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MODEL PAPER OF IIT JEE

ANSWERS

PAPER I

1.(b) **2.**(b) **3.**(c) **4.**(d) **5.**(b) **6.**(d) **7.**(b) **8.**(b, d) **9.**(b, d) **10.**(a, b, c)

11.(b, c, d) **12.**(a) **13.**(c) **14.**(a) **15.**(c) **16.**(a) **17.**(c) **18.**(5) **19.**(4) **20.**(12)

21.(5) **22.**(1) **23.**(3) **24.**(5)

PAPER II

1.(d) **2.**(b) **3.**(a) **4.**(c) **5.**(b) **6.**(a) **7.**(c) **8.**(a) **9.**(b, c, d) **10.**(c)

11.(b) **12.**(d) **13.**(1) **14.**(2) **15.**(6) **16.**(4) **17.**(1) **18.**(≥ 2)

19. (A) $\rightarrow$(r);(B) $\rightarrow$(r);(C) $\rightarrow$(p),(q),(r),(s);(D) $\rightarrow$(p) **20.** (A) $\rightarrow$(r);(B) $\rightarrow$(p);(C) $\rightarrow$(s);(D) $\rightarrow$(q)

HINTS & SOLUTIONS

PAPER I

Sol.1

$\sin(A-B) = \sin A\cos B - \cos A\sin B$ …… (i)

Substituting the values of $\cos A$ and $\sin A$ from the given eq. (i), we have

$\sin(A-B) = \dfrac{\sin B\cos B}{\sqrt{2}}$ ……….. (ii)

On squaring and adding given equation, we get $\cos 2B = 1/3$

$\sin(A-B) = \pm 1/3$

Sol.2

Obviously $\alpha = -i$ where $i^2 = -1$

So, $\omega = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -i & i \\ i & 1 & 0 \end{vmatrix} = -1-i, \arg(\omega) = -3\pi/4$

Sol.3

We have , $\vec{a}\vec{x}=0$

$x=b+(x\times a)$

$x\times a=b\times a+(x\times a)\times a$

$x\times a=b\times a+(x.a)a-(a.a)x$

$x-b=b\times a-x$

$|2x|=|b+b\times a|$

$\Rightarrow |x|=\dfrac{1}{\sqrt{2}}$

Sol.4

The required point is the point of intersection of

$x^2+y^2=2a^2$

And $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

Sol.5

We have,

$$\frac{x^2}{\tan 9^\circ}+\frac{y^2}{\tan 27^\circ}+\frac{z^2}{\tan 63^\circ}+\frac{w^2}{\tan 81^\circ}\geq\frac{25}{4\sqrt{5}}\geq\frac{5\sqrt{5}}{4}$$

Sol.6

$\sin^{-1}\sin\sqrt{1-\alpha}=\sqrt{1-\alpha}$

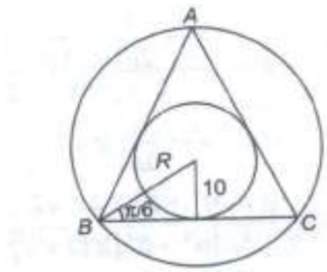
$\Rightarrow \alpha\leq 1$ and $\alpha\leq\sqrt{1-\alpha}\leq\pi/2$

$1-\alpha\leq\pi/4$

$1-\pi/4\leq\alpha$

$\Rightarrow \alpha\geq\dfrac{4-\pi}{4}$

Sol.7



$\sin \pi / 6 = 10 / R$

$\Rightarrow R = 20$

$\Rightarrow R = 2r$

Sol.8

$$f\left(\frac{1}{x}\right) = \int_1^{1/x} \frac{Int}{1+t} dt$$

Put t = 1/u

$$= \int_1^x \left(\frac{Ln\ u}{1+u}\right)\left(-\frac{1}{u^2}\right) du = \int_1^x \frac{In\ t}{t(1+t)} dt$$

Now, $f(x) + f\left(\frac{1}{x}\right) = \frac{(In\ x)^2}{2}$

Sol.9

$f(1) = a + 1 + c - a > 0$ [$\because$c>-1]

