Practice Paper 5

Section-I

Straight Objective Type

Q1

The number of positive integers satisfying $\left[\frac{x}{99}\right] = \left[\frac{x}{101}\right]$ must be

a. 2491

b. 2495

c. 2498

d. 2499

Q2

The foci of the ellipse $x^2 + 4x + 4\lambda^2 y^2 = 0$ will lie on the line $x = -2$, if

a. $\lambda \in (-2, 2)$

b. $\lambda \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

c. $\lambda = 1$

d. None of these

Q3

The point in the closed di$D = \{(x, y): x^2 + y^2 \leq 1\}$sc at which the function $x + y$ attains its maximum is

a. $\left(\frac{1}{2}, \frac{3}{4}\right)$

b. $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

c. (0, 1)

d. (1, 0)

Q4

Let $P(x) = x^4 + ax^3 + bx^3 + cx + d$, where a, b, c, d are integers. Then sum of pairs of roots of $P(x)$ are given by 1, 2, 5, 6, 9, 10 then $P"\left(\frac{1}{2}\right)$ must be

a. 33

b. 28

c. −28

d. None of these

Q5

Let two arithmetic means and two geometric means between ∝ and β (∝ > β > 0) be A_1, A_2 and $G_1,\ G_1$ respectively then

a. $A_1A_2 < G_1G_1$

b. $A_1A_2 > G_1G_1$

c. $A_1A_2 = G_1G_1$

d. None of these

Q6

The number of integral values of a for which the lines $x - 4y = 1$ and ax+3y=1 intersect at an integer point must be

a. 0

b. 1

c. 3

d. infinite

Q7

The range of the function $\sin^{-1}\left(\frac{x^2+x+1}{x^4+1}\right)$ is

a. $\left[0, \frac{\pi}{4}\right]$

b. $\left[0, \frac{\pi}{2}\right]$

c. $\left[0, \frac{\pi}{2}\right]$

d. None of these

Q8

If m and n are arbitrary positive integers then $(1 + \omega + \omega^2 + \ldots + \omega^n)^m$ (ω being a non-real complex cube root of unity) will take

a. 3 values

b. 5 values

c. 7 values

d. mn values

Q9

If $f(x)$ monotonically increasing and is differentiable on $[\propto, \beta,]$ $then$ $\int_{\propto}^{\beta} f(x)dx + \int_{f(\propto)}^{f(\beta)} f^{-1}(x)dx$ is equal to:

a. $f(\propto) - f(\beta)$

b. $\beta f(\propto) - \propto f(\beta)$

c. $\beta f(\beta) - \propto f(\propto)$

d. $f(\propto) + f(\beta)$

Section-II

Multiple Objective Type

Q10

If P is a point inside a convex quadrilateral $ABCD$ such that $PA^2 + PB^2 + PC^2 + PD^2$ is twice the area of the quadrilateral, then the correct statement is/are

a. PA, PB, PC, PD all are equal

b. $ABCD$ must be a square and P must be its Centre

c. $ABCD$ must be a square but P may not be its Centre

d. $ABCD$ may not be a square

Q11

If $f(x)$ and $g(x)$ are two monotonically increasing functions such that $g(x) = f(x)\sqrt{1-2(f(x))^2}$ are then for values of x in the domain of $f(x)$

a. $|f(x)| \leq 1$

b. $|f(x)| < \frac{2}{3}$

c. $|f(x)| < \frac{1}{2}$

d. $|f(x)| < \frac{1}{\sqrt{2}}$

Q12

Eight players of equal strength participate in a round robin tournament (Each player plays against any other player exactly once). Suppose no game ends in a draw then

a. Total results in the tournament can be 256

b. Total results in the tournament can be 2^{28}

c. The number of cases in which each team has won different number of games must be 56

d. The number of cases in which each team has won different number of games must be 8!.

Q13

The equation of an ellipse is $(x + 2y - 3)^2 + 4(2x - y - 4)^2 = 10$ then

a. Centre of the ellipse is $\left(\frac{11}{5}, \frac{2}{5}\right)$

b. eccentricity is $\frac{\sqrt{3}}{2}$

c. foci is $\left(\frac{11}{5} + \sqrt{\frac{3}{5}}, \frac{2}{5} - \frac{3}{2\sqrt{5}}\right)$

d. foci is $\left(\frac{11}{5} - \sqrt{\frac{3}{5}}, \frac{2}{5} + \frac{3}{2\sqrt{5}}\right)$

Q14

Which of the following is (are true in a triangle)

a. $R^2 \geq \frac{abc}{a+b+c}$

b. if the triangle is right angled at C then $r + 2R = s$

c. $\sin 2A + \sin 2B + \sin 2C \leq \frac{3\sqrt{3}}{2}$

d. $\frac{2}{R} \leq \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

Q15

In a cube of side a let A and F be the opposite corners such that AF is a solid diagonal, then

a. length of any solid diagonal is a$\sqrt{3}$

b. length of any plane diagonal is a$\sqrt{2}$

c. shortest distance between A and F through plane faces of the cubic $a(\sqrt{2}+1)$

d. shortest distance between A *and F through plane faces of the cube is* $a\sqrt{5}$

Q16

If $x \in \left(0, \frac{\pi}{2}\right)$ then the minimum value of $\tan x + \cot x + sec\, x + cosec\, x$ must be

a. more than 4

b. $2(\sqrt{2}+1)$

c. less than 5

d. $2\sqrt{2}$.

