JEE MAIN 2014 Solutions- Math (CODE-G)

1. **If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + Bx^2 + x$ then**
   a. $\alpha = -6, \beta = -\frac{1}{2}$
   b. $\alpha = -2, \beta = -\frac{1}{2}$
   c. $\alpha = -6, \beta - \frac{1}{2}$
   d. $\alpha = -6, \beta = \frac{1}{2}$

Sol. X = -1   x = 2

Are maxima & minima

$\Rightarrow \alpha \log|x| + \beta x^2 + x = f(x)$

Taking x> 0

$F(x) = \alpha \log x + \beta x^2 + x$

$F'(x) = \frac{\alpha}{\beta} + 2\beta x + 1 = 0$

$2\beta x2 + x + \alpha = 0$

Now x = -1 & 2

Must satisfy this as these are critical points

X = -1

$2\beta - 1 + \alpha = 0$

X = 2

$2\beta + 2 + \alpha = 0$

Solving $\beta = -1/2$

$\alpha = 2$

2. **The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is:**
   a. $(x^2 - y^2)^2 = 6x^2 - 2y^2$
   b. $(x^2 + y^2)^2 = 6x^2 + 2y^2$
   c. $(x^2 + y^2)^2 = 6x^2 - 2y^2$

d. $(x^2 - y^2)^2 = 6x^2 + 2y^2$

Sol. Foot of perpendicular is given by :

$$\frac{h-x}{x} = \frac{k-y}{b} = -\frac{[ax+by+c]}{a^2+b^2}$$

X, y = 0, 0 eqn tangent:

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

Putting values:

$$\frac{ah}{\cos\theta} = \frac{bk}{\sin\theta} = \frac{1}{\frac{\cos\theta}{a^2} + \frac{\sin^2\theta}{b^2}}$$

$\Rightarrow$ h = $\frac{ab^2\cos\theta}{b^2\cos^2\theta + a^2\sin^2\theta}$

K = $\frac{a^2 b\sin\theta}{b^2\cos^2\theta + a^2\sin^2\theta}$

Now it is difficult to eliminate θ so we check option

Answer = $(x^2 + y^2)^2 = 6x^2 + 2y^2$

3. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals :

a. $\frac{1}{3}$

b. $\frac{1}{4}$

c. $\frac{1}{12}$

d. $\frac{1}{6}$

Sol. $f_4 = \frac{\sin^4 x + \cos^4 x}{4}$

$= 1 - \frac{2\sin^2 x\cos^2 x}{6}$

$f_6 = \frac{\sin^6 x + \cos^6 x}{6}$

$= 1.(\frac{\sin^4 x - \sin^2 x\cos^2 x + \cos^4 x}{6})$

By formula $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= [\frac{1-3sin^2 x\, cos^2 x}{6}]$

$r_4 - r_6 = \frac{1-2sin^2 x\, cos^2 x}{4} - \frac{(1-3sin^2 x\, cos^2 x}{6}$

$6 - \frac{12sin^2 cos^2 x - 4 + 12sin^2 cos^2 x}{24}$

$= \frac{2}{24} = \frac{1}{12}$ Ans.

4. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural, then $X \cup Y$ is equal :

a. Y – X

b. X

c. Y

d. N

Sol. X = 4n – 3n – 1

Rewriting:

X = $(3+1)^n$ – 3n – 1

Expanding $(1+3)^n$

X = $(1 + 3n + \frac{3n.(n-1)}{1.2} \ldots\ldots 3^{n.n}c_n) - 3^{n\,1}$

$= \frac{3.n(n-1)}{1.2} \ldots\ldots 3^{n.n}c_n$

⇒ All the multiple of 9 are in x which is represented by y as well but y will exceed x at some point.

So X ∪ Y has to be Y & not X.

5.) If A is a 3×3 non – singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals:

a. 1

b. B^{-1}

c. $(B^{-1})'$

d. $1 + B$

Sol. $BB^1 = (A^{-1}A^1)\ (A^{-1}A^1)^1$

$(A^{-1}A^1)\{(A^1)1\ (A^{-1})1\}$

$= A^{-1} (A^{t} A) (A^{t} \cdot)$

$= A^{-1} (AA^{t}) (A^{t}) 1$

$= A^{-1} A \; A^{-1} A^{t}$

I . I = I Ans.

6.) The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx$ is equal to :

e. $x\, e^{x+\frac{1}{x}} + c$

f. $(x + 1)\, e^{x+\frac{1}{x}} + c$

g. $-x\, e^{x+\frac{1}{x}} + c$

h. $(x - 1)\, e^{x+\frac{1}{x}} + c$

Sol. $\int \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}} . dx$

$= \int (e^{x+\frac{1}{x}} . dx + \int (x - \frac{1}{x}) e^{x+\frac{1}{x} dx}$

By pasts

$= xe^{x+\frac{1}{x}} - \int e^{x+\frac{1}{x}} (1 - \frac{1}{x^2}) x.dx + \int (x - \frac{1}{x}) e^{x+\frac{1}{x}} . dx$

$= xe^{x+\frac{1}{x}}$ Ans.