Also, $f\left(\frac{1}{2}\right) = \frac{\propto}{4} + \frac{1}{2} + c - a > 0$

$\Longrightarrow 2 + 4c > 3a$

Sol.10

Now, $\sum \cos^2 \alpha_1 \sin^2 ß_1 \geq (\sum cos\alpha_1 sinß_1)^2$

$\because \sin^2 \Theta_1 \leq 1$

$\Longrightarrow \sin^2 \Theta_1 = 1$

Similarly, $\cos^2 \Theta_2 = 1$

Sol.12

Equation of tangent at P is $\frac{x+y-4}{\sqrt{2}} = \frac{3x+4y+5}{5}$

$5x + 5y - 20 = \sqrt{2x} + 4\sqrt{2}y + 5\sqrt{2}$

$x(5 - 3\sqrt{2}) + y(5 - 4\sqrt{2}) - (20 + 5\sqrt{2}) = 0$

Sol.13

$2ac = 13\sqrt{2}$

$\therefore 2a = 26$

$a = 13$

$\Rightarrow 2 \times 13 \times e = 13\sqrt{2}$

$\Rightarrow e = \frac{1}{\sqrt{2}}$

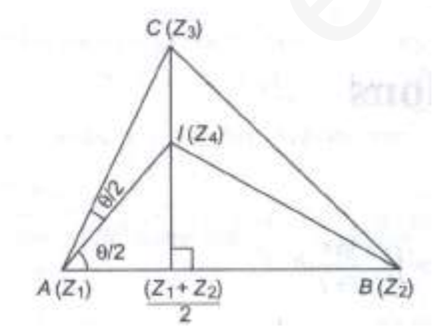
Sol.15

$$\frac{z_2 - z_1}{|z_2 - z_1|} = \frac{z_4 - z_1}{|z_4 - z_1|}.e^{\frac{i\Theta}{2}}$$

$$\frac{z_3 - z_1}{|z_3 - z_1|} = \frac{z_4 - z_1}{|z_4 - z_1|}.e^{\frac{i\Theta}{2}}$$

$$\Rightarrow \frac{(z_2 - z_1)(z_4 - z_1)}{|z_2 - z_1||z_3 - z_1|} = \frac{(z_4 - z_1)^2}{|z_4 - z_1|^2}.e^{\Theta}$$

$$\Rightarrow \frac{(z_2 - z_1)(z_4 - z_1)}{(z_4 - z_1)^2} = \frac{AB.AC}{(IA)^2}$$



Similarly, others

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Sol.18

Since, the equation has unequal real roots, the discriminant is positive , that is

$4(a+b)^2 > 4(a-b+8)$

$\Rightarrow \mathrm{a}^2 + 2ab + b^2 > a - b + 8$

$\Rightarrow \mathrm{a}^2 + (2b-1)a + (b^2+b-8) > 0, \forall\, a \in R$

$\therefore$ Discriminant should be negative.

$\Rightarrow (2\mathrm{b}-1)^2 < 4(b^2+b-8)$

$\Rightarrow 4\mathrm{b}^2 - 4b + 1 < 4b^2 + 4b - 32$

$\Rightarrow 33 < 8\mathrm{b} \quad \therefore \quad \mathrm{b} > \dfrac{33}{8}$

Hence, smallest natural number b=5.

Sol.19

Put $x = rcos\Theta, y = rsin\Theta$

Hence, we have to minimize r^2?

Now $r^2 cos\Theta sin\Theta r^2(\cos^2\Theta - \sin^2\Theta = r^2)$

$r^2 sin2\Theta cos2\Theta = 2$

$r^2 \frac{sin4\Theta}{4} = 1$

$r^2 = \frac{4}{sin4\Theta}$

$r^2 = 4cosec4\Theta$

$\therefore \qquad r^2|\_max = 4$

Sol.20

Equation of normal at $(2t^2, 4t)$ is

$$y = -tx + 4t + 2t^3$$

Since, it passes through (α,ß)

$\therefore ß = -\alpha t + 4t + 2t^3$

$\therefore t_1 + t_2 + t_3 = 0$

And $t_1t_2 + t_2t_3 + t_3t_1 = \frac{4-\alpha}{2}$

The normal intersects the axis at $4 + 2t^2, 0$

$\therefore OQ + OR + OS = \Sigma(4 + 2t_1^2) = 12 + 2(t_1^2 + t_2^2 + t_3^2)$

$= 12 + 2[(t_1 + t_2 + t_3)^2 - 2(t_1t_2 + t_2t_3 + t_3t_1)]$

$= 12 + 2\left[0 - 2\frac{4-\alpha}{2}\right] = 12 - 2(4 - \alpha)$

$=2(\alpha + 2) > 12$ $\quad$ (∵α>4)

Hence, greatest value of k is 12.

