \frac{dt}{\sqrt{5-4t-t^2}}$$ (½)

$$= \int \frac{dt}{\sqrt{-\left(t^2+4t-5\right)}} = \int \frac{dt}{\sqrt{-\left[(t+2)^2-(3)^2\right]}}$$

$$= \int \frac{dt}{\sqrt{(3)^2-(t+2)^2}}$$ (1)

Q. No.	Value Points	Marks
	$= \sin^{-1}\left(\frac{t+2}{3}\right) + c$	(1)
	$= \sin^{-1}\left(\frac{e^x+2}{3}\right) + c$	(½)
18.	area $\Delta\, ABC = \frac{1}{2}\left\|\vec{AB}\times\vec{AC}\right\|$	(1)
	$= \frac{1}{2}\left\|\left(\vec{b}-\vec{a}\right)\times\left(\vec{c}-\vec{a}\right)\right\|$	(½)
	$= \frac{1}{2}\left\|\vec{b}\times\vec{c}-\vec{a}\times\vec{c}-\vec{b}\times\vec{a}\right\| = \frac{1}{2}\left\|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right\|$	(1)
	If A, B, C are collinear then area $\Delta\, ABC = 0$	(½)
	$\Rightarrow \quad \frac{1}{2}\left\|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right\| = 0$	
	$\Rightarrow \quad \left\|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right\| = 0$	(½)
	$\Rightarrow \quad \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a} = \vec{0}$	(½)
	OR	
	We have, $\vec{AB} = -\hat{i}-2\hat{j}-6\hat{k}$, $\vec{BC} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{CA} = -\hat{i}+3\hat{j}+5\hat{k}$	(1)
	$\left\|\vec{AB}\right\| = \sqrt{41}$, $\left\|\vec{BC}\right\| = \sqrt{6}$, $\left\|\vec{CA}\right\| = \sqrt{35}$	
	Since $\left\|\vec{BC}\right\| + \left\|\vec{CA}\right\| > \left\|\vec{AB}\right\|$ $\therefore$ A, B, C form a triangle.	(½)
	Also, $\vec{BC}.\vec{CA} = -2-3+5 = 0$ $\therefore$ BC ⊥ CA $\Rightarrow$ $< BCA = \frac{\pi}{2}$.	(½)
	i.e. ABC is a right triangle.	

Q. No.	Value Points	Marks

$$\cos A = \frac{\vec{AB}.\vec{AC}}{|\vec{AB}||\vec{AC}|} = \frac{(-\hat{i}-2\hat{j}-6\hat{k}).(\hat{i}-3\hat{j}-5\hat{k})}{\sqrt{41}.\sqrt{35}} = \frac{35}{\sqrt{41}.\sqrt{35}} = \sqrt{\frac{35}{41}}$$

$\therefore$ $A = \cos^{-1}\left(\sqrt{\frac{35}{41}}\right)$ (1)

$$\cos B = \frac{\vec{BA}.\vec{BC}}{|\vec{BA}||\vec{BC}|} = \frac{(\hat{i}+2\hat{j}+5\hat{k}).(2\hat{i}-\hat{j}-5\hat{k})}{\sqrt{41}.\sqrt{6}} = \frac{6}{\sqrt{41}\sqrt{6}} = \sqrt{\frac{6}{41}}$$

$\Rightarrow$ $B = \cos^{-1}\sqrt{\frac{6}{41}}$ (1)

19. Let $I = \int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x}dx$, $a, b > 0$

$I = \int_0^{\pi} \frac{\pi - x}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}dx$, $\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$I = \int_0^{\pi} \frac{\pi - x}{a^2\cos^2 x + b^2\sin^2 x}dx$

$\Rightarrow$ $2I = \pi\int_0^{\pi} \frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx$ (½)

$\Rightarrow$ $2I = 2\pi\int_0^{\pi/2} \frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx$ $\left(\text{as } \int_0^{2p} f(x)dx = 2\int_0^{p} f(x) \text{ if } f(2p - x) = f(x)\right)$ (1)

$\Rightarrow$ $I = \pi\int_0^{\pi/2} \frac{\sec^2 x}{a^2 + b^2\tan^2 x}dx$ $\left(\text{Div Num and Den. by } \cos^2 x\right)$ (½)

$= I = \pi\int \frac{dt}{a^2 + t^2b^2}$ $\left(\text{put } \tan x = t \Rightarrow \sec^2 x\,dx = dt\right)$ (½)

Q. No. **Value Points** **Marks**

$$= \frac{\pi}{b^2}\int_0^{\alpha}\frac{dt}{\left(\frac{a}{b}\right)^2 + t^2} = \left[\frac{\pi}{b^2}\times\frac{1}{\frac{a}{b}}\tan^{-1}\left(\frac{t}{\frac{a}{b}}\right)\right]_0^{\alpha}$$ (1)

$$I = \frac{\pi}{ab}\left|\tan^{-1}\left(\frac{bt}{a}\right)\right|_0^{\infty} = \frac{\pi^2}{2ab} \qquad (\text{as } a, b > 0)$$

$\Rightarrow \quad I = \frac{\pi^2}{2ab}$ (½)

20. $\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$

$$I.F. = e^{\int \sec^2 x . dx} = e^{\tan x}$$ (1)

$\Rightarrow \quad y.e^{\tan x} = \int e^{\tan x} \tan x \sec^2 x \; dx + c$ (1)

$= I + c$

$I = \int e^t , t\, dt \qquad (\text{put } \tan x = t \quad \Rightarrow \quad \sec^2 x\, dx = dt)$

$\Rightarrow \quad I = t\, e^t - \int e^t \; dt$

$\Rightarrow \quad I = te^t - e^t = (\tan x - 1)e^{\tan x}$ (½)

Solution is $\Rightarrow \quad y\; e^{\tan x} = (\tan x - 1)e^{\tan x} + c$

Now $y(0) = 1$

$\Rightarrow \quad 1.e^0 = (0 - 1)e^0 + c$

$c = 2$ (1)

$$\Rightarrow \begin{cases} y\; e^{\tan x} = (\tan x - 1)e^{\tan x} + 2 \\ \text{or} \quad y = (\tan x - 1) + 2.e^{-\tan x} \end{cases}$$ (½)

OR

$$2x^2\frac{dy}{dx} = (2xy + y^2)$$

Q. No. **Value Points** **Marks**

$\Rightarrow \quad \frac{dy}{dx} = \frac{(y^2 + 2xy)}{2x^2}$ (½)

Put $y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$ (½)

$\Rightarrow \quad v + x\frac{dv}{x} = + \frac{(v^2x^2 + 2vx^2)}{2x^2}$

$\Rightarrow \quad 2v + 2x\frac{dv}{dx} = 2v + v^2$

$\Rightarrow \quad 2x\frac{dv}{dx} = v^2 \quad \Rightarrow \quad \frac{2}{v^2}dv = \frac{dx}{x}$ (1)

$\Rightarrow \quad 2\int \frac{1}{v^2}dv = \int \frac{1}{x}dx$

$\Rightarrow \quad \frac{-2}{v} = \log|x| + c$

$\Rightarrow \quad \frac{-2x}{y} = \log|x| + c$ (1)

$x = 1, \quad y = 2$

$\Rightarrow \quad c = -1$ (½)

$\therefore \quad \begin{cases} \frac{-2x}{y} = \log|x| - 1 \\ y = \frac{2x}{1 - \log|x|} \end{cases}$ (½)

