JEE Advanced Paper – 2 Solutions (Code-7)

PART III: MATHEMATICS

SECTION – 1. (Only One Option Correct Type)

Q41. For $x \in (0, \pi)$, the equation $\sin x + 2 \sin 2x - \sin 3x = 3$ has

(a) Infinitely many solutions
(b) Three solutions
(c) One solution
(d) No solution

Sol) $\sin x + 2 \sin 2x - \sin 3x = 3$

$\Rightarrow \sin x\,(1 + 2 \cos x - 3 + 4 \sin^2) = 3$

$\Rightarrow 4 \sin^2 x + 2 \cos x - 2 = 3/\sin x$

$\Rightarrow 2 - 4 \cos^2 x + 2 \cos x - 2 = 3/\sin x$

$\Rightarrow 9/4 - (2 \cos x - 1/2)^2 = 3/\sin x$

L.H.S $\leq 9/4$ R.H.S ≥ 3

(D) No solution

Q42. The following integral

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2 \operatorname{cosec} x)^{17} dx$

is equal to

(a) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$

(b) $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{1}$

(c) $\int_0^{\log(1+\sqrt{2})} (e^u - e^{-u})^{17} du$

(d) [illegible]

Sol) 42 $\int_{45^\circ}^{90^\circ}(2cosec\ x)^{17}\,dx$

Put 2 cosec x = $e^4 + e^{-4}$

2 cosec x ωtx dx = ($e^4 - e^{-4}$) dx ...(1)

Wehaur $cot^2 1$= $cosec^2$ x x – 1

$4cot^2 x = 4\ cosec^2 x - 4$

⇒ $(2\cot x)^2 = (e^4 + e^{-4})^2 - 4$

⇒ $(2\cot x)^2 = (e^4 - e^{-4})^2$

⇒$2\cot x = e^4 - e^{-4}$

From eqn (1) , 2 cosec x cot x dx = 2 cot x dx

Dx = 2du/$e^4 + e^{-4}$ (2)

∴ $\int_{40^\circ}^{90^\circ}(2\ cosec\ x)^{17}dx = (e^4 + e^{-4})^{17}\ 2dx\ /\ (e^4 + e^{-4})$

$2\ (e^4 + e^{-4})^{16}$ dx

$E^4 + e^{-4} = 2$ cosec x

$E^4 - e^{-4} = 2$ cot x

$2e^4 = 2$ cosec x + 2 cot x

e^4 cosec + cot x

at x = 45° 4 = ln (j2 +1)

at x = 90° 4 = ln (0+1) = 0

Q43. The quadratic equation p(x) = 0 with real coefficients has purely imaginary then the equation

p(p(x)) = 0has

(A) Only purely imaginary roots
(B) Al real roots
(C) Two real and two purely imaginary roots
(D) Neither real nor purely imaginary roots

Sol) p (x) = $ax^2 + bx + c$

Since roots are purely imaginary

b = 0 (as $x = -b+\sqrt{b^2 - 4ac} / 2a$)

$p(x) = ax^2 + c$

and $-4ac < 0$ (roots are imaginary $d < 0$)

$4ac > 0$

$Ac > 0$

$P(p(x)) = 2 [2x^2 + tc]^2 + c$

$= a [a^2 x^4 + c^2 + 2acx^2] + c$

$= a^3 x^4 + 2a^2 cx^2 + (ac^2 + c)$

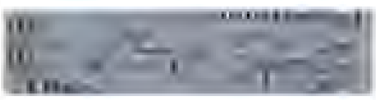
$D = 4a^4 c^2 - 4 a^3 (ac^2 + c)$

$= -4a^2 (ac) < 0$

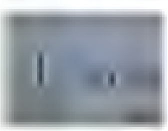
$X^2 = -2a^2 C + \sqrt{-4a^2ac} / 3$ ($ac > 0$)

So all roots are imaginary

Q44. The function y = f (x) is the solution of the differential equation



in (-1, 1) satisfying f(0) = 0. Then



is

(A) $\pi/3 - \sqrt{3}/2$
(B) $\pi/3 - \sqrt{3}/4$
(C) $\pi/6 - \sqrt{3}/4$
(D) $\pi/6 - \sqrt{3}/2$