Sol.21

$$\int_0^8 (\sqrt{\cot^{-1}(\cot \pi x)} + \cot^{-1}(cot\pi\sqrt{x})dx$$

$=8\ \int_0^1 \sqrt{\cot^{-1}(cot\pi x)}\, dx + \int_0^1 \cot^{-1}(cot\pi\sqrt{x})dx + \int_1^4 \cot^{-1}(cot\pi\sqrt{x})dx + \int_4^8 \cot^{-1}(cot\pi\sqrt{x})dx$

$=\int_0^1 \sqrt{\pi}\sqrt{x}\, dx + \int_0^1 \pi\sqrt{x}\, dx + \int_1^4 (\pi\sqrt{x} - \pi)dx + \int_4^8 (x\sqrt{x} - 2\pi)dx + \int_4^8 (x\sqrt{x} - 2x)dx = \frac{16}{3}\sqrt{\pi} + \frac{32\sqrt{2}\pi}{3} - 11\pi$

$\therefore A = \frac{16}{3}, b = 11\ \therefore\ 3\mathrm{a} - \mathrm{b} = 5$

Sol.27

Roots of equation $ax^3 + x - 1 - a = 0$ are 1, α, ß

$\therefore \alpha + ß = -1$ and $\alpha ß = \frac{1+a}{a}$

$$= \lim_{x \to \frac{1}{\alpha}} \frac{(1+a)x^3 - x^2 - a}{(e^{1-\alpha x} - 1)(x-1)}$$

$$= \frac{\lim_{x \to \frac{1}{\alpha}} (x-1)\ \ (1+a)x^2 + ax + a}{(e^{1-\alpha x} - 1)(x-1)}$$

$$= \frac{\lim_{x \to \frac{1}{\alpha}} a\alpha ß x^2 + ax + a}{(e^{1-\alpha x} - 1)}$$

$$= \frac{\lim_{x \to \frac{1}{\alpha}} a(1 - \alpha x - ß x + \alpha ß x^2)}{1 - \alpha x}$$

$$= \frac{\lim_{x \to \frac{1}{\alpha}} a(1 - \alpha x)(1 - ß x)}{1 - \alpha x}$$

$$= \lim_{x \to \frac{1}{\alpha}} a(1 - ß x) = \frac{a(\alpha - ß)}{\alpha}$$

$\therefore$ k=1 and l=1 and so kl=1

Sol.23

$$\alpha^2 + ß^2 = (\alpha + ß)^2 - 2\alpha ß = a^2 - 2\left(-\frac{1}{2a^2}\right) = a^2 + \frac{1}{a^2}$$

$$\alpha^4 + ß^4 = (\alpha^2 + ß^2)^2 - 2\alpha^2 ß^2$$

$$= \left(a^2 + \frac{1}{a^2}\right)^2 - 2\left(\frac{1}{4a^2}\right) = a^4 + \frac{1}{a^4} + 2 - \frac{1}{2a^4}$$

$$= a^4 + \frac{1}{2a^4} + 2$$

$$\frac{a^4 + \frac{1}{2a^4}}{2} \geq \sqrt{a^4 . \frac{1}{2a^4}} = \frac{1}{\sqrt{2}}$$

$$\therefore a^4 + \frac{1}{2a^4} \geq \sqrt{2}$$

$\therefore\ \alpha^4 + ß^4 \geq 2 + \sqrt{2}$

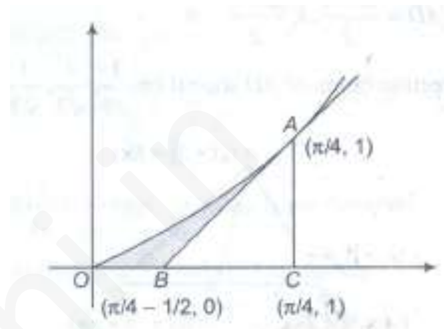
$\therefore [\alpha^4 + ß^4] \geq [2 + \sqrt{2}] = 3$

