PRACTICE PAPER 4

SECTION-I

Straight Objective Type

This section contains 9 multiple choice question numbered 1 to 9. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Q1

Let a, b, c be three real numbers such that a< b< c. Let *f*(x) be continuous in [a, c] and differentiable is (a, c). If *f'*(x) is strictly increasing in (a, c), then

a. $(c-b)f(a)+(b-a)\,f(c)>(c-a)\,f(b)$

b. $(c-b)f(a)+(b-a)\,f\,(c)<(c-a)\,f(b)$

c.$f(a)<\ f(b)<\ f(c)$

d. None of the above

Q2

The number of rational points on the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is

a. none

b. two

c. four

d. Infinite

Q3

The number of non-negative continuous functions on [0, 1] satisfying $\int_0^1 f(x)\,dx = 1, \int_0^1 xf(x)\,dx = a \text{ and } \int_0^1 x^2\,f(x)\,dx = a^2, (a \neq 0)$ is

a. none

b. two

c. four

d. Infinite

Q4

The hypotenuse of a right angled triangle passes through the point (2, 4) and the sides are along x and y-axis. The number of such triangles having area 18 units must be

a. 1

b. 2

c. 3

d. 4

Q5

Three numbers are drawn from the set $\{1, 2, 3, \dots, n\}$ with replacement. The probability that their sum is 2n, is

a. $\frac{1}{2_n}$

b. $\frac{1}{2}$

c. $\frac{(n-1)(n+4)}{2n^3}$

d. $\frac{(n-1)(n+2)}{n^3}$

Q6

Among the following points there is only point from which tangents can be drawn to the ellipse $3x^2 + 4y^2 = 12$ are perpendicular. The point must be

a. $(3, 4)$

b. $(3, \sqrt{2})$

c. $(-2, \sqrt{3})$

d. $(1, 2)$

Q7

The minimum value of $\sin 3A + \sin 3B + \sin 3C$ in a triangle must be

a. -1

b. -2

c. $-\frac{\sqrt{3}}{2}$

d. None of the above

Q8

If the perimeter of a triangle is 2 then the expression $E = ab + bc + ac - abc - 1$

a. is essentially positive

b. is essentially negative

c. may or may not be positive

d. None of these

Q9

Let $S = \sum_{r=1}^{2000} \sqrt{1 + \frac{1}{r^2} + \frac{1}{(r+1)^2}}$. Then S must be equal to

a. $2000\left(1 + \frac{1}{2001}\right)$

b. $2000\left(1 + \frac{1}{1999}\right)$

c. $\sqrt{2000}\left(1 + \frac{1}{2001}\right)$

d. None of these

Section-II

Multiple Objective Type

Q10

The values of a for which $x^4 - 2ax^2 + x + a^2 - a = 0$ has all real roots are

a. -1

b. 1

c. 2

d. 3

Q11

If y is a function of x given by $2\log(y-1) - \log x - log(y-2) = 0$, then

a. domain is[4, ∞]

b. domain is [0, ∞]

c. range is (2, ∞)

d. range is (0, ∞)

Q12

If ordered pair (∝, β) where ∝, β ∈ 1 satisfy the equation $2x^2 - 3xy - 2y^2 = 7$, then value of ∝ + β can be

a. 5

b. 4

c. −4

d. 3

Q13

For the parabola $y^2 = 4x$, let P be the point of concurrency of three normals and S be the focus. If $\propto_1$ be the sum of the angles made by three normals from the positive direction of x-axis and $\propto_2$ be the angle made by PS with the positive direction of x-axis then can be equal to

a. 1

b. 2

c. 1/2

d. 3/2

Q14

Let $\left(\frac{p_1}{q_1}, \frac{p_2}{q_2}\right)$ and $\left(\frac{a_1}{b_1}, \frac{a_2}{b_2}\right)$ be any two rational points on the circle $x^2 + y^2 = 1$ where $p_1, p_2, q_1, q_2, a_1, a_2, b_1$ and b_2 are integers and H.C.F. of $(p_1, q_1), (p_2, q_2), (a_1, b_1)$ and (a_2, b_2) is 1. Then the statements which are always correct are

a. $q_1 = q_2$

b. $p_1 = \pm 1 \; or \; 0$

c. $b_1 = b_2$

d. $a_1 = \pm 1 \; or \; 0$

Q15

If all the roots of the equation $(x^2 - mx + n)(x^2 - nx + m) = 0$ are positive integers then m + n can be equal to

a. 8

b. 9

c. 10

d. 11

Q16

Let a, b, c be the lengths of the sides of a triangle ABC such that $b+c \neq 1, c-b \neq 1$. If $log_{b+c}\, a + log_{c-b}\, a = 2log_{c+b} a log_{c-b} a$, then

a. $sin^2 A + sin^2 B = sin^2 C$

b. $\tan A + \tan B = 1$

c. $A + B = C$

d. $cos^2 A + cos^2 B = 1$

Q17

If $f(x) = \int_{x"}^{x"} \frac{dt}{In\, t}, x > 0 \;\; and\; n > m$, then

a. $f'(x) = \frac{x^{m-1}(x-1)}{In\, x}$

b. $f(x)$ is decreasing for$x > 1$

c. $f(x)$is increasing in (0, 1)

d. $f(x)$is increasing for $x > 1$

Section-III

Assertion-Reason Type

Q18

Statement-1:

