eVidyarthi
FREE Education

Practice Set for IIT JEE

Paper I

Objective Questions I [Only one correct option]

Q 1.

The number of lines in the xy-Plane, Whose distance from (-1, 2) is 2 and from (2, 6) is 3, is

a. 2

b. 3

c. 4

d. None of the above

Q 2.

Let p, q, r be rational numbers, p≠0. If $\begin{vmatrix} p & \pi \\ r & q \end{vmatrix}$= 0, then q3 + r^2 is equal to

a. -1

b. 1

c. Cannot be decided

d. 0

Q 3.

The maximum value of x, for which there is no real y, satisfying y^2+ 2y = 2x + 1

a. is 0

b. is -1

c. is -∞

d. does not exist

Q 4.

Let f (x) = $\begin{cases} e^{x^{\frac{1}{2}}},\ x \neq 0\ f'(0) is\ equal\ to \\ \qquad 0, x = 0 \end{cases}$

a. 0

b. 1

c. -1

d. does not exist

Q 5.

Set of value of x satisfying [$\sin^{-1}$x]= [$\cos^{-1}$x+is (*+ denotes greatest integer function

a. (cos 1, sin 1)

b. (sin 1, cos 1)

c. (cos 1, 1)

d. (sin 1, 1)

Q 6.

If u =$\cot^{-1}\sqrt{\tan \propto}\ - tan^{-1}\sqrt{\tan \propto}, then\ tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$is equal to

a. $\sqrt{\tan \propto}$

b. $\sqrt{\cot \propto}$

c. tan ∝

d. cot ∝

Q 7.

The equation $2\sin^2 x-(p+3)\sin x + 2p - 2 = 0$ possesses a real solution, if

a. $0 \le p \le 1$

b. $-1 \le p \le 3$

c. $4 \le p \le 6$

d. $p \ge 6$

Objective questions II {One or more than one correct option}

Q 8.

Which of the following functions have their range equal to R (the set of real numbers)?

a. $x\sin x$

b. $\frac{[x]}{\tan 2x}\cdot x \in \left(-\frac{\pi}{4}\cdot\frac{\pi}{4}\right)-$,0-, [·+ denotes the greatest integer function \

c. $\frac{x}{\sin x}$

d. $[x]+\sqrt{\{x\},[\cdot] and\ \{\cdot\}}$ respectively denote the greatest integer function and fractional part function

Q 9.

Let f :R → R be an even continuous function satisfying f $(x) = f\ (1-x) \forall\ x\ \in R, then$

a. graph of y = f $(x)$is symmetric about the line $(x) = \frac{1}{2}$

b. $\int_0^1 f(x)\,dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} f\left(\frac{1}{2}+x\right)\,dx$

c. f is periodic

d. $\int{}^{1+n}_{0} f(x)dx = n\int{}^{1}_{0} f(x)dx, n \in N\ , a\ \in R$

Q 10.

Tangents drawn from the point ($\propto, \propto^2$) to the curve $x^3 + 3y^2 = 9$ include an acute angle between them, if $\propto$ is equal to

a. tan4

b. cosec $\left(\frac{1}{\sqrt{3}}\right)$

c. $\sqrt{e+\frac{1}{e}}$

d. $\sqrt{2}+\frac{1}{\sqrt{2}}$

Q 11.

$(sin\theta + icon\theta)" \ = \ \sin\ n\theta, \forall\ \theta\ \in\ R, if\ n$ is equal to

a. −7

b. 4

c. 9

d. 17

Passage Based Problems

Direction (Q. No. 12 to 14) Let f : N→ N (N being the set of positive integers) be a function defined by f (x)= the biggest positive integer obtained by reshuffling the digits of x. For example f (296) = 962.

$\Rightarrow 2(2\cos^2\theta-1)^2 - 1 = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow 8\cos^4\theta - 4\cos^3\theta - 8\cos^2\theta + 3\cos\theta + 1 = 0$

Now, answer the following questions. Thus, cos θ is a root of the equation

P (x) ≡ $8x^4 - 4x^3 - 8x^3 + 3x + 1 = 0, \theta = \frac{2n\pi}{7}, n \in I$

Q 12.

f is

a. one-one and onto

b. one-one and into

c. many-one and onto

d. many-one and into

Q 13.

The biggest positive integer which divides $f(n) - n, for\ all\ n \in N, is$

a. 3

b. 9

c. 18

d. 27

Q 14.