Sol) $dx/dx + x\,4 / \sqrt{x^2-1} = x^4 + 2x / \sqrt{1-x^2}$

(co) = 0

$\sqrt{x} / \sqrt{x^2} 1 dx$

I.F.E $= \sqrt{x^2 - 1}$

So solution

4. $\sqrt{x^2 - 1} \mid x^4 + 2x / \sqrt{1 - x^2} \cdot \sqrt{x^2 - 1}\, dx$

$\mid i (x^4 + 2x) dx + c$

$4 \sqrt{x^2 - 1} = i (x5/5 + x^2) + c$

Now $f(0) = 0$

$4 \sqrt{0^2 - 1} = i (0+0) + c$

$\Rightarrow |c = 0|$

$\therefore 4 = i (x5/5 + x^2) / \sqrt{x^2 - 1}$

So, $I = \int_{\sqrt{3}/2}^{\sqrt{3}/2} \frac{\lambda\left(\frac{x^5}{5}+x^3\right)dx}{\lambda \sqrt{1-x^2}} = \underbrace{\int x^5 dx / 5 \sqrt{1 - x^2}}_{odd} = \int \underbrace{\frac{x^2\, dx}{\sqrt{1-x^2}}}_{Even}$

$\Rightarrow I = 2 \int_0^{\sqrt{3}/2} \frac{x^2\, dx}{\sqrt{1 - x^2}}$

Now $x = \sin\theta$

$I = 2 \int_0^{\pi/3} \frac{\sin^2\theta}{\cos\theta} \cos d\theta$

$= 2 \int \sin^2\theta\, d\theta = \int \not 2 (1 - \cos\theta/\not x)\, d\theta$

$= [(\theta - \sin 2\theta / 2)]_0^{\pi/3}$

$= (\pi/3 - \sin 2\pi/3) - (0)$

$= \pi/3 - \sqrt{3}/4$

Ans (B)

Q45. Coefficient of x^{17} in the expansion of $(1 + x^2)^5 (1 + x^3)^7 (1 + x^4)^{11}$ is

(A) 1051
(B) 1106
(C) 1113
(D) 1120

Sol) $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$

Coefficient of x^{11} will come from

Coefficient of $[(x^2)^4 (x^3)^1 + (x^2)^1 (x^3)^3 + (x^2) (x^3)^1 (x^4)^1 + (x^3 (x^4)^2]$

$= {}^4C_4\,{}^7C_1 + {}^4C_1\,{}^7C_3 + {}^4C_0 + {}^7C_1\,{}^{12}C_1 + {}^7C_1\,{}^{12}C_2$

= 1x 7 + 4 x 7 x 6 x 5 / 3 x 2 / 4 x 3 / 2x 7 x 12 +

7x 12 x 11 /2

On solving we get coefficient 1113

Q46. Let f: [0, 2] →ℝ be a function which I is continuous on [0, 2] and is differentiable on (0, 2) with f(0) = 1. Let

[illegible]

For x ∈[0, 2]. If $F^1(x) = f^1(x)$ for all x ∈ (0, 2) then F (2) equals

(A)$e^2 - 1$

(B)$e^4 - 1$

(C)e- 1

(D)e^4

Sol)

$F(x) \int_0^{x^2} f(\sqrt{t})\,dt$

Using iebinity formula differing both sides wrt x

$F'(x) = 2(x^2)/\ 2x\ f(\sqrt{x^2}) + 2(0)/2\ f(50)$

$F'(x) = f'(x) = 2\ xx\ f(x)$

$f'(x) = 2\,x\,f(x)$

$f(x) = 2\,x$

$f(x)$

f (x) / f (x) dx = 2x dn

integration both sides

in f (x) = x^2

f (x) = e^{x^2}

$f(x) = \int_0^{x^2} e^t\, dt = e^{x^2} - 1$

|f (2) = e^4 - 1|

Q47.The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is

(A) 3
(B) 6
(C) 9
(D) 15

Sol) 47

⇒ |1/r = cot θ |- (1)

Area of trapezium = (rs+ pq)/2 (AB)