21. Let E_1 = the bolt is manufactured by machine A

E_2 = the bolt is manufactured by machine B

E_3 = the bolt is manufactured by machine C

A = the bolt is defective

Then $P(E_1) = \frac{25}{100}, P(E_2) = \frac{35}{100} \; ; \; P(E_3) = \frac{40}{100}$ (½)

Q. No. **Value Points** **Marks**

(i) Now defective bolt could be from machine A, B or C

$\therefore \quad P(A|E_1) = \frac{5}{100}, P(A|E_2) = \frac{4}{100}, P(A|E_3) = \frac{2}{100}$ (½)

$\Rightarrow \quad P(A) = \frac{25}{100}\times\frac{5}{100} + \frac{35}{100}\times\frac{4}{100} + \frac{40}{100}\times\frac{2}{100}$ (1)

= 0.0345

(ii) We require probability that the bolt is manufactured by machine B given that bolt drawn is defective i.e., $P(E_2/A)$, So by Baye's theorem

$$\Rightarrow \quad P(E_2/A) = \frac{\frac{35}{100}\times\frac{4}{100}}{\frac{25}{100}\times\frac{5}{100} + \frac{35}{100}\times\frac{4}{100} + \frac{40}{100}\times\frac{2}{100}}$$ (1)

$$= \frac{140}{345} = \frac{28}{69}$$ (1)

22. Here x = 0, 1 or 2

$P(x=0) = \frac{5}{6}\times\frac{5}{6} = \frac{25}{36}$ (½)

$P(x=1) = \frac{5}{6}\times\frac{1}{6}\times 2 = \frac{10}{36}$

$P(x=2) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$

probability distribution table of x is

X :	0	1	2
P(x) :	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

(1½)

Q. No. **Value Points** **Marks**

x_i	$p_i = P(x = x_i)$	$p_i x_i$	$p_i x_i^2$
0	$\frac{25}{36}$	0	0
1	$\frac{10}{36}$	$\frac{10}{36}$	$\frac{10}{36}$
2	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$
		$\Sigma p_i x_i = \frac{2}{36} = \frac{1}{3}$	$\Sigma p_i x_i^2 = \frac{14}{36} = \frac{7}{18}$

(1)

Now, mean (x) = $\Sigma p_i x_i = \frac{1}{3}$ (½)

var (x) $= \Sigma p_i x_i^2 = -(\Sigma p_i x_i)^2 = \frac{7}{18} - \frac{1}{9}$

$= \frac{5}{18}$ (½)

SECTION - C

23. (i) '*' is a binary operation on P(X)

as it is a function from P(X) × P(X) to P(X)

'*' is a binary operation on P(X) as (1)

A, B, P(X) ; A * B = A ∩ B also belongs to P(X)

(ii) '*' is commutative as,

A * B = A ∩ B = B ∩ A = B * A ∀ A, B ∈ P(X) (1)

(iii) '*' is associative as,

$$\left.\begin{aligned} A * (B * C) &= A \cap (B \cap C) \\ &= (A \cap B) \cap C \\ &= (A * B) * C \end{aligned}\right\} \forall A, B, C \in P(x)$$ (1)

(iv) X is the identity element is P(x) as, X * A = X ∩ A = A = A ∩ X = A * X ∀ A ∈ P(X) (1)

(v) Let A be an invertible element in P(x) w.r.t *

⇒ $\exists B \in P(x)$ s.t $A * B = B * A = X$ .... (i)

∴ A ∩ B = B ∩ A = X ⇒ X ⊂ A

⇒ X = A (as A ⊂ X)

Since no element of P(X) other than X satisfies (i), X is the only invertible element of P(X) w.r.t. * (1)

(vi) o is another binary operation of P(x) with A o B = A ∪ B

Now, $\left.\begin{aligned} A \text{ o } (B * C) &= A \cup (B \cap C) \\ &= (A \cup B) \cap (A \cup C) \\ &= (A \circ B) * (A \circ B) \end{aligned}\right] \forall A, B, C \in P(x)$ (1)

[or, (B * C) o A = (B o A) * (C o A)]

OR

Sol. f is 1 – 1

Let $x_1, x_2 \in R_+$ such that $f(x_1) = f(x_2)$

∴ $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$9(x_1^2 - x_2^2) + 5(x_1 - x_2) = 0$

$(x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$ (2)

⇒ $x_1 - x_2 = 0$ as $9x_1 + 9x_2 + 6 > 0$

∴ $x_1 = x_2$

f is onto

Let $y = \in [-5, \ ]$

Suppose $f(x) = y$ i.e. $9x^2 + 6x - 5 = y$

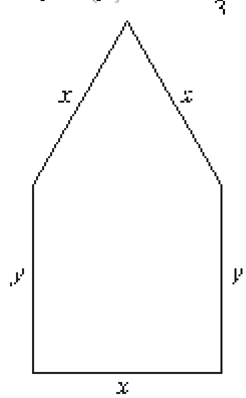
Solving for x to get $x = \frac{\sqrt{y+6} - 1}{3}$ (2½)

Since, $x = \frac{\sqrt{y+6} - 1}{3} \in R_+$ for $y \in (-5, \infty)$

∴ f is onto

∴ $f^{-1} : [-5, \infty) \to R_+$ is given as $f^{-1}(y) = \frac{\sqrt{y+6} - 1}{3}$ (1½)

24.



Q. No. **Value Points** **Marks**

$$3x + 2y = 12 \text{ (given)} \Rightarrow y = \frac{12 - 3x}{2}$$ (1)

Maximize $A = xy + \frac{\sqrt{3}}{4}x^2$

$$\Rightarrow \quad A = x\left(\frac{12 - 3x}{2}\right) + \frac{\sqrt{3}\,x^2}{4}$$ (1)

$$\Rightarrow \quad A = 6x - \frac{3x^2}{2} + \frac{\sqrt{3}}{4}x^2$$

$$\Rightarrow \quad \frac{dA}{dx} = 6 + \left(\frac{-6 + \sqrt{3}}{2}\right)x$$ (½)

$$\frac{dA}{dx} = 0 \quad \Rightarrow \quad x = \frac{-12}{-6 + \sqrt{3}} = \frac{12}{6 - \sqrt{3}}$$ (1)

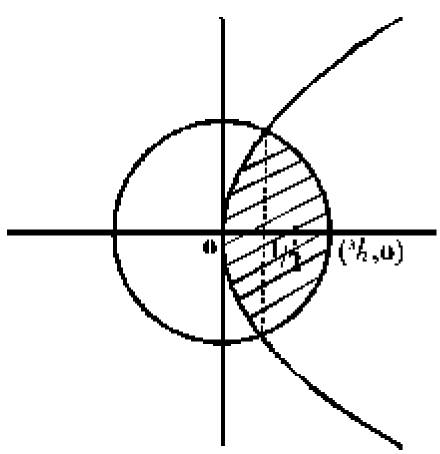
$$\frac{d^2A}{dx^2} = \frac{-6 + \sqrt{3}}{2} < 0$$ (1)

$$\Rightarrow \quad y = \frac{12 - 3x}{2} = \frac{1}{2}\left(12 - \frac{36}{6 - \sqrt{3}}\right)$$

$$= 6 - \frac{18}{6 - \sqrt{3}} = \frac{18 - 6\sqrt{3}}{6 - \sqrt{3}}$$ (1)