If n is odd then the product$P = (1-i_1)(2-i_2)(3-i_3)\ldots(n-i_n)$ where $i_1, i_2, i_3, \ldots, i_n$are distinct integers taken from the set {1, 2, 3, …, n} is certainly even. Because

Statement-2:

P can be zero for some choice of $i_1, i_2, \ldots, i_n$.

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

c. Statement-1 is True, Statement-2 is False

d. Statement-1 is True, Statement-2 is True

Q19

Statement-1:

For any large positive integer n, the integer next to $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{2n-1}-\frac{1}{2n}$ is 2.

Statement-2:

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots-\frac{1}{2n}=\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n-1}$$

Q20

Statement-1:

If n leaves remainder 2 when divided by 3 then 3^n-1 leaves remainder 8 when divided by 13.
because

Statement-2:

$3^5-1=242$ leaves remainder 8 when divided by 13.

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1 c. Statement-1 is True, Statement-2 is False
d. Statement-1 is True, Statement-2 is True

Q21

Statement-2:

$\tan^{-1}\frac{1}{5}$Is approximately equal to$\frac{\pi}{16}$. because

Statement-1:

If$5+i=\sqrt{26}(\cos\theta+i\sin\theta)$, then$(5+i)^4=476+480\,i=676\,(\cos 4\theta+i\sin 4\theta)$

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
c. Statement-1 is True, Statement-2 is False
d. Statement-1 is True, Statement-2 is True

Section-IV

Linked Comprehension Type

$M_{22\text{-}24}$: Paragraph for Question Nos. 22 to 24

Let $I_{m,\ n} = \int_0^{\pi/2} cos^m\, x \cos nx\, dx$

Q22

If m and n are non-negative integers, then

a. $I_{m,\ n} > 0\ for\ all\ m\ and\ n$

b. $I_{m,\ n} = 0\ for\ some\ m\ and\ n$

c. $I_{m,\ n} < 0\ for\ some\ m\ and\ n$

d. None of these

Q23

$\frac{I_{m,\ n}}{I_{m-2,\ n}}$ must be equal to

a. $\frac{m(m-1)}{m^2+n^2}$

b. $\frac{m(m-1)}{m^2-n^2}$

c. $\frac{m(m-2)}{m^2-n^2}$

d. None of these

Q24

$I_{n,\ n}$ must be equal to

a. $\frac{\pi}{4}$

b. $\frac{\pi}{2n}$

c. $\frac{\pi}{2^{n+1}}$

d. None of these

$M_{25\text{-}27}$: Paragraph for Question Nos. 25 to 27

Let ∝, β, y be positive roots of the equation $x^3 + ax^2 + bx + c = 0$. answer the following questions

Q25

If $c = -1/64$ then minimum value of $\propto + \beta + y$ must be

a. 1/3

b. ¼

c. ½

d. ¾

Q26

If a = −1 then maximum value of $\propto \beta^2 y^3$ must be

a. 3/2

b. 1/2

c. 1/432

d. 1/64

Q27

If $c = -1/64$ such that $(\propto +\beta)^3 - 27 \propto \beta\, \gamma \leq 0$ then (a + b) must be equal to

a. $-\frac{9}{16}$

b. $\frac{9}{16}$

c. $-\frac{9}{32}$

d. $-\frac{3}{4}$

Section-V

Subject Type

This section contains 3 questions. Write the answer of the questions (28-31) from the following combinations:

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Q28

If a, b, c are positive integers which are in increasing G.P. If $log_6\, a + log_6\, b + log_6\, c = 6\ and\ b - a$ is a perfect cube then the numerical value of a + b + c must be equal to

Q29

If $x, y, z > 0$ then minimum value of $\frac{x^4+y^4+z^4}{xyz}\ is\ \sqrt{\lambda}$ the λ must be equal to

Q30

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} \sin 60° & \cos 60° \\ -\cos 60° & \sin 60° \end{bmatrix}$. Then $(BB^T A)^{2007} = \begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix}$, the numerical quantity λ must be

Q31

If the roots of the equation $x^2 + ax + b + 1 = 0$ are distinct positive integers, then min. value $a^2 + b^2$ must be equal to