The range of f is

a. N

b. set of positive integers whose digits are non-increasing from left to right

c. set of positive integers whose digits are non- decreasing from left to right

d. set of positive integers whose digits decrease from left to right

Directions (Q. No. 15 to17) Let θ = $\frac{2n\pi}{7}, n \in I$

⇒ $2n\pi - 4\theta = 3\theta$

⇒ $\cos 4\theta = \cos 3\theta$

⇒ $2\cos^2 2\theta - 1 = 4\cos^3\theta - 3\cos\theta$

⇒ $2(2\cos^2\theta - 1)^2 - 1 = 4\cos^3\theta - 3\cos\theta$

⇒ $8\cos^4\theta - 4\cos^3\theta - 8\cos^2\theta + 3\cos\theta + 1 = 0$

Now, the following a root of the equation. Thus cos θ is a root of the equation

$P(x) \equiv 8x^4 - 4x^3 - 8x^2 + 3x + 1 = 0, \theta = \frac{2n\pi}{7}, n \in I$

Q 15.

Which of the following a root of the equation $p(x) = 0$?

a. – 1

b. $\sin\frac{3\pi}{14}$

c. $\sec\frac{2\pi}{7}$

d. $\cos\frac{3\pi}{7} + cos\,\frac{\pi}{7}$

Q 16.

The value of $\cos\frac{2\pi}{7} \cdot cos\,\frac{4\pi}{7} \cdot cos\,\frac{8\pi}{7}$ is equal to

a. $\frac{1}{8}$

b. $\frac{3}{8}$

c. $\frac{1}{2}$

d. None of the above

Q 17.

Let p be the number of positive roots and N be the number of negative roots of the equation $p(x) = 0$, then |P-Q| is

a. 4

b. 2

c. 1

d. 0

Integer Answer Type Questions

Q 18.

If the variable line 3x – 4y + k= 0 lies between the circles $x^2 + y^2 - 2x - 2y + 1 = 0$ without intersecting or touching either circle, then the range of k is (a, b) where a, b ∈ I. Find the value of (b – a).

Q 19.

The locus of a point the divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1: 2 is a parabola with vertex (a, b) and length of latusrectum l. Then, the value of 3(a + b –l) is..... .

Q 20.

If $\sum_{p=1}^{n} \sum_{m=p}^{n} {}^{n}C_{m}.\, {}^{m}C_{p} = 19$, then find value of n.

Q 21.

If f (x) = In $(x^2 - x + 2)$; $R^+ \to R$ and g (x) = ,x- + 1 ; *1,2+ → *1, 2+where ,x- denotes frectional part of x. If the domain and range of f (g(x)) are [a, b] and [c, d] respectively (a<b, c< d), then find the value of $\frac{b}{a} + \frac{d}{c}$.

Q 22.

Iff (x) = $\begin{cases} \frac{\tan[x^2]\pi}{ax^2} + ax^3 + b, & 0 \le x \le 1 \\ 2\cos \pi x + tan{-}^1 x, 1 < x \le 2 \end{cases}$ is differentiable in [0, 2], then a = $\frac{1}{k_1}$ $and\ b = \frac{\pi}{4} - \frac{26}{k_2}$. Then, $k_2 - k_1$ is equal to …….. ,Where [·] denotes greatest integer function-.

Q 23.

If all values of x ∈ $\left(0, \frac{\pi}{2}\right)$, then find the maximum value of $\frac{36}{\pi}$ (b – a)

Q 24.

The set of all points where f(x) is increasing is (a, b) ∪ (c, ∞), then find *a + b +c+……. ,where *.+ denotes G.I.F} . Given that f(x) = 2F $\left(\frac{x^2}{2}\right) + f(6 - x^2)\ \forall\, x\ \in R$ and $f''^{(x)} > 0, \forall\, x\ \in R$.

Paper II

Objective Questions I [Only one correct option]

Q 1.

The maximum power of 7, present in 2, 4, 6, 8,…, 998. 1000, is

a. 82

b. 92

c. 102

d. 81

Q 2.

The locus of the middle point of the part of a line through (1, − 3) which lies between the lines y = x and y = 3x is

a. a parabola

b. an ellipse

c. a hyperbola

d. None of these

Q 3.

Secants are drawn from the point $p(-1, 3)$ to the curve $x^2 + y^2 - 2x + 4y - 8 = 0$, which meets the circle at A and B. The minimum value of *PA +PB* is

a. 0

b. 4

c. 8

d. 16

Q 4.

$\lim_{n\to\infty} (\sin^n 1+\cos^n 1)^n$ is equal to

a. cot 1

b. tan 1

c. cos 1

d. sin 1

Q 5.

The number of ordered pairs (x, y)such that cos (x + y) = cos (x − y), where x, y ∈ $[-2\pi, 2\pi]$, is

a. 0

b. 5

c. 10

d. not finite

Q 6.

If $f'(a) = \frac{1}{4}$, then $\lim_{h\to 0} \frac{f(a+2h^2)-f(a-2h^2)}{f(a+h^3-h^2)-f(a-h^3+h^2)}$ equal to

a. 0

b. 1

c. -2

d. None of these

Q 7.