7) The area of the region described by $A = \{(x, y) : x^2 + y^2 \le 1 \; and \; y^2 \le 1 - x\}$ is :

i. $\frac{\pi}{2} - \frac{4}{3}$

j. $\frac{\pi}{2} - \frac{2}{3}$

k. $\frac{\pi}{2} + \frac{2}{3}$

l. $\frac{\pi}{2} + \frac{4}{3}$

Sol. Req. Area = Area ACB + Area BCD

$= \int_0^1 1 - y^2 . dy + \frac{\pi r^2}{2}$

$= 2(y - \frac{y^3}{3}) + \frac{\pi}{2}$

$= \frac{\pi}{2} + \frac{4}{3}$ Ans.

8] The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line :

a. $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

b. $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

c. $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

d. $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

Sol. Plane and line are parallel.

Eqn of normal to plane

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = K.$$

Point → 2K+1 ,3-K, 4 + K

$\Rightarrow \frac{2K+2}{2}, \frac{6-K}{2}, \frac{8+K}{2}$

Lies on plane

$2(K+1) - \frac{(6-K)}{2} + \frac{8+K}{2} + 3 = 0$

K = -2

Point through which image passes (-3, 5, 2)

Hence, $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

9] The variance of first 50 even natural numbers is :

a. **833**

b. **437**

c. $\frac{437}{4}$

d. $\frac{833}{4}$

Sol. Even Natural No.

= 2, 4, 6, 8---------- 100

Variance = $\sum \frac{(x-\bar{x})^2}{n}$

$\bar{x}$ = mean = 5

$n = 50$

$x = 2, 4, 6, \text{------} 100$

$$= \frac{(2-51)^2 + (4-51)^2 \text{-----------} (100-\bar{x})^2}{50}$$

= 833 Ans.

10) If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$:

Sol. $|Z| \geqslant 2$ represents a circle with

Radius $\geqslant 2$

$|z + ½|$ represent distance

From point)-1/2, 0)

[image]

$|2 - ½| = 3/2$

11) Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new number is in A.P. Then the common ratio of the G.P> is :

Sol. Let GP be,

a ,ar , ar^2

also a 2ar ar^2 (are in ap)

$\Rightarrow 4ar = a + ar^2$

$4r = 1 + r^2$

$r = \frac{4 \pm \sqrt{-12}}{2}$

$= 2 \pm \sqrt{3}$

$r = 2 - \sqrt{3}$ doesn't satisfy A.P condition

$\Rightarrow r = 2 + \sqrt{3}$ Ans.

12) **If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to :**

Sol. $(1 + ax + bx^2)(1 - 2x^{18})$

x^3 terms:

$= 18_{C_3} + (-2x)^3 + 18_{C_2}(-2x)^2 . ax + 18_{C_1}(-2x)\, bx^2 = 0$

$-18_{C_3} . 8 + 18_{C_2} . 4a - 18_{C_1} . 2b = 0$---------- (1)

x^4 terms :

$18_{C_4}(-2x)^4 + 18_{C_3} + (-2x)^3 . ax + 18_{C_2}(-2x)^2 \; bx^2 = 0$

$16 . 18_{C_4}$ -8a $. 18_{C_3} + 18_{C_2}$. 4b = 0------------(2)

Solving (1) & (2) for

a& b :

a = 16 $\quad b = \frac{272}{3}$

13) **Let a, b, and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then:**

Sol. Putting the condition

Solving: 4ax – 2ax + c = 0

2ax +c = 0______________(1)

5bx – 2bx +d = 0

3bx d = 0______________(2)

Putting value of x

2a. $(\frac{-d}{3b}) + c = 0$

3bc – 2ab = 0

14) If $[\vec{a} \times \vec{b}\vec{b} \times \vec{c}\vec{c} \times \vec{a}] = \lambda[\vec{a}\vec{b}\vec{c}]^2$ then λ is equal to

Sol. $[\vec{a} \times \vec{b}\vec{b} \times \vec{c}\vec{c} \times \vec{a}] = (\vec{a} \times \vec{b}).\left[(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})\right]$

$= \left[(\vec{a} \times \vec{b}.(\vec{p} \times (\vec{c} \times \vec{a})\right]$