$\Rightarrow$ Area is maximum and dimensions of the window are $\frac{12}{6 - \sqrt{3}}$ m and $\frac{18 - 6\sqrt{3}}{6 - \sqrt{3}}$ m (½)

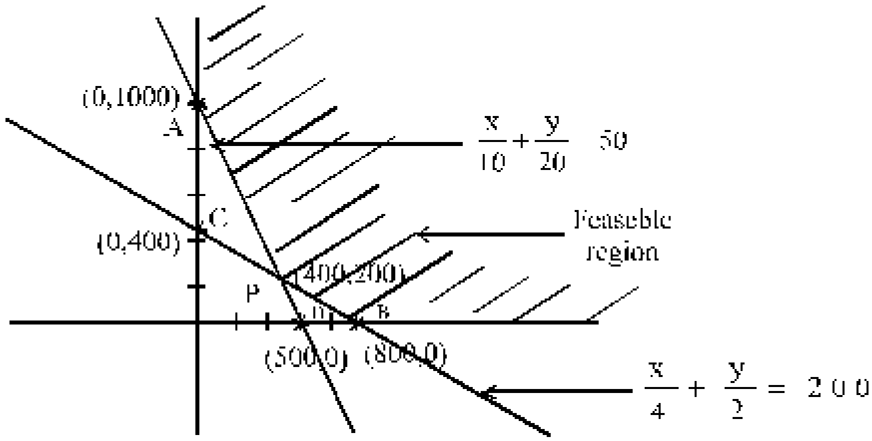
25.



(1½)

Q. No.	Value Points	Marks
	Point of intersection as $x = \frac{1}{2}$	(1)
	Required area = $\int_0^{\frac{1}{2}} 2\sqrt{x}\, dx + \frac{1}{2} \int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{9-4x^2}\, dx$	(1½)
	$= 2\left[4\frac{x^{3/2}}{3}\Big|_0^{1/2} + \frac{1}{4}\left(x\sqrt{9-4x^2} + \frac{9}{2}\text{Sin}^{-1}\frac{2x}{3}\right)_{1/2}^{3/2}\right]$	(1)
	$= 2\left[\frac{4}{3}\frac{1}{2\sqrt{2}} + \frac{1}{4}\left(\frac{9}{2}\frac{\pi}{2} - \sqrt{2} - \frac{9}{2}\text{Sin}^{-1}\frac{1}{3}\right)\right]$	
	$= 2\left[\frac{\sqrt{2}}{12} + \frac{9}{16}\pi - \frac{9}{8}\text{Sin}^{-1}\frac{1}{3}\right]$	(1)
26.	Suppose x gms of wheat and y gm of rice are mixed in the daily diet.	
	As per the data, x gms of wheat and y grams of rice will provide $0.1\,x + 0.5\,y$ gms of proteins	
	$\Rightarrow \quad 0.1x + 0.05y \geq 50$	
	$\Rightarrow \quad \frac{x}{10} + \frac{y}{20} \geq 50$	
	Similarly $0.25\,x + 0.5 \geq 200$	
	$\Rightarrow \quad \frac{x}{4} + \frac{y}{2} \geq 200$	
	Hence L.P.P. is	
	Minimize $Z = \frac{4x}{1000} + \frac{6y}{1000}$	(½)
	subject to the constraints	
	$\frac{x}{10} + \frac{y}{20} \geq 50$ $\frac{x}{4} + \frac{y}{2} \geq 200$ and $x \geq 0,\ y \geq 0$	(2)

Q. No.	Value Points	Marks



2

The feasible region is unbounded and has vertices A (0, 1000), B(800, 0) and P(400, 200)

Point Value of objective function, Z

(800, 0) $\frac{4}{1000}\times 800+\frac{8}{1000}\times 0=3.2$

(400, 200) $\frac{4}{1000}\times 400+\frac{6}{1000}\times 200=2.8$ (1)

(0, 1000) $\frac{4}{1000}\times 0+\frac{6}{1000}\times 1000=6$

Clearly Z is minimum for $x = 400$ (½)

i.e., Wheat = 400 gm and rice = 200 gm

27. We have

$$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$$

Now, $|A| = 1(1+3)+1(2+3)+1(2-1)=10\neq 0$ (1)

$\Rightarrow$ A^{-1} exists

$\Rightarrow$ matrix of cofactors $C=\begin{bmatrix}4 & -5 & 1\\ 2 & 0 & -2\\ 2 & 5 & 3\end{bmatrix}$ (1½)

$\Rightarrow$ adj $A = C^t = \begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$ (½)

$$\Rightarrow A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$ (½)

Now the given system of equations is expressible as

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$\Rightarrow A^t X = B$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ (½)

and, $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$$\Rightarrow X = (A^t)^{-1}.B$$ (½)

$$= (A^{-1})^t.B \qquad \left(\text{as } (A^t)^{-1} = (A^{-1})^t\right)$$ (½)

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}^t \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$ (½)

$\Rightarrow x = \frac{9}{5}, y = \frac{2}{5}, z = \frac{7}{5}$ in the required solution. (½)

28. $\vec{n} = (\hat{i} + 2\hat{j} - \hat{k}) \times (-\hat{i} + \hat{j} - 2\hat{k})$ is normal to the plane containing

$\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ .... (1)

and $\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ .... (2)

Q. No. **Value Points** **Marks**

$$\Rightarrow \quad \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(-4+1) - \hat{j}(-2-1) + \hat{k}(1+2)$$

$$\vec{n} = -3\hat{i} + 3\hat{j} + 3\hat{k}$$ (2)

Both lines (1) and (2) pass through $\hat{i} + \hat{j}$

$\Rightarrow$ eqns. of the plane containing (1) & (2)

is, $\vec{r}.\vec{n} = (\hat{i} + \hat{j}).\vec{n}$ (1)

is, $\vec{r}(-3\hat{i} + 3\hat{j} + 3\hat{k}) = (\hat{i} + \hat{j}).(-3\hat{i} + 3\hat{j} + 3\hat{k})$

$\Rightarrow$ $\vec{r}(-3\hat{i} + 3\hat{j} + 3\hat{k}) = -3 + 3$

$\Rightarrow$ $\vec{r}(-3\hat{i} + 3\hat{j} + 3\hat{k}) = 0$ (1)

or, $-3x + 3y + 3z = 0$ or $x - y - z = 0$

Since this plane contains the origin, its distance from origin is zero. (1)

Distance of the plane $\vec{r}.(-3\hat{i} + 3\hat{j} + 3\hat{k}) = 0$ from (1, 1, 1) i.e $(\hat{i} + \hat{j} + \hat{k})$, is

$$= \left| \frac{(\hat{i} + \hat{j} + \hat{k}).(-3\hat{i} + 3\hat{j} + 3\hat{k})}{|-3\hat{i} + 3\hat{j} + 3\hat{k}|} \right| = \left| \frac{-3+3+3}{\sqrt{9+9+9}} \right| = \frac{3}{\sqrt{27}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ units}$$ (1)

OR

Equation of family of planes passing through the intersection of $2x + 3y - z + 1 = 0$; and $x + y - 2z + 3 = 0$ is,

$$2x + 3y - z + 1 + \lambda(x + y - 2z + 3) = 0$$ .... (i) (1)

as plane (i) is perpendicular to plane $x + y - 2z + 3 = 0$

$\therefore$ $(2+\lambda).3 + (3+\lambda)(-1) + (-1-2\lambda)(-2) = 0$ (1½)