= ($2x^2$ - cosq) (4 r + sin θ)

= (2 r^2 - √2 1/√r^2 +1)(√2 r /√r^2 +1)

Check for r = 1

= 3 x 5

= |15|

Ans (D)

Q48. Six cards and six envelopes are numbered 1,2,3,4,5,6, and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelop bearing the same umber and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it ways it can be done is

(A) 264

(B) 265
(C) 53
(D) 67

Sol) cards Envelops

1	1
2	2
3	3
4	4
5	5
6	6

→ If '2' goes in '1' then it is derangement of 4 things which can be done in 4! (1/2! -1/3! + 1/4!) = 9 ways

→ if '2' doesn't go in 1 it is derangement of 5 thingswhich can be done in 44 ways

→ hence total 53 ways

Option (c) is correct

SECTION- 2: Comprehension Type (Only One Option Correct)

Q51. The value of r is

(A) -1/t
(B) t^2+1/t
(C) 1/t
(D) t^2-1/t

Sol) P(at^2 , 2at)

Q (aq^2, 2aq)

P (a,2a) K (2a,0)

Q (a , 2a) R (ar^2, 2ar)

Slope of pk = slope of QR

0-2a/2a-a = 2ar + 2a / ar^2 -a

-2a / a = 2a (r+1)/a(r_2-1)

$R^2 - 1 = -r - \lambda$

$r^2 + r = 0$

r = 0, -1 but 2c ≠ 0

∴ r = -1

Now t = 1 so r = -1/t is correct

Q52. If st = 1, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(A) $(t^2 + 1)^2/2t^3$
(B) $a(t^2 + 1)^2/2t^3$
(C) $a(t^2 + 1)^2/t^3$
(D) $a(t^2 + 2)^2/t^3$

Sol) eqn of normal : $y = -5x + 2as + as^3$

Eqn of tangent : $x = ty - at^2$

$Y = -5\,(ty - at^2) + 2as + as^3$

$Y = -st\,y + ast^2 + 2as + as^3$

Now st= 1 ⇒ y = - y + at + 2as + as^3

⇒ 2y = a [t + 1/t + 1/t^3]

⇒ y = a(1+t^2)^2 / 2t3

So, (B) is correct

Paragraph For Questions 53 and 54

Given that for each a ∈ (0, 1)

$$\lim \int \ldots$$ [illegible]

Exists. Let this limit be g(a). In addition, it is given that the function differentiable on (0, 1).

Q53. The value of g(1/2) is

(A) π
(B) 2π
(C) $\pi/2$
(D) $\pi/4$

Sol) $\lim_{h \to 0} \int_h^{1-h} t^{-a} (1-t)^{a-1}\, dt$

T t h → 1

h → 0

$g(1/2) \int_0^1 t^{-1/2}(1-t)^{-1/2}\, dt$

$\int_0^1 1/\sqrt{t}\, \sqrt{1-t}\, dt$

$T = \sin^2 x \quad t \to 0 \quad \sin x \to 0$

$dt = 2 \sin x \cos x \; dx \quad t \to 0 \quad x \to \pi/2$

$$\int_0^{\pi/2} \frac{2 \sin x \cos x \, dx}{\sin x \cos x}$$

= 2

$\int_0^{\pi/2} dx = 2 \times \pi/2 = \pi$

Q54. The value of g'(1/2) is

(A) $\pi/2$
(B) π
(C) $\pi/2$
(D) $\pi/4$

Sol) $g(a) = \lim_{h \to 0^+} \int_0^{1-h} t^{-a} (1-t)^{a-1}\, dt$

$2g(a)/2a = 2/2a\,(1-h)\, t^{-(1-0)}\,(1-t)^{(1-0)-1}$

$+ 2(0)/2a\; t^{-a}\,(1-t)^{a-1}$

$g^{(1/2)} = 0$ using Leibnitz therein

Paragraph For Questions 55 and 56

Box 1 contains three cards bearing numbers 1,2,3; box 2 contains five cardsbearing numbers 1, 2, 3, 4, 5; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box i = 1, 2, 3.