Let $\vec{p} = (\vec{b} \times \vec{c})$

$= (\vec{a} \times \vec{b}).((p.a)\vec{c} - (p.c)\vec{c}$

$= (\vec{a} \times \vec{b})(\left((\vec{b} \times \vec{c}).\vec{a}\vec{c}\right) - (\left((\vec{b} \times \vec{c}).\vec{c}\right)\vec{a})$

= (a × v) [(bca) $\vec{c}$) – 0

$\because (\vec{b} \times \vec{c}).\vec{c} = 0$

$\Rightarrow \quad (\vec{a} \times \vec{b}).\vec{c}[\vec{b}\,\vec{c}\,\vec{a}]$

$= [\vec{a}\,\vec{b}\,\vec{c}] = 1$

15) Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cup B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{6}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are

Sol. $P(\overline{A \cup B}) = \frac{1}{6}$

$P(A \cap B) = \frac{5}{6}$

$P(\overline{A}) = \frac{1}{4}$

$P(A) = \frac{3}{4}$

$P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$= \frac{5}{6} - \frac{3}{4} + \frac{1}{4}$

$P(B) = \frac{5}{6} - \frac{1}{2} = \frac{1}{3}$

Also P (A). (PB) = $(A \cap B)$

= Independent events

P (A) ≠ P(B) Unlikely

16) Let PS be the median of the triangle with vertices $P(2,2)$, $Q(6,-1)$ and $R(7,3)$. The equation of the line passing through $(1,-1)$ and parallel to PS is :

Sol. Coordinate of S = $(\frac{13}{2}, 1)$ by mid point formula

Slope PS = $\frac{2-1}{2-\frac{13}{2}} = \frac{-2}{9}$

$y = \frac{-2}{9}x + C$

Putting (1, -1)

$-1 = \frac{-2}{9}\ C$

$C = \frac{-7}{9}$

$2x + 9y - 7 = 0$

17) $\lim_{x\to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to :

Sol. Applying Hospitals Rules

$= \lim_{x\to 0} \frac{\cos(\pi \cos^2 x)\pi 2\cos x(-\sin x)}{2x}$

$= \lim_{x\to 0} \frac{\cos(\pi \cos^2 x)}{2} \pi\ 2\cos x \frac{(-\sin x)}{x}$

$= \pi \lim_{x\to 0} \frac{\sin x}{x} = 1$

18) Let ∝ and β be the roots of equation $px^2 + qx + r = 0, p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is:

Sol. A - B $= \sqrt{(\alpha + \beta^2 - 4\alpha\beta}$

$= \sqrt{\frac{q^2}{p} - 4 \cdot \frac{r}{p}}$

$= \frac{\alpha + \beta}{\alpha\beta} = 4$

$= -\frac{\frac{q}{p}}{\frac{r}{p}} = \frac{-q}{r} = 4$ ------ (1)

2q = r + p

$2 = \frac{r}{q} + \frac{p}{q}$

$2 = \frac{-1}{4} + \frac{p}{q}$

$= \frac{p}{q} \cdot \frac{9}{4}$ ——— (2)

From (1) & (2)

$= \frac{r}{p} = \frac{-1}{9}$ ———— (3)

Putting the values from (2) & (3)

$(\alpha - \beta) = \sqrt{\frac{4^2}{9} + 4 \cdot \frac{1}{9}}$

$= \sqrt{\frac{52}{81}} = \frac{2\sqrt{13}}{9}$ Ans.

19) A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is :

Sol. In Δ AOB

$= \frac{AB}{OB} = \frac{20}{OB} = \tan 45 = 1$

OB = 20

Similarly in $\Delta A'OB'$

$OB^1 = 20\sqrt{3}$

Distance moved:

$OB^1 - OB = 20(\sqrt{3} - 1)$

Velocity $= \frac{20(\sqrt{3}-1)}{1}$

$= 20(\sqrt{3} - 1)$

20) If a ∈ R and the equation $-3(x - [x])^2 + 2(x + a^2 = 0$ (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval :

Sol -3 (x − [x]) ^{2} + 2(x − [x]) + a^2

(x − [x]) = {x}

R = [0, 1]

for {x} = 0

$a^2 = 0$

for {x} = 1

$-1 + a^2 = 0$

for the eqn. not to hold:

a * (0, 1) u (-1, 0)

(-1, 0) u (0, 1)

21) The integral $\int_0^{\pi} \sqrt{1 + 4sin^2\frac{x}{2} - 4sin\frac{x}{2}\, dx}$ equals :

Sol. $\int_0^{\pi} \sqrt{1 + 4sin^2\frac{x}{2} - 4sin\frac{x}{2}\, dx}$

$= \int_0^{\pi} \sqrt{(1 - 2sin\frac{x}{2})^2}$. dx

$= \int_0^{\pi} ((1 - 2sin\frac{x}{2})$. dx

X + 2 cos$\frac{x}{2}$. 2

= (π - 4)