$\Rightarrow$ $6 + 3\lambda - 3 - \lambda + 2 + 4\lambda = 0$

$\Rightarrow$ $5 + 6\lambda = 0 \Rightarrow \lambda = -\frac{5}{6}$ (½)

Q. No.	Value Points	Marks

Substituting in (i), are obtain,

$$\left(2-\frac{5}{6}\right)x+\left(3-\frac{5}{6}\right)y+\left(-1+2\times\frac{5}{6}\right)z+1+3\left(\frac{-5}{6}\right)=0$$

$\Rightarrow$ $7x+13y+4z=9$ is the required plane. (1)

Let θ be the inclination of this plane with *xy* plane (i.e. z= 0) (½)

$\Rightarrow$ $\cos\theta=\left|\dfrac{7\times0+13\times0+4\times1}{\sqrt{234}\ \ \sqrt{1}}\right|=\dfrac{4}{3\sqrt{26}}$ (1)

$\Rightarrow$ $\theta=\cos^{-1}\left(\dfrac{4}{3\sqrt{26}}\right)$ (½)

29. Equations of the line perpendicular the plane $3x-y+4z=2$ .... (i)

and passing through (3, –2, 1)

is $\dfrac{x-3}{3}=\dfrac{y+2}{-1}=\dfrac{z-1}{4}=\lambda$ .... (ii) (½)

Now, $(3\lambda+3,-\lambda-2,4\lambda+1)$ is a general point in (ii)

The foot of perpendicular from (3, –2, 1) to the plane (i) .... (i)

is the point of intersection of (ii) and (i) (1)

$\Rightarrow$ $3(3\lambda+3)-1(-\lambda-2)+4(4\lambda+1)=2$

$\Rightarrow$ $26\lambda=-13$ $\Rightarrow$ $\lambda=\dfrac{-1}{2}$ (1)

$\Rightarrow$ foot of perpendieular $\left(\dfrac{3}{2},-\dfrac{3}{2},-1\right)$ (½)

Let Image of (3, –2, 1) in the given plane be (x_1, y_1, z_1)

$\Rightarrow$ $\dfrac{x_1+3}{2}=\dfrac{3}{2},\ \dfrac{y_1-2}{2}=\dfrac{-3}{2}\ ;\ \dfrac{z_1+1}{2}=-1$ (1)

$\Rightarrow$ $x_1=0,\ y_1=-1,\ z_1=-3$

$\Rightarrow$ Image of (3, –2, 1) in the plane $3x-y+4z=2$ is $(0,-1,-3)$ (1)

which lies on the plane $x+y+z+4=0$ (as $0-1-3+4=0$) (1)

BLUE PRINT - III
MATHEMATICS - XII

S.No.	Topic	VSA	SA	LA	TOTAL
1. (a)	Relations and Functions	1 (1)	4 (1)	-	10 (4)
(b)	Inverse trigonometric Functions	1 (1)	4 (1)	-	
2. (a)	Matrices	2 (2)	-	6 (1)	-
(b)	Determinants	1 (1)	4 (1)	-	13 (5)
3. (a)	Continuity and Differentiability	-	12 (3)	-	}18 (4)
(b)	Applications of Derivatives	-	-	6 (1)	44 (11)
(c)	Integrals	2 (2)	4 (1)	6 (1)	
(d)	Applications of Integrals			6 (1)	}18 (5)
(e)	Differential equations		8 (2)		8 (2)
4. (a)	Vectors	3 (3)	4 (1)	-	
(b)	3 - dimensional geometry	-	4 (1)	6 (1)	17 (6)
5.	Linear - Programming	-	-	6 (1)	6 (1)
6.	Probability	-	4 (1)	6 (1)	10 (2)
	Total	10 (10)	48 (12)	42 (7)	100 (29)

Sample Question Paper - III

Time : 3 Hours **Max. Marks : 100**

General Instructions :

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 mark each, section B is of 12 questions of 4 marks each and section C is of 7 questions of 6 marks each.
3. There is no overall choice. However, an internal choice has been provided in four questions of 4 marks each and two questions of six marks each.
4. Use of calculators is not permitted. However, you may ask for Mathematical tables.

SECTION - A

1. Let $f : R - \left\{-\frac{3}{5}\right\} \to R$ be a function defined as $f(x) = \frac{2x}{5x+3}$, find f^{-1} : Range of $f \to R - \left\{-\frac{3}{5}\right\}$

2. Write the range of one branch of $\sin^{-1}x$, other than the Principal Branch.

3. If $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$, find x, $0 < x < \frac{\pi}{2}$ when $A + A' = I$

4. If B is a skew symmetric matrix, write whether the matrix (ABA') is symmetric or skew symmetric.

5. On expanding by first row, the value of a third order determinant is $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$. Write the expression for its value on expanding by 2nd column. Where A_{ij} is the cofactor of element a_{ij}.

6. Write a value of $\int \frac{1+\cot x}{x + \log \sin x}\,dx$

7. Write the value of $\int_0^{\pi/2} \log\left[\frac{3+5\cos x}{3+5\sin x}\right] dx$

8. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector. Then what is the angle between $\vec{a}$ and $\vec{b}$?

9. Write the value of $\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{k}\times\hat{i}) + \hat{k}.(\hat{j}\times\hat{i})$

10. For two non zero vectors $\vec{a}$ and $\vec{b}$ write when $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right|$ holds.

SECTION - B

11. Show that the relation R in the set $A = \{x \mid x \in W, 0 \le x \le 12\}$ given by $R = \{(a, b) : (a - b) \text{ is a multiple of } 4\}$ is an equivalence relation. Also find the set of all elements related to 2.

OR

Let * be a binary operation defined on N × N, by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Also find the identity element for * on N × N, if any.

12. Solve for x :

$$\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}, \quad |x|<1$$

13. If a, b and c are real numbers and

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Show that either a + b + c = 0 or a = b = c.

14. If $f(x) = \begin{cases} \dfrac{x-5}{|x-5|} + a, & \text{if } x < 5 \\ a + b, & \text{if } x = 5 \\ \dfrac{x-5}{|x-5|} + b, & \text{if } x < 5 \end{cases}$

is a continuous function. Find a, b.

15. If $x^y + y^x = \log a$, find $\frac{dy}{dx}$.

16. Use lagrange's Mean Value theorem to determine a point P on the curve $y = \sqrt{x-2}$ where the tangent is parallel to the chord joining (2, 0) and (3, 1).

17. Evaluate : $$\int \frac{1}{\cos(x-a)\cos(x-b)}\,dx$$

OR

Evaluate : $$\int \frac{2+\sin x}{1+\cos x} \cdot e^{x/2} \cdot dx.$$

18. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then prove that $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$

OR

If are the dia gonals of a parallelogram with sides , $\vec{a}$ and $\vec{b}$ ind the area of parallelogram in terms of and hence find the area with $\vec{d_1} = i + 2\hat{j} + 3\hat{k}$ and $\vec{d_2} = 3i - 2\hat{j} + k$.