$\therefore$ Least value of $[\alpha^4 + ß^4] = 3$

Sol.24

$\frac{dy}{dx} = \sec^2 x$

$\Rightarrow \frac{dy}{dx}\Big|_{x=\frac{\pi}{4}} = \sec^2 \frac{\pi}{4} = 2$



Thus, the equation of the tangent to the curve C at

$\left(\frac{\pi}{4}, tan\frac{\pi}{4}\right) = \left(\frac{\pi}{4}, 1\right)$ is $y - 1 = 2\left(x - \frac{\pi}{4}\right)$

This meets the x-axis at $\left(\frac{\pi}{4} - \frac{1}{2}, 0\right)$. Therefore, the required area is area of ΔOAB = area of ΔOAC-area of $\Delta ABC = \int_0^{\frac{\pi}{4}} \tan x dx - \frac{1}{2}(BC) \times AC$

=In sec $x \quad |^{\pi/4}{}_0 - \frac{1}{2} \times \left[\frac{\pi}{4} - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right] \times 1 = \frac{1}{2}\left(In\ 2 - \frac{1}{2}\right)$

Hence, k=5

PAPER -II

Sol.1

$$\begin{vmatrix} 1 & -k & -1 \\ k & -1 & k \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow$ k=±1

Sol.2

$A^2 = A \Rightarrow A^3 = A^4 = A^n = A$

Also,$(I + A)^n$

$=I^n + {}^nC_1 I^{n-1} A + {}^nC_2 I^{n-2} A^2 + \cdots + {}^nC_n I^n A^n$

$=I + A\left[{}^nC_1 + {}^nC_2 + \cdots + {}^nC_n\right] = i + A(2^n - 1)$

Sol.3

$f'(x) - pf(x) \leq 0$

$\Longrightarrow \quad \frac{d}{dx} e^{-pf} f(x) \leq 0 \qquad \ldots(1)$

Let $\quad g(x) = e^{-pf} f(x)$

$\quad g'(x) \leq 0, \forall x \geq 1 \qquad$ [from Eq.(i)]

$\Longrightarrow \quad g(x) \leq g(1), \forall x \geq 1$

$\quad g(x) \leq 0 \Longrightarrow \mathrm{f(x)} \leq 0$

But given $f(x) \geq 0$

$\Longrightarrow \quad f(x) = 0, \forall x \geq 1$

Sol.4

We have,

$$\frac{s^2}{P\left(\frac{A}{B}\right)} + \frac{(s-a)^2}{P\left(\frac{A}{B}\right)} + \frac{(s-b)^2}{P\left(\frac{B}{A}\right)} + \frac{(s-c)^2}{P\left(\frac{B}{A}\right)} \geq 2s^2 \geq 6\sqrt{3}\Delta$$

Sol.5

$$s_n = \frac{0}{\left(n_{C_0}\right)^k} + \frac{1}{\left(n_{C_1}\right)^k} + \frac{2}{\left(n_{C_2}\right)^k} + \cdots + \frac{n}{\left(n_{C_{0n}}\right)^k}$$

$$s_n = \frac{n}{\left(n_{C_0}\right)^k} + \frac{n-1}{\left(n_{C_1}\right)^k} + \frac{n-2}{\left(n_{C_2}\right)^k} + \cdots + \frac{0}{\left(n_{C_{0n}}\right)^k}$$

$2S_n = nt_n$

$\Longrightarrow \quad \frac{S_n}{nt_n} = \frac{1}{2}$

Sol. 6

√2+√3>π(1.414+1.732=3.146>π)

$\therefore \frac{\sqrt{2}+\sqrt{3}}{2} > \frac{\pi}{2}, also\ 0 <\downarrow 13 - \sqrt{2} < \frac{\pi}{4}$

$\left(\sqrt{3} - \sqrt{2} = 0.318 < \frac{\pi}{4}\right)$

Now, sin √2-sin√3=2 cos $\frac{\sqrt{2}+\sqrt{3}}{2}$ sin $\frac{\sqrt{2}-\sqrt{3}}{2} > 0$

And cos $\sqrt{2} - cos\sqrt{3} = 2\sin\frac{\sqrt{2}+\sqrt{3}}{2}\sin\frac{\sqrt{3}-\sqrt{2}}{2} > 0$

Sol.7

$\pi \log_3\left(\frac{1}{x}\right) = k\pi, k \in I$

$\log_3\left(\frac{1}{x}\right) = k \Rightarrow x = 3^{-k}$

Possible values of k are -1,0,1,2,3…..