Q55. The probability that $x_1 + x_2 + x_3$ is odd, is

(A) 29/105
(B) 53/105
(C) 57/105
(D) ½

Sol)

box 1		box 2	box 3
(I) odd	<	even	− even
		odd	− odd
(II) even	<	even	− odd
		odd	− even

2/3 [2/5 . 3/7 + 3/5 . 4/7] = 36/105

1/3 [2/5 . 4/7 + 3/5 3/7] = 17/105

Total = 53/105

Q56. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(A) 9/105
(B) 10/105
(C) 11/105
(D) 7/105

Sol) $2x_2 = x_1 + x_3$

Possible	x_2	x_1	x_3
	1	1	1
	2	1	3

2	2	2
2	3	1
3	1	5
3	2	4
3	3	3
4	1	7
4	2	6
4	3	5
5	3	7

$(A \times 1) = 1/3\ p(x_2) = 1/5\ p(x_3) = 1/7$

And there are II cases

$\Rightarrow 11 \times [1/3 \times 1/5 \times 1/7] = 11/105$

SECTION – 3: Matching List Type (Only One Option Correct)

This section contains four questions, each having two matching lists. Choices for the correct combination of elements from List – I and List – II are given as options (A), (B), (C) and (D), out of which one is correct.

Q57. Let $z_k = \cos(2k\pi/10) + i \sin(2k\pi/10)$; k = 1,2,...,9.

List 1	List II
P. For each z_k there exists a z_j such that $z_k . z_j = 1$	1. True
Q. There exists a $k \in \{1,2,\ldots,9\}$ such that $z_1 . z = z_k$ has no solution z in the set of complex numbers.	2. False
R. 	3. 1
S. 	4. 2

P Q R S

(A) 1 2 4 3
(B) 2 1 3 4
(C) 1 2 3 4
(D) 2 1 4 3

Sol) $\sum_{k=1}^{9} as(25\pi/10) = \cos(2\pi/10) + \cos(9\pi/10) + \cos(6\pi/10) + \cos(8\pi/10) + \omega^{?}(10\pi/10)$

$+ \cos(12\pi/10) + \cos(14\pi/10) + \cos(16\pi/10) + \cos(18\pi/10)$

Now $= \cos(18\pi/10)\cos(2\pi - 18\pi/10) = \cos(2\pi/10)$

Similarity $\cos(16\pi/10) = \cos(4\pi/10)$

$\cos(11\pi/10) = \cos(6\pi/10)$

$\cos(12\pi/10) = \cos(8\pi/10)$

$\therefore \sum_{k=1}^{9} \cos(2k\pi/10) = 2[\cos(4\pi/10) + \cos(6\pi/10) + \cos(8\pi/10)] + \cos\pi$

$= 2[\underbrace{2\sin\pi\sin(6\pi/10)}_{=0} + \underbrace{2\sin\pi\sin(2\pi/10)}_{=0} + -1$

$= -1$

$= -\sum_{k=1}^{9} \cos(2k\pi/10) = 2$

$\therefore s \to 4$

(Q) $z_1 z = z_k$

$e^{i(2\pi/10)} e^{i(2n\pi/10)} = e^{i(2k\pi/10)}$

$1 + n = k$ for K - 1 , 2 , ... 9

N = 0, 1 , ... 0

∴ (Q) is false

This is (C)

Q. 58. Let $f_1 : R \to R$, $f_2 : [0, \infty) \to R$, $f_3 : R \to R$ and $f_4 : R \to [0, \infty)$ be defined by

$f_1(x) = \begin{cases} |x| & \text{if } x < 0, \\ e^x & \text{if } x \geq 0; \end{cases}$

$f_2(x) = x^2;$

$f_3(x) = \begin{cases} \sin x & \text{if } x < 0, \\ x & \text{if } x \geq 0 \end{cases}$

and

$f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0, \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0 \end{cases}$

List I	List II
P. f_4 is	1. onto but not one – one
Q. f_3 is	2. neither continuous nor one – one
R. $f_2 0 f_1$ is	3. differentiable but not one – one
S. f_2 is	4. continuous and one – one