19. Find the shortest distance between the lines, whose equations are

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{10-z}{-7} \text{ and } \frac{x-15}{3} = \frac{58-2y}{-16} = \frac{z-5}{-5}.$$

20. A bag contains 50 tickets numbered 1, 2, 3, ..., 50 of which five are drawn at random and arranged in ascending order of the number appearing on the tickets $(x_1 < x_2 < x_3 < x_4 < x_5)$. Find the probability that $x_3 = 30$.

21. Show that the differential equation

$$2y\, e^{x/y} dx + \left(y - 2x\, e^{x/y}\right) dy = 0 \text{ is}$$

homogeneous and find its particular solution given that $x = 0$ when $y = 1$.

OR

Find the particular solution of the differential equation $\frac{dx}{dy} + y\cot x = 2x + x^2 \cot x, \ \ x \neq 0$

given that y = 0, when $x = \frac{\pi}{2}$

22. Form the differential equation representing the family of ellipses having foci on x-axis and centre at origin.

SECTION - C

23. A letter is known to have come from either TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probabiity that the letter has come from

(1) Tata nagar (ii) Calcutta

OR

Find the probability distribution of the number of white balls drawn in a random draw of 3 balls without replacement from a bag containing 4 white and 6 red balls. Also find the mean and variance of the distribution.

24. Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.

25. Using integration, compute the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$

OR

Find the ratio of the areas into which curve $y^2 = 6x$ divides the region bounded by $x^2 + y^2 = 16$.

26. Evaluate :

27. A point on the hypotenuse of a right triangle is at a distance 'a' and 'b' from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left[a^{2/3}+b^{2/3}\right]^{\frac{3}{2}}$.

28. Using elementary transformations, find the inverse of the matrix

$$\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$$

29. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically.

$$\int \frac{e^{\tan^{-1} x}}{\left(1+x^2\right)^2}\,dx.$$

Marking Scheme

Sample Paper - III

Q. No. **Value Points** **Marks**

SECTION - A

1. $f^{-1}(x) = \frac{3x}{2-5x}$ 2. $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (or any other equivalent) 3. $x = \frac{\pi}{3}$

4. Skew symmetric 5. $a_{12}A_{12} + a_{22}A_{22} + A_{32}A_{32}$ 6. log | x + log sin x | + c

7. Zero. 8. $\frac{\pi}{4}$ 9.1

10. $\vec{a}$ and $\vec{b}$ are like parallel vectors. $1 \times 10 = 10$

SECTION - B

1. $f^{-1}(x) = \frac{3x}{2-5x}$ 2. $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (or any other equivalent) 3. $x = \frac{\pi}{3}$

11. (i) since (a – a) = 0 is a multiple of 4, ∀ a ∈ A ∴ R is reflexive ½

(ii) (a, b) ∈ R ⇒ (a – b) is a multiple of 4 ⇒ (b – a) is also a multiple of 4

⇒ (b, a) ∈ R ∀ a, b ∈ A ⇒ R is Symmetric 1

(iii) (a, b) ∈ R and (b, c) ∈ R ⇒ (a – b) = 4k, k ∈ z

(b – c) = 4m, m ∈ z ∀ a, b, c ∈ A 1

⇒ (a – c) = 4(k + m), (k + m) ∈ z ∴ (a, c) ∈ R

⇒ R is transitive ½

Set of all elements related to 2 are

{2, 6, 10} 1

OR

(i) ∀ a, b, c, d ∈ N, (a, b) * (c, d) = (a + c, b + d)

= (c + a, d + b)

= (c, d) * (a, b)

⇒ * is commutative 1

(ii) [(a, b) * (c, d)] * (e, *f*) = (a + c, b + d) * (e, *f*)

= ((a + c) + e, (b + d) + *f*)

= (a + c + e, b + d + *f*)

= (a + (c + e), b + (d + *f*)) ∀ a, b, c, d, e, f, ∈ N

⇒ = (a, b) * [(c, d) * (e, *f*)] * is associative 1½

Q. No. | **Value Points** | **Marks**

(iii) Let (e, *f*) be the identity element, then

(a, b) * (e, *f*) = (a, b) — ½

$\Rightarrow$ (a + e, b + *f*) = (a, b) $\Rightarrow$ e = 0, *f* = 0

but (0, 0) $\notin$ N × N

So, identity element does not exist — 1

12. We have $\tan^{-1}\left\{\dfrac{\dfrac{x-1}{x-2}+\dfrac{x+1}{x+2}}{1-\dfrac{x-1}{x-2}\cdot\dfrac{x+1}{x+2}}\right\}=\dfrac{\pi}{4}$ — 1½

$\Rightarrow$ $\tan^{-1}\left\{\dfrac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}\right\}=\dfrac{\pi}{4}$ — 1

$\Rightarrow$ $\dfrac{2x^2-4}{-3}=1$ — 1

$\Rightarrow$ $2x^2=1 \Rightarrow x^2=\dfrac{1}{2},\ x=\pm\dfrac{1}{\sqrt{2}}$ — ½

13. $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}=0$ $\quad c_1\to c_1+c_2+c_3 \Rightarrow \quad 2(a+b+c)\begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix}=0$ — 1

$R_2\to R_2-R_1$
$R_3\to R_3-R_1$ $\Rightarrow$ $2(a+b+c)\begin{vmatrix} 1 & c+a & ab \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix}=0$ — 1½

$\Rightarrow$ $2(a+b+c)\left(-a^2-b^2-c^2+ab+bc+ca\right)=0$

$\Rightarrow$ $-(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]=0$ — ½

$\Rightarrow$ $a+b+c=0$ or $a=b=c$ — 1

14. $\underset{x\to 5^-}{\text{dim}}\, f(x)=-1+a$ — 1

$f(5) = a + b$

$\underset{x\to 5^+}{\text{dim}}\, f(x)=7+b$ — 1

$\Rightarrow$ $-1 + a = a + b = 7 + b$ — 1

$\Rightarrow$ $b = -1,\ a = 7$ — ½,½

Q. No.	Value Points	Marks
15.	Let $u = x^y$ and $v = y^x \Rightarrow u + v = \log a$	
	$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$ .... (i)	½
	$\log u = y \log x$	
	$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{y}{x} + \log x \frac{dy}{dx}$	
	$\Rightarrow \frac{du}{dx} = x^y\left[\frac{y}{x} + \log x \frac{dy}{dx}\right]$	1½
	$\log v = x \log y$	
	$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$	
	$\Rightarrow \frac{dv}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right]$	1
	(i) $\Rightarrow yx^{y-1} + x^y \log x \frac{dy}{dx} + xy^{x-1}\frac{dy}{dx} + y^x \log y = 0$	½
	$\Rightarrow \frac{dy}{dx} = -\left[\frac{y.x^{y-1} + y^x.\log y}{x.y^{x-1} + x^y.\log x}\right]$	½
16.	(i) Since $(x-2) \geq 0$ in [2, 3] so $f(x) = \sqrt{x-2}$ is continuous	1½
	(ii) $f'(x) = \frac{1}{2\sqrt{x-2}}$ exists for all $x \in (2, 3)$ $\therefore$ $f(x)$ is differentiable in (2, 3)	
	Thus lagrang's mean value theorem is applicable;	
	$\therefore$ There exists atleast one real number c in (2, 3) such that	
	$f'(c) = \frac{f(3) - f(2)}{3-2}$ or $\frac{1}{2\sqrt{c-2}} = \frac{(1) - 0}{1} \Rightarrow 2\sqrt{c-2} = 1$	1½
	$c = 2 + \frac{1}{4} = 2.25 \in (2, 3) \Rightarrow$ LMV is verified and the	
	req. point is (2.25, 0.5)	1
17.	$I = \int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(b-a)}\int \frac{\sin((x-a)-(x-b))}{\cos(x-a)\cos(x-b)}dx$	1
	$= \frac{1}{\sin(b-a)}\int [\tan(x-a) - \tan(x-b)]dx$	1½