$S = (3+1) + \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots \infty\right)$

=4+$\frac{(1/3)}{1-(1/3)} = 4 + \frac{1}{2} = \frac{9}{2}$

Sol.8

$\sin^2 x + \sin^4 x + \sin^6 x + \cdots . \infty = \frac{\sin^2 x}{1-\sin^2 x} = \tan^2 x$

∴We have,$e^{\tan^2 x . In\ 3 = 3^{\tan^2 x}}$ satisfies the equation

$(t-27)(t-1) = 0$

$\therefore 3^{\tan^2 x = 27\ or\ 3^{\tan^2 x}} = 1$

$\tan^2 x = 3$ as $\tan^2 x = 0$ (rejected think!)

$\tan x = \sqrt{3} or x \in \left(0, \frac{\pi}{2}\right) \Rightarrow x = \frac{\pi}{2}$

Now,$\frac{1}{\sin x + \cos x} = \frac{\sec x}{1+\tan x} = \frac{\sqrt{1+\tan^2 x}}{1+\tan x} = \frac{2}{\sqrt{3}+1} = \frac{2\sqrt{3}-1}{2} = (\sqrt{3}-1)$

Sol.9

L.H.S=$(1+2x^2+x^4)(1+C_1x+C_2x^2+C_3x^3+\cdots)$

RHS=$a_0+a_1x+a_2x^2+a_3x^3+\cdots$

Comparing the coefficients of x, x^2, x^3 ....

Now, $2a_2=a_1+a_3$

$2\left(n_{C_2+2}\right)=n_{C_1+\left(n_{C_3}+2^nC_1\right)}$

$2\frac{n(n-1)}{2}+4=3n+\frac{n(n-1)(n-2)}{6}$

Or $\quad n^3-9n^2+26n-24=0$

$\therefore (n-2)(n^2-7n+12)=0$ (∵8+52+=36+24)

Or $(n-2)(n-3)(n-4)=0$

$\therefore n=2,3,4$

Sol.10

$f(x)=\int_0^x e^{|t-x|}dt+\int_x^1 e^{|t-x|}dt=e^x+e^{1-x}-2$

$\Rightarrow f'(x)=e^x-e^{1-x}$

Clearly, $x=\frac{1}{2}$ is the point of minima of $f(x)$

Also,$f'(0)=f(1)=e-1$

Sol.11

$f(x)\frac{\sin(\pi\{x\})}{x^4+3x^2+7}$

Hence, $f\left(\frac{1}{2}\right)=f\left(-\frac{1}{2}\right)$ ⇒Clearly, $f(x)$ is not one-one and also it is dependent on $=x$.

Sol.12

Since, $f(x)$and $g(x)$are one-one and onto and are also the mirror images of each other which respect to the line $y=a$.It clearly indicates that $h(x)=f(x)+g(x)$ will be a constant function and will always be equal to 2a.

Sol.13

Let $B = 2A$ andBD be the bisector of angle B, then

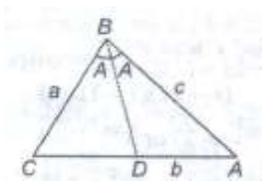
$CD = \frac{ab}{a+c}$ and $AD = \frac{ac}{a+c}$

Now, ΔABC and ΔBDC are similar,so

$$\frac{BC}{AC} = \frac{CD}{BC} \Rightarrow a^2 = \frac{ab}{a+c}b \Rightarrow b^2 = a(a+c) \dots (i)$$

Since, b>a⇒ Either $b = a + 1 \; or \; b = a + 2, if \; b = a + 1$,then

[from Eq.(i)]



$(a+1)^2 62 = (a+c)a \Rightarrow c = 2 + \frac{1}{a}$

∵c is integer $\Rightarrow a = 1, b = 2, c = 3$ but then, no triangle will from.

If $b = a + 2$, then obviously $c = a + 1$ and then, [from Eq. (i)]

$(a+2)^2 = a(2a+1)$

$\Rightarrow a^2 - 3a - 4 = 0 \; or \; a = 4$

∴ $a = 4, b = 6, c = 5$ is the only possible solution.