P Q R S

(A) 3 1 4 2

(B) 1 3 4 2

(C) 3 1 2 4

(D) 1 3 2 4

Sol) $f_1 \equiv x \rightarrow$ continues and one – one

∴ x → 4

f_4 —

f_3 differentiable but not one –one

Q. 59.

List I	List II
P. Let $y(x) = \cos(3\cos^{-1}x)$, $x \in [-1, 1]$, $x \neq \pm\sqrt{3}/2$. Then $1/y(x)\{(x^2 - 1)\, d^2y(x)/dx^2 + x\, dy(x)/dx\}$ equals	1. 1
Q. Let $A_1, A_2, \ldots, A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\overrightarrow{a_k}$ be the position vector of the point A_k, $k = 1, 2, \ldots, n$. If $\left\|\sum_{k=1}^{n-1}(\vec{a}_k * \overrightarrow{a_{k+1}})\right\| = \left\|\sum_{k=1}^{n-1}(\vec{a}_k \cdot \overrightarrow{a_{k+1}})\right\|$, then the minimum value of n is	2. 2
R. If the normal from the point $P(h,1)$ on the ellipse $x^2/6 + y^2/3 = 1$ is perpendicular to the line $x + y = 8$, then the value of h is	4. 9
S. Number of positive solutions satisfying the equation $\tan^{-1}(1/2x+1) + \tan^{-1}(1/4x+1) = \tan^{-1}(2/x^2)$ is	4. 9

(A) 4 3 2 1
(B) 2 4 3 1
(C) 4 3 1 2
(D) 2 4 1 3.

Sol) (p) $y = 4x^3 - 3x$ where $\cos\theta = x$

$Dy/dx = 12x^2 - 3$

$d^2/dx^2 + x dy/dx = (x^2 - 1) 24x = x(12x^2 - 3)$

$= 36x^3 - 27x = 9(4x^3 - 3x) = 9y$

Hence, $1/y \{(x^2 - 1) d^2y/dx^2 + x\, dy/dx\} = 9$

(R) Equation of normal $6x/b - 3y/1 = 3$ (Equation of normal is $a^2 x/x_1 - b^2 y/y_1 = a^2 - b^2$)

Slope $= 6/3b = 1$ cos it is perpendicular to $x + y = 1$

$\Rightarrow R = 2$

Q. 60.

List I	List II
P. The number of polynomials f(x) with non – negative integer coefficients of degree ≤2, satisfying f(0) = 0 and $\int_0^1 f(x)dx = 1$, is	1. 8
Q. The number of points in the interval $[-\sqrt{13}, \sqrt{13}$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	2. 2
R. $\int_{-2}^{2} 3x^2/(1+e^x)\, dx$ equals	3. 4
S. $(\int_{-1/2}^{1/2} \cos 2x \log (1+x)(1-x)dx / (\int_0^{1/2} \cos 2x \log(1+x)/(1-x)dx)$ equals	4. 0

P Q R S

(A) 3 2 4 1
(B) 2 3 4 1
(C) 3 2 1 4
(D) 2 3 1 4.

Sol) Q =

$f(x) = \sin(x^2) + \cos(x^2)$

$x \in [-\sqrt{13}, \sqrt{13}]$

$x^2 \in [0, 13]$

let $x^2 = t$

$\Rightarrow t \in [0, 13]$

F (x) sin t + cos t

$f(x) = 52 \sin(\pi/4 + t)$

it is max when

$\pi/4 + t = \pi/2\ \pi$

$\sin(\pi/4 + t) = \sin(\pi/2)$

$\pi/4 + t = n\pi + (-1) + (-1)^n \pi/2$

$\pi/4 + t = n\pi + (-1)^n \pi/2$

for n = 1

$\pi/4 + t = \pi - \pi/2$

$t = \pi/4$ also $\pi - \pi/4$

i.e. $t = 3\pi/4$ will satisfy

for n = 2

$\pi/4 + t = 2\pi + \pi/2$

$t = 2\pi + \pi/4 + 9\pi/4$

for n = 3

$\pi/4 + t = 3\pi - \pi/2 = 3\pi - \pi/2 - \pi/4$

$12\pi-$

Also $t = 2\pi + (\pi - \pi/4) = 11\pi/4$ will satisfy

So 4 solution in the interval [0,13]