Q. No.	Value Points	Marks
	$= \frac{1}{\sin(b-a)}\left[\log\left|\sec(x-a)\right| - \log\left|\sec(x-b)\right|\right] + c$	1½
	or, $= \frac{1}{\sin(b-a)}\left[\log\left|\frac{\sec(x-a)}{\sec(x-b)}\right|\right] + c$	

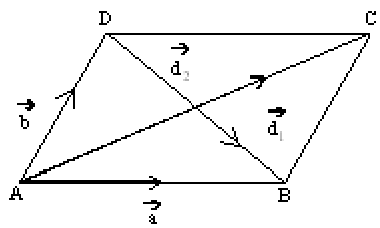
OR

	$I = \int \frac{2+\sin x}{1+\cos x} e^{x/2}dx$	
	$= \int \left[\frac{2}{1+\cos x} + \frac{\sin x}{1+\cos x}\right] e^{x/2}dx$	½
	$= \int \left[\frac{2}{2\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right] e^{x/2}dx$	1
	$= \int \left(\sec^2\frac{x}{2} + \tan\frac{x}{2}\right) e^{x/2}dx$	1
	$2\tan\frac{x}{2}.e^{x/2} + c \qquad \left[\because \int f(x) + f'(x)e^x dx = f(x)e^x + c\right]$	1½
18.	$\left\|\vec{a}+\vec{b}\right\|^2 = \left(\vec{a}+\vec{b}\right)^2 = \vec{a}^2 + \vec{b}^2 + 2\,\vec{a}.\vec{b}$	1
	$= \left\|\vec{a}\right\|^2 + \left\|\vec{b}\right\|^2 + 2\left\|\vec{a}\right\|\left\|\vec{b}\right\|\cos\theta$	
	$= 1 + 1 + 2.1.1.\cos\theta$	1
	$= 2(1+\cos\theta) = 2.2\cos^2\frac{\theta}{2}$	1
	$\Rightarrow \quad \frac{1}{4}\left\|\vec{a}+\vec{b}\right\|^2 = \cos^2\frac{\theta}{2} \quad \Rightarrow \quad \cos\frac{\theta}{2} = \frac{1}{2}\left\|\vec{a}+\vec{b}\right\|$	1

OR

Q. No. **Value Points** **Marks**

Let ABCD be the parallelogram with sides $\vec{a}$ and $\vec{b}$



$\therefore$ $\vec{d}_1 = \vec{AC} = \vec{a} + \vec{b}$ and $\vec{d}_2 = \vec{DB} = \vec{a} - \vec{b}$ 1

Now $\left|\vec{d}_1 \times \vec{d}_2\right| = \left|\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)\right| = 2\left|\vec{a} \times \vec{b}\right|$

$\Rightarrow$ area| |gm $= \left|\vec{a} \times \vec{b}\right| = \frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right|$ 1

When $\vec{d}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{d}_2 = 3\hat{i} = 2\hat{j} + \hat{k}$

$\Rightarrow$ $\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \tilde{k} \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = 8\hat{i} + 8\hat{j} - 8\hat{k}$ 1

$\therefore$ area of 11 gm $= \frac{1}{2}\left[\sqrt{8^2 + 8^2 + 8^2}\right] = 4\sqrt{3}$ sq. u 1

19. The given equations can be written as

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$ 1

The shortest distance between two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is given by

$$\text{S.D.} = \frac{\left(\vec{a}_2 - \vec{a}_1\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$ ½

Here $\vec{a}_1 = (8, -9, 10)$, $\vec{a}_2 = (15, 29, 5) \Rightarrow \vec{a}_2 - \vec{a}_1 = (7, 38, -5)$

and $\vec{b}_1 = (3, -16, 7)$ and $\vec{b}_2 = (3, 8, -5) \Rightarrow \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$

Q. No.	Value Points	Marks
	$= 24\hat{i} + 36\hat{j} + 72\hat{k}$	1
	$\therefore$ S.D. $= \dfrac{168 + 1368 - 360}{\sqrt{576 + 1296 + 5184}} = \dfrac{1176}{84} = 14$ units.	1½
20.	Since $x_3 = 30$ $\therefore$ $x_1, x_2 < 30$ and $x_4, x_5 > 30$	1
	$\therefore$ Required Probability is	2
	$= {}^{29}c_2 \cdot {}^{1}c_1 \cdot {}^{20}c_2 / {}^{50}c_5$	
	$= \dfrac{\frac{29.28}{2.1} \cdot \frac{1}{1} \cdot \frac{20.19}{2.1}}{\frac{50.49.48.47.46}{5.4.3.2.1}}$	
	$= \dfrac{551}{15134}$	1
21.	Here $\dfrac{dx}{dy} = \dfrac{2x\, e^{x/y} - y}{2y\, e^{x/y}}$	
	Let $F(x, y) = \dfrac{2x\, e^{x/y} - y}{2y\, e^{x/y}}$ then $F(\lambda x, \lambda y) = \dfrac{\lambda\left(2x e^{x/y} - y\right)}{\lambda\left(2y e^{x/y}\right)}$	½
	$\Rightarrow F(x, y)$ is a homogeneous function of degree zero, thus the given differential equation is a homogeneous differential equation	½
	Put $x = vy$ to get $\dfrac{dx}{dy} = v + y\dfrac{dv}{dy}$	
	$\therefore$ $v + y\dfrac{dv}{dy} = \dfrac{2v\, e^{v} - 1}{2\, e^{v}}$ or $y\dfrac{dv}{dy} = \dfrac{2v\, e^{v} - 1 - 2v\, e^{v}}{2\, e^{v}} = -\dfrac{1}{2e^{v}}$	1
	$\therefore$ $2\, e^{v}\, dv = -\dfrac{dy}{y}$ $\Rightarrow$ $2\, e^{v} + \log\lvert y\rvert = c$	1
	$x = 0, y = 1$ or, $2\, e^{x/y} + \log\lvert y\rvert = c$	
	$\Rightarrow$ $c = 2$ $\therefore$ $2e^{x/y} + \log\lvert y\rvert = 2$	1
	OR	
	Here integrating factor $= e^{\int \cot x\, dx} = e^{\log \sin x} = \text{Sin } x$	1
	$\therefore$ the solution of differential equation is given by	
	y. sin x $= \int \left(2x + x^2 \text{Cot } x\right) \text{Sin } x\, dx$	1

Q. No.	Value Points	Marks

$$= \int 2x \operatorname{Sin} x\,dx + \int x^2 \operatorname{Cos} x\,dx$$

$$= \int 2x \operatorname{Sin} x\,dx + x^2 \sin x - \int 2x \sin x\,dx + c$$

$$= x^2 \sin x + c$$ ....(1) 1

Substituting y = o and x = $\pi/2$, we get

$$o = \frac{\pi^2}{4} + Lc \quad \text{or} \quad c = -\pi^2/4$$ ½

$\therefore$ (i) => $y \sin x = x^2 \sin x - \pi^2/4$

or $y = x^2 - \frac{\pi^2}{4} \operatorname{Cosec} x$ ½

22. Equation of the said family is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ 1

Differentiating w.r.t x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0 \text{ or } \frac{y}{x}\frac{dy}{dx} = -\frac{b^2}{a^2}$$ y.sin x 1½

$$\left.\begin{aligned} &\left(\frac{y}{x}\right)\frac{d^2y}{dx^2} + \frac{x\frac{dy}{dx} - y}{x^2}\left(\frac{dy}{dx}\right) = 0 \\ &\text{or} \quad xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0 \end{aligned}\right\}$$ 1½