Minimum distance between two points P and Q, where P lies on the parabola $y^2 - x + 2 = 0$ and Q, where $x^2 - y + 2 = 0$ is

a. $7\sqrt{2}$ unit

b. 4 unit

c . $\frac{7}{2\sqrt{2}}$ unit

d. None of these

Q 8.

A plane mirror and a point source of light are situated at the origin O and a point on OX respectively. A ray of light along x – axis from the source strikes the mirror and is reflected. If the d. r.' s (direction ratio) of the normal of the plane of the mirror are 1, -1, 1 and d. c. 's (direction cosines of the reflected ray are

a. $\frac{1}{2}, \frac{2}{3}, \frac{2}{3}$

b. $-\frac{1}{2}, \frac{2}{3}, \frac{2}{3}$

c. $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$

d. $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

Objective Questions II [One or more than one correct option]

Q 9.

The function f (x) = a (x^2 −1) (ax + b) (a ≠ 0) has

a. a local maxima at certain x ∈ R^+

b. a local minima at certain x ∈ R^+

c. a local maxima at certain x ∈ R^-

d. a local minima at certain x ∈ R^-

Q 10.

If p (∝, β), the point of intersection of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$ and the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{a^2(E^2-1)} = \frac{1}{4}$, is equidistant from the foci of the two curves (all lying in the right of y-axis), then

a. 2∝ = a (2e + E)

b. a − e∝ = E∝ − a/2

c. E = $\frac{\sqrt{e^2+24-3e}}{2}$

d. E = $\frac{e^2+12-3e}{2}$

Q 11.

The solution set of |sin x|≤|cos 2x | contains

a. $\bigcup_{n\in I}\left\{\left[n\pi - \frac{\pi}{6}, n\pi + \frac{\pi}{6}\right]\right\}$

b. $\bigcup_{n\in I}\left\{n\pi + \frac{\pi}{2}\right\}$

c. $\bigcup_{n\in I}\left\{\left[n\pi - \frac{\pi}{8}, n\pi + \frac{\pi}{8}\right]\right\}$

d. $\bigcup_{n\in I}\left\{\left[n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{4}\right]\right\}$

Q 12.

∝, β, y are the roots of $x^3 - 3x^2 + 3x + 7 = 0$ (ω is cube root of unity), then $\left(\frac{\propto-1}{\beta-1} + \frac{\beta-1}{y-1} + \frac{y-1}{\propto-1}\right)$ is

a. $\frac{3}{\omega}$

b. ω^2

c. $2\omega^2$

d. $3\omega^2$

Integer Answer Type Questions

Q 13.

Let P $(()\propto_1, \beta_1)$, Q $(\propto_2, \beta_2)$ and R $(\propto_3, \beta_3)$ be the centroid, orthocenter and circumcentre of a scalene triangle having its vertices on the curve $y^2 = x^3$, then $\frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3}$ is equal to ....

Q 14.

If the number of ordered pairs of (x, y) satisfying the system of equation $5x\left(1 + \frac{1}{x^{2+y^2}}\right) = 12$and 5 $y\left(1 - \frac{1}{x^{2+y^2}}\right) = 4$ is n, then n is ....

Q 15.

Consider the sequence $a_n$given by $a_1 = \frac{1}{3}, a_{n+1} = a_n^2 + a_n$. Let S = $\frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{a_{2008}}$,then *S+is equal to ....(where[.] represents greatest integer function)

Q 16.

Let y=g(x) be the image of f (x) = x + sin x about the line x + y = 0. If the area bounded by y = g (x), x-axis, x = 0 and x = 2π is A, then $\frac{A}{\pi^2}$ is ....

Q 17.

If $\int_0^1 \frac{(1-x^2)dx}{(1+x^2+x^4)} = \text{In } b$, then find $2b^2$

Q 18.

If trace (A)> 0 and abc = 1 where $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$and $AA' = I$, then find the value of $a^3 + b^3 + c^3$.

Match the columns

Q 19.

Match the statements of Column I with values of Column II.

	Column I		Column II
(A)	Three vectors are collinear	(p)	The volume of the parallelopiped formed by the vectors
(B)	Three vectors are coplanar	(q)	The volume of the parallelopiped formed by the vectors is non-zero
(C)	Three vectors are non-coplanar	(r)	There is a plane which contain all the three vectors
(D)	Three non-zero vectors are such that exactly two of them are collinear	(s)	The vectors are position vectors of three collinear points

Q 20.

Match the statements of Column I with values of Column II.

	Column I		Column II
(A)	The maximum value of sin (cos x)+cos (sin x), x $\epsilon\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ is	(p)	cos (cos 1)
(B)	The minimum value of sin (cos x)+cos(sin x), x $\epsilon\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ is	(q)	1+cos 1
(C)	The maximum value of cos (cos (sin x)) is	(r)	cos 1
(D)	The minimum value of cos (cos (sin x)) is	(s)	1+ sin 1