Sol.14

PV of $D = \frac{\lambda-1}{2}\hat{\imath} + 4\hat{\jmath} + \frac{\mu+2}{2}\hat{k}$

DR of $AD = \frac{\lambda-4}{2}, 1 \frac{\mu+8}{2}$

But direction ratios of AD should be $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

$$\Rightarrow \frac{\lambda-4}{2} = 1 = \frac{\mu+8}{2}$$

$\lambda = 6, \mu = 10$

$2\lambda - \mu = 2$

Sol.15

Provided that $x_1 + x_2 + x_3 = 24$

$$x_1^2 + x_2^2 + x_3^3 = 210$$

$$x_1 x_2 x_3 = 440$$

$\Rightarrow$ According to given question

$$x_1x_2 + x_2x_3 + x_3x_1 = \frac{(x_1 + x_2 + x_3)^2 - (x_1^2 + x_2^2 + x_3^2)}{2}$$

$\Rightarrow x_1x_2 + x_2x_3 + x_3x_1 = 183$

So, x_1, x_2, x_3 are the roots of the equation

$x^3 - 24x^2 + 183x - 440 = (x-5)(x-8)(x-11) = 0$

$\Rightarrow x = 5,8,11$

Number of possible permutations=6

Sol.16

$f(3) = 3, f(1) = 3, f(4) = f(2+1+1) = 2+1+1 = 4$ and so one.

In general, we get $f(r) = r \forall\, r \in N$

$\Rightarrow \lim_{n\to\infty} \frac{\sum_{r=1}^{n}(4r)f(3r)}{n^3} = \lim\limits_{n\to\infty} \frac{12n(n+1)(2n+1)}{6n^3} = 4$

Sol.17

In given ΔABC both $\frac{A}{2}$and $\frac{B}{2}$ lie strictly between $\left(0, \frac{\pi}{2}\right)$ and

$\sin x$ is always increasing in $\left(0, \frac{\pi}{2}\right)$whereas $\cos x$is always decreasing throughout $\left(0, \frac{\pi}{2}\right)$.

So, if $\frac{A}{2} > \frac{B}{2}$

$\Rightarrow \sin\frac{A}{2} > \sin\frac{B}{2}$ or $x_1 > x_2$

And $\frac{1}{x_3} > \frac{1}{x_4}$

So, $x_1^{2007} x_4^{2006} = x_2^{2007} x_3^{2006}$ is not valid.

Similarly , for $\frac{A}{2} < \frac{B}{2}$

$\Rightarrow \sin\frac{A}{2} < \sin\frac{B}{2}$

$\Rightarrow \quad x_1 < x_2$

And $\frac{1}{x_3} < \frac{1}{x_4}$ again equality is not possible,

Therefore, only possible case is when

$$\frac{A}{2} = \frac{B}{2} \Rightarrow \mathrm{x}_1 = \mathrm{x}_2$$

And $\quad \frac{1}{x_3} = \frac{1}{x_4}$

Hence, in that case ΔABC is isosceles with

$$\angle ABC = \angle CAB$$

$\Rightarrow BC = AC = 1unit$

Sol.19

(A)P (two faulty identified)+ P(two correct identified)

$$= 2\left(2\frac{C_2}{{}^4C_2}\right) = 2\left(\frac{1.2}{4.3}\right)$$

$$= 2\left(\frac{2}{4}.\frac{1}{3}\right) = \frac{1}{3}$$

(B) $n(s) = 6 \times 6 = 36$

$$n(E) = 12$$

$$p = \frac{12}{36}$$

$$= \frac{1}{3}$$

(C) k may be any real constant.

(D) $n(E) = 1, n(s) = 2$

Sol.20

(A) $x = \frac{a+b+c}{2}$

(B) Reationalize ans solve

(C) $\lim_{x\to 0} \frac{1}{3}\left(\frac{a^x-1}{x} + \frac{b^x-1}{x} + \frac{c^x-1}{x}\right)$

$= e^{\frac{1}{3}(\log_e a + \log_e b + \log_e c)} = \sqrt[3]{abc}$

(D) $f(1) = (a+b-c) + (b+c-a) + (c+a-b)$

$= a + b + c = 0$

As $\quad a^3 + b^3 + c^3 = 3abc$

$\Rightarrow (a+b+c)(a^2+b^2+c^2-ab-bc-ca) = 0$

$\Rightarrow \quad a + b + c = 0$