SECTION - C

23. Let E_1 : Letter has come from tatanagar $\therefore$ $P(E_1) = \frac{1}{2}$ ½

E_2 : Letter has come from calcutta $P(E_2) = \frac{1}{2}$

A: Obtaining two consecutive letters "TA"

$\therefore$ $P(A|E_1) = \frac{2}{8} = \frac{1}{4}$ {Total possiblities TA, AT, TA, AN, NA, AG, GA, AR = 8, favourable = 2} 1 + 1

$P(A|E_2) = \frac{1}{7}$ {Total possiblities CA, AL, LC, CU, UT, TT, TA = 87 favourable = 1} 1

Q. No.	**Value Points**	**Marks**

$\therefore \quad P(E_1 | A) = \dfrac{P(E_1)\,P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$ ½

$$= \frac{\frac{1}{2}.\frac{1}{4}}{\frac{1}{2}.\frac{1}{4}+\frac{1}{2}.\frac{1}{7}} = \frac{7}{11}$$ 1

$\therefore \quad P(E_2 | A) = 1 - \dfrac{7}{11} = \dfrac{4}{11}$ 1

OR

Let x = Number of white balls.

$\therefore \quad P(x=0) = \dfrac{{}^{6}C_3}{{}^{10}C_3} = \dfrac{6.5.4}{10.9.8} = \dfrac{1}{6}$

$P(x=1) = \dfrac{{}^{4}C_1 . {}^{6}C_2}{{}^{10}C_3} = \dfrac{4.3.6.3}{10.9.8} = \dfrac{1}{2}$

$P(x=2) = \dfrac{{}^{4}C_2 . {}^{6}C_1}{{}^{10}C_3} = \dfrac{4.3.6.3}{10.9.8} = \dfrac{3}{10}$

$P(x=3) = \dfrac{{}^{4}C_3}{{}^{10}C_3} = \dfrac{4.3.2}{10.9.8} = \dfrac{1}{30}$

½ × 4 = 2

Thus we have

x	$P(x)$	$x\,P(x)$	$x^2 P(x)$
0	$\frac{1}{6}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{12}{10}$
3	$\frac{1}{30}$	$\frac{3}{30}$	$\frac{9}{30}$
	1	$36/30$	$60/30 = 2$

correct $\Sigma x\,P(x)$ 1

correct $\Sigma x^2 P(x)$ 1

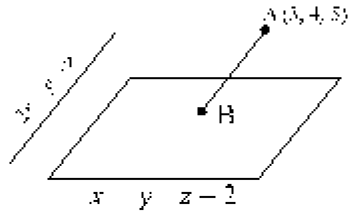
Mean $= \Sigma\, xP(x) = \dfrac{36}{30} = \dfrac{18}{15} = \dfrac{6}{5} = 1.2$

Variance $= \Sigma x^2 P(x) - [\Sigma xP(x)]^2$ 1

$= 2 - \dfrac{36}{25} = \dfrac{14}{25}$ or 0.56 1

Q. No. **Value Points** **Marks**

24.



AB is parallel to the line $2x = y = z$

or $\frac{x}{1/2} = \frac{y}{1} = \frac{z}{1}$ ½

or $\frac{x}{1} = \frac{y}{2} = \frac{z}{2}$

$\Rightarrow$ Equation of AB is $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ 1

For some value of λ, B is $(\lambda + 3, 2\lambda + 4, 2\lambda + 5)$ 1

B lies on plane $\therefore$ $\lambda + 3 + 2\lambda + 4 + 2\lambda + 5 - 2 = 0$ $\Rightarrow$ $5\lambda = -10$, $\lambda = 2$ 1

$\therefore$ B is (1, 0, 1) 1

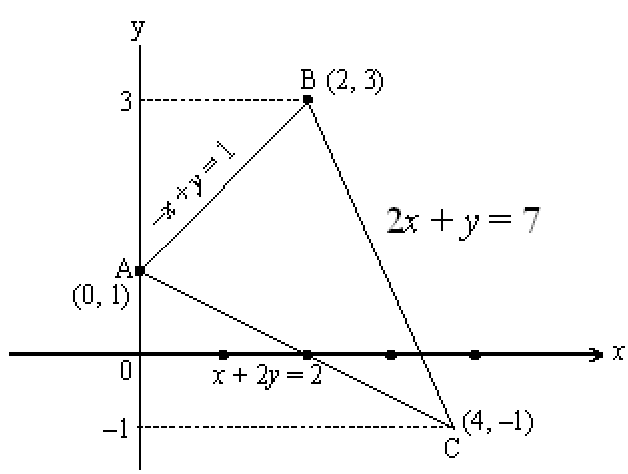
$AB = \sqrt{(3-1)^2 + (4-0)^2 + (5-1)^2}$ 1

= 6 units. ½

25. Solving the equations in pairs to get the vertices of Δ as

(0, 1), (2, 3) and (4 – 1) ½ × 3 = 1½

For correct figure 1



Q. No.	Value Points	Marks

Required area

$$= \frac{1}{2}\int_{-1}^{3}(7-y)dy - \int_{-1}^{1}(2-2y)dy - \int_{0}^{3}(y-1)dy$$ 1½

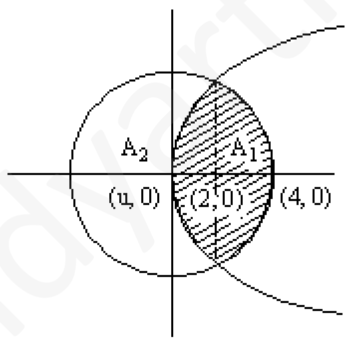
$$= \frac{1}{2}\left[7y-\frac{y^2}{2}\right]_{-1}^{3} - \left[2y-y^2\right]_{-1}^{1} - \left[\frac{y^2}{2}-y\right]_{1}^{3}$$ 1½

= 12 – 4 – 2 = 6 sq. U. ½

OR

Correct figure 1

Area $A_1 = 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16-x^2}\,dx\right]$ 2



$$= 2\left[\left(\sqrt{6}.\frac{2x^{3/2}}{3}\right)_0^2 + \left(\frac{x}{2}\sqrt{16-x^2}+8\sin^{-1}\frac{x}{4}\right)_2^4\right]$$

$$= \frac{4\sqrt{3}+16\pi}{3} \text{ sq. U.}$$ 1

A_2 = Area of circle – shaded area ½

$$16\pi - \frac{4\sqrt{3}+16\pi}{3} = \frac{32\pi-4\sqrt{3}}{3}$$ ½

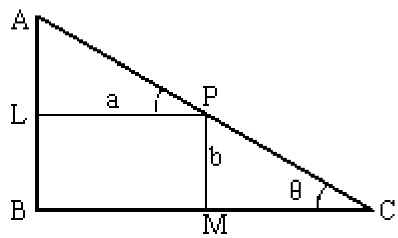
$\therefore$ $$\frac{A_1}{A_2} = \frac{16\pi+4\sqrt{3}}{32\pi+4\sqrt{3}} = \frac{4\pi+\sqrt{3}}{8\pi-\sqrt{3}}$$ 1