Q17

The equation $x^4 + 4x + a = 0$, where a is real has

a. no real roots if $a > 2$

b. no real roots if $a > 3$

c. two equal roots if $a = 3$

d. two distinct real roots if $a < 3$

Section-III

Assertion-Reason Type

Q18

Statement-1:

If $a > b > 0$ then eccentricity of the ellipse $ax^2 + by^2 + cx + dy + e = 0\ is\ \sqrt{\frac{a-b}{b}}$ because

Statement-2:

Eccentricity e of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ is given by $b^2 = a^2(1 - e^2)$

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

c. Statement-1 is True, Statement-2 is False

d. Statement-1 is True, Statement-2 is True

Q19

Statement-1:

If a tangent to the curve $x^3 + y^3 = a^3$ at (x_1, y_1) passes through (a, a) then (x_1, y_1) lies on a circle of radius a because

Statement-2:

The curve $x^3 + y^3 = a^3$ and $x^2 + y^2 = a^2$ cut orthogonally.

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

c. Statement-1 is True, Statement-2 is False

d. Statement-1 is True, Statement-2 is True

Q20

Statement-1:

The curve $x^3 + y^3 = a^3$ and $x^3 + y^3 = a^2$ cut orthogonally.

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

c. Statement-1 is True, Statement-2 is False

d. Statement-1 is True, Statement-2 is True

Statement-2:

The solution to the differential equation $yy" + xy'^2 = 3yy'$ is $y = C_1x^3 + C_2$

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

c. Statement-1 is True, Statement-2 is False

d. Statement-1 is True, Statement-2 is True

Q21

Statement-1:

The inequality $3x^3 - 4x^2 - 3x - 2 < 0$id equivalent to $x - 2 < 0$.

Statement-2:

$ax^3 + bx + c > 0$ for all x of $a > 0, b^2 - 4ac > 0$.

a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

c. Statement-1 is True, Statement-2 is False

d. Statement-1 is True, Statement-2 is True

Section-IV

Linked comprehension type

M22-24: Paragraph for Question Nos. 22 to 24

Consider the differential equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$ and answer the following questions:

Q22

By putting z = log x, the differential takes the form.

a. $\frac{d^2y}{dz^2} - 2\frac{dy}{dz} + y = 0$

b. $\frac{d^2y}{dz^2} - y = 0$

c. $\frac{d^2y}{dx^2} + y = 0$

d. None of these

Q23

The solution of the differential equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$ is

a. $y = (c_1 + c_2x)ex$

b. $y = C_1 \sin(x + C_2)$

c. $y = C_1x + \frac{C_2}{x}$

d. None of these

Q24

The family of curves satisfying the given differential equations cannot be

a. circles

b. lines

c. rectangular hyperbolas

d. $y = C_1x + \frac{C_2}{x}$

M25-27: Paragraph for Question Nos. 25 to 27

Let a_1, a_2, a_3 ... be a sequence of complex numbers defined by $a_1 = 0 a_{n+1} = a_n^2 - i \; for \; n > 1$.

Q25

a_{2n} Must be

a. 1 + i

b. 1 – i

c. − 1 – i

d. i

Q26

a_{2n+1} must be

a. 1 + i

b. 1 – i

c. − 1 – i

d. i

Q27

The distance between the points represented by complex numbers d. a_{2008} *and* a_{2009} must be

a. $\sqrt{2}$

b. $\sqrt{3}$

c. 2

d. $\sqrt{5}$

Section-V

Subjective Type

This section contains 3 Question. Write the answer of the questions (28-31) from the following combinations

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Q28

The value of definite integeral $\frac{4}{\pi}\int_0^\infty \frac{xdx}{(1+x)(1+x^2)}$ must be

Q29

The angle of intersection of curves $y = (x-2)^2$ and $y = 4x - x^2 + 4$ must be $\tan^{-1}\frac{8}{\lambda}$, the numerical quantity λ should be

Q30

A normal to the curve $y = x\log x$ parallel to the line $2x - 2y + 3 = 0 is\ x - y = \frac{\lambda}{e^2}$, the numerical quantity λ should be

Q31

If the shortest distance between the circle $x^2 + y^2 - 3x - \sqrt{8}\,y + 4 = 0$ and the parabola $y = 1 - \frac{x^2}{2}$ is $\frac{\sqrt{\lambda}-1}{2}$ then λ should be

Section-VI

Matrix-Match Type

Q32

If $a + b = 1, a^2 + b^2 = 2$, then match the following:

Column I	**Column II**
a. $a^3 + b^3$	p. 19/4
b. $a^4 + b^4$	q. 7/2
c. $a^5 + b^5$	r. 5/2
d. $a^7 + b^7$	s. 71/8

Q33

Let $ABCDEF$ be a convex Hexagon in2 dimensional plane where A is origin $AB \| DE, BC \| EF$ *and* $CD \| FA$. They coordinates of vertices A, B, C, D, E, F *are the five distinct elements of the set* $\{2, 4, 6, 8, 10\}$.

Match the x-co-ordinates of these vertices.

Column I	**Column II**
a. A	p. $6\sqrt{3}$
b. B	q. 2
c. C	r. $\frac{10}{\sqrt{3}}$
d. D	s. 0
e. E	t. $-\frac{8}{\sqrt{3}}$

Q34

Match the domain of following function [0, 2π]:

Column I	**Column II**
a. $\sqrt{\tan x}$	p. $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \cup \left(\pi, \frac{3\pi}{2}\right)$
b. $\tan x + \cot x$	q. $\left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$
c. $\log(1 + tanx)$	r. $[0, 2\pi]$
d. $\tan(\sin x)$	s. $\left(\frac{3\pi}{4}, \frac{3\pi}{2}\right) \cup \left[\frac{7\pi}{4}, 2\pi\right]$