22) If f and g are differentiable functions in $[o, 1]$ satisfying $f(0) = g(1), g(0) = 0$ and $f(1) = 6$, then for some $C\epsilon]0, 1[$:

Sol. x = 0 1

f(x) 2 6

g(x) 2 6

By Rolles theorem:

f^1 (x) = $\frac{6-2}{1}$= 4

g^1 (x) = $\frac{2-0}{1}$ = 2

f^1 (x) = 2g^1 (x)

23) If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$ then $g'(x0$ is equal to :

Sol. f^1 (x) = $\frac{1}{1+x^5}$

f(x) = $\int_0^x \frac{1}{1+x^5}$. dx

→ Inverse fun:

X = $\int_0^{g(x)} \frac{1}{1+xg^{5(x)}}$. d($d_{(x)}$)

Differentiating:

1 = $\frac{1}{1+xg^{5(x)}}$. g(x)

G(x) = 1 +g^5x

24) If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$, then k is equal to :

Sol. $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$

1 = 2 ($\frac{11}{10}$) + 3($\frac{11}{10}$)^2 — 10 $\frac{11^9}{10^9}$ = K

Subtracting:

$1+\frac{11}{10}+(\frac{11}{10})^2 - - - - (\frac{11}{10})^9 - 10.(\frac{11}{10})^{10} = \frac{-K}{10}$

$= \frac{(\frac{11}{10})^{10}-1}{\frac{1}{10}} - 10.(\frac{11}{10})^{10} = \frac{K}{10}$

$10.(\frac{11}{10})^{10}$ - $10\ 10.(\frac{11}{10})^{10} = \frac{-K}{10}$

K = 100 Ans.

25) If $\propto, \beta \neq 0$, and $f(n0 = \propto^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\propto)^2(1-\beta)^2(\propto-\beta)^2$, then K is equal to

Sol. $\begin{vmatrix} 3 & 1+\alpha+\beta) & 1+\alpha(2)+\beta(2) \\ 1+\alpha+\beta) & 1+\alpha(2)+\beta(2) & 1+\alpha(3)+\beta(3) \\ 1+\alpha(2)+\beta(2) & 1+\alpha(3)+\beta(3) & 1+\alpha(4)+\beta(3) \end{vmatrix}$

$\begin{matrix} 3 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{matrix}$

Solving we get K = 1

26) The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

Sol. $y^2 = 4x$

Tangent y = mx + $\frac{1}{m}$

Touches $x^2 = -32y$

$= x^2 = -32\ (mx + \frac{1}{m})$

$= x^2 + 32\ mx + \frac{32}{m} = 0$

$D = 0 \rightarrow (32m)^2 - 4.\frac{32}{m} = 0$

$= m^3 = \frac{1}{8}$

m = $\frac{1}{2}$ Ans.

27) The statement $\sim(P \leftrightarrow \sim q)$ is :

Sol.

P	Q	~P	~q	P↔q	P↔~q	~P↔q	~(P↔~q)
T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	F
T	T	F	F	T	F	F	T
F	F	T	T	T	F	F	T

28) Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $P(0) = 100$, then $P(t)$ equals :

Sol. $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$

$= \int_{100}^{p} \frac{dp(t)}{\frac{1}{2}p(t)-200} = \int_0^t dt$

$2\,[\log(\frac{1}{2}p(t) - 200) - \log(-150)]$

$\text{Log}\,\frac{\frac{1}{2}p(t)-200}{-150} = \frac{t}{2}$

$= \frac{1}{2}p(t) - 200 = -150\,e^{\frac{t}{2}}$

$P(t) = 400 - 300\,e^{\frac{t}{2}}$ Ans.

29) Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to :

Sol. (Image)

$= r_1 + r_2 = \sqrt{(1-0)^2}(1-y)^2$

$1 + \sqrt{(y-0)^2 + (0-0)^2} = \sqrt{1 + (1-y)^2}$

$= (1+y)^2 = 2 + y^2 - 2y$

$1 + y^2 + 2y = 2 + y^2 - 2p$

$4y = 1$

Y = $\frac{1}{4}$ Ans.

30) The angle between the lines whose direction cosine satisfy the equations $l + m + n = 0\ and\ l^2 = m^2 + n^2$ is :

Sol. (l + n) = -m

l^2 = $(\text{l} + \text{n})^2 + n^2$

l^2= l^2+ n^2 2ln + n^2

2n^2 +2ln = 0

2n (n + l) = 0

N = 0

N = -l

L = -m

M = o

dr's$l, -l, o$

l, 0, -l.

cosθ = $\frac{1}{\sqrt{2}\sqrt{2}}$ = $\frac{1}{2}$

θ = $\frac{\pi}{3}$ Ans.