Q. No.	Value Points	Marks
26.	$I=\int e^{\tan^{-1}x}.\frac{1}{(1+x^2)^2}dx$	
	Put $x=\tan\theta$ to get $I=\int e^{\theta}.\cos^2\theta d\theta$	
	$=\frac{1}{2}\int e^{\theta}(1+\cos 2\theta)d\theta$	1
	$=\frac{1}{2}e^{\theta}+\frac{1}{2}\int e^{\theta}.\cos 2\theta d\theta$	
	$=\frac{1}{2}e^{\theta}+I_1$ .... (i)	½
	$I_1=\frac{1}{2}\int e^{\theta}.\cos 2\theta\, d\theta$	
	$=\frac{1}{2}.\left[e^{\theta}.\cos 2\theta-\int -2\sin 2\theta.e^{\theta}d\theta\right]$	1
	$=\frac{1}{2}\left[e^{\theta}\cos 2\theta+2\left(\sin 2\theta.e^{\theta}-\int 2\cos 2\theta.e^{\theta}d\theta\right)\right]$	½
	$=\frac{1}{2}\left[e^{\theta}\cos 2\theta+2\sin 2\theta e^{\theta}\right]-4.\frac{1}{2}\int \cos 2\theta e^{\theta}\,d\theta$	
	$I_1=\frac{1}{2}e^{\theta}\cos 2\theta+\sin 2\theta e^{\theta}-4I_1$	1
	$\Rightarrow \quad I_1=\frac{1}{10}e^{\theta}\cos 2\theta+\frac{1}{5}\sin 2\theta e^{\theta}$	½
	Putting in (i) we get	
	$I=\frac{1}{2}e^{\theta}+\frac{1}{10}e^{\theta}\cos 2\theta+\frac{1}{5}\sin 2\theta e^{\theta}+c$ $=\frac{1}{10}e^{\theta}\left[5+\cos 2\theta+2\sin 2\theta\right]+c$	½
	$=\frac{1}{10}e^{\tan^{-1}x}.\left[5+\frac{1-x^2}{1+x^2}+\frac{4x}{1+x^2}\right]+c$	1

Q. No.	Value Points	Marks

For correct figure 1

27.



Let $\angle c = \theta$.

$\therefore$ AC = AP + PC = S (say)

$\therefore$ $S = a \sec\theta + b \operatorname{cosec}\theta$ 1

$\therefore$ $\frac{ds}{d\theta} = a\sec\theta\tan\theta - b\operatorname{cosec}\theta \cot\theta$

$\frac{ds}{d\theta} = 0 \Rightarrow \frac{a\sin\theta}{\cos^2\theta} = \frac{b\cos\theta}{\sin^2\theta}$ or $\tan^3\theta = \frac{b}{a}$ or $\tan\theta = \left(\frac{b}{a}\right)^{1/3}$ 1

$$\frac{d^2s}{d\theta^2} = a\left[\sec^3\theta + \sec\theta\tan^2\theta\right] + b\left[\operatorname{cosec}^3\theta + \operatorname{cosec}\theta\cot^2\theta\right]$$

Which is +ve as a, b > 0 and θ is acute 1

$\therefore$ S is minimum when $\tan\theta = \left(\frac{b}{a}\right)^{1/3}$

$\therefore$ Minimum $S = AC = a\sqrt{1+\tan^2\theta} + b\sqrt{1+\cot^2\theta}$ 1

$$= a\sqrt{1+\left(\frac{b}{a}\right)^{2/3}} + b\sqrt{1+\left(\frac{a}{b}\right)^{2/3}}$$

$$= a^{2/3}\sqrt{a^{2/3}+b^{2/3}} + b^{2/3}\sqrt{b^{2/3}+a^{2/3}}$$ 1

$$= \left(a^{2/3}+b^{2/3}\right)^{3/2}$$

28. Let $A = \begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$

Q. No.	Value Points	Marks

Writing $A = IA$ or $\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A.$ 1

Operating $R_2 \to R_2 + 3R_1$, we have
$R_3 \to R_3 - 2R_1$

$$\begin{pmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} A$$

$$R_1 \to R_1 + 3R_3 \begin{pmatrix} 1 & 0 & 10 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 3 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} A$$

$$R_2 \to R_2 + 8R_3 \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{pmatrix} A$$

$$R_3 \to R_3 + R_2 \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{pmatrix} A$$

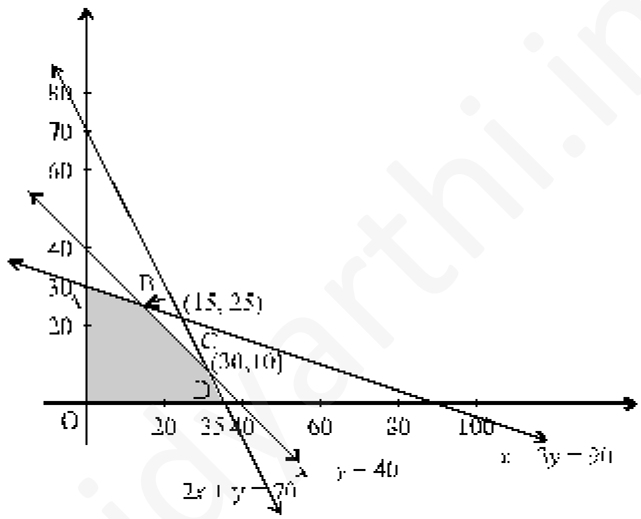
$$R_3 \to \frac{1}{25} R_3 \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -\frac{15}{25} & \frac{1}{25} & \frac{9}{25} \end{pmatrix} A$$

$$\begin{matrix} R_1 \to R_1 - 10R_3 \\ R_2 \to R_2 - 21R_3 \end{matrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{-10}{25} & \frac{-15}{25} \\ \frac{-10}{25} & \frac{4}{25} & \frac{11}{25} \\ \frac{-15}{25} & \frac{1}{25} & \frac{9}{25} \end{pmatrix} A$$

$8 \times ½ = 4$

$$\therefore \quad A^{-1} = \begin{pmatrix} 1 & \frac{-10}{25} & \frac{-15}{25} \\ \frac{-10}{25} & \frac{4}{25} & \frac{11}{25} \\ \frac{15}{25} & \frac{1}{25} & \frac{9}{25} \end{pmatrix} \text{ or } \frac{1}{25}\begin{pmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{pmatrix}$$ 1

Q. No.	Value Points	Marks
29.	Let number of chairs = x	
	number of tables = y	
	∴ LPP is Maximise P = 30 x + 60y	½
	Subject to $\left\{\begin{array}{l} 2x+y\le 70 \\ x+y\le 40 \\ x+3y\le 90 \end{array}\right\}$	1½
	$x\ge 0 \quad y\ge 0$	
	For correct graph	3



P = 30 (x + 2y)

$P_{(A)}$ = 30 (60)

P_B = 30(65)

P_C = 30(50)

P_D = 30(35)

∴ For Max Profit (30 × 65) — No. of chairs = 15, No. of tables = 25 — 1