ANSWER KEYS

CHEMISTRY

1. a	2. c	3. c	4. b	5. d	6. d	7. a	8. a	9. a	10. d	11. b	12. b	13. d
14. d	15. a	16. d	17. a	18. d	19. c	20. d	21. b	22. b	23. b	24. a	25. d	26. a
27. c	28. b	29. b	30. b									

PHYSICS

1. b	2. b	3. a	4. c	5. b	6. a	7. a	8. c	9. a	10. c	11. d	12. b	13. a
14. a	15. c	16. d	17. b	18. d	19. a	20. b	21. a	22. a	23. b	24. d	25. d	26. b
27. b	28. b	29. c	30. b									

MATHEMATICS

1. c	2. a	3. a	4. b	5. c	6. b	7. a	8. b	9. a	10. c	11. c	12. a	13. a
14. a	15. c	16. a	17. b	18. d	19. a	20. a	21. b	22. a	23. d	24. d	25. a	26. a
27. c	28. b	29. b	30. c									

HINTS AND EXPLANATIONS

CHEMISTRY

Sol 1.

Amount of A ($t_{½}$ = 20 min.) left after one hour or 3 hlf life periods = 1 → 1/2 → ¼ → 1/8 Amount of A ($t_{½}$= 10 min.) left after one hour or 6 half life periods =

1 → 1/2 → ¼ → 1/8 → 1/16 → 1/32 → 1/64

Mole ratio of A and B after one hour = $\frac{1}{8} : \frac{1}{64}$ or 64 : 8 or 8 :1

Sol 2.

NaOH HCl

$M_1 V_1 – M_2 V_2 = M_3 V_3$

0.45 x 10 – 0.1 x 40 = M_3 x 50

4.5 – 4.0 = 0.5 = M_3 x 50

M_3 = 0.5 / 50 = 0.01 or 1 x 10^{-2}

$[OH^-] = 10^{-2}$;∴ $[H^+] = 10^{-12}$ and pH = - log 10^{-12} = 12

Sol 3.

SO_2 → S; change in oxidation number of

S (+ 4 → 0) = 4 Equivalent mass of SO_2 =

Molecular mass/4 = 64 / 4 = 16

Sol 4.

$N_2O_4 \rightleftharpoons 2NO_2$

1	0
1 – 0.2	2 x 0.2;

Total moles = 0.8 + 0.4 = 1.2

When temperature becomes double at constant volume, pressure will also become double, i.e., 2atm. Since number of moles are also changing,

Total pressure = 1.2 x 2 = 2.4 atm.

Sol 5.

Chromatographic technique is based on differential adsorption of different constituents of a mixture on a stationary phase.

Sol 6.

One Faraday will liberate 0.5 g mole of Be from Be^{2+} ; 0.5 g mol of Cu from Cu^{2+} ; 0.33 g mol of Al from Al^{3+} and 1 g mole of Na from Na^{+}.

Sol 7.

On passing H_2S in dilute HCI solution, cations of group II (Hg^{2+}, Pb^{2+}, Cu^{2+} Cd^{2+}) are precipitated so cations from options (2), (3) and (4) can be separated. Al^{3+} (group III cation) and Sn^{2+} are no precipitate.

Sol 8.

Element A can lose one of its valence electron easily and element B can accept one electron easily to achieve stable noble gas electronic configuration. So the correct formula of the compound formed between A and B is $A^{+}B^{-}$.

Sol 9.

Total number in NO_3^{-} = 7 + 24 + 1 = 32; CO_3^{2-} = 6 + 24 + 2 = 32; ClO_3^{-} = 17 + 24 + 1 = 42 and in SO_3 = 16 + 24 = 40. Therefore, ions NO_3^{-} and CO_3^{2-} are isoelectronic; also these two have same structures, i.e., are isostructural.

Sol 10.

$Cu^{2+} + e \rightarrow Cu^{+}$ E° = 0. 15 V (1)

$Cu^{+} \rightarrow Cu^{2+} + e$ E° = - 0.15 V (2)

$Cu^{2+} + 2e \rightarrow Cu$ E° = 0.34 V (3)

On adding equations 2 and 3, we get.

$Cu^{+} + e \rightarrow Cu$ E° = (2 x 0.34 – 0.15) = 0.53V (4)on adding equations (2) and (4), we get the desired reaction;

$2Cu^{+} \rightarrow Cu^{2+} + Cu$ E° = 0.53 – 0.15 = 0.38 V

Sol 11.

$K_{sp} = [Ag^{+}] [Cl^{-}] = 1.1 \times 10^{-10}$; given $[Ag^{+}] = 1.1 \times 10^{-7}$mol / L.

For precipitation to occur $C_{ionic} > K_{sp}$

Thus when conc. Of $[Cl^{-}] > 1.1 \times 10^{-3}$; precipitation will occur.

Sol 12.

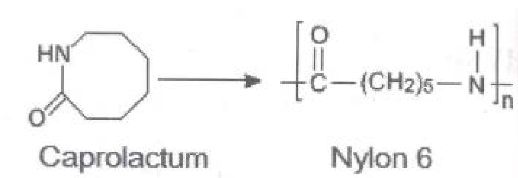
ΔG

Sol 13.

Na_2SO_4 is not suitable for use in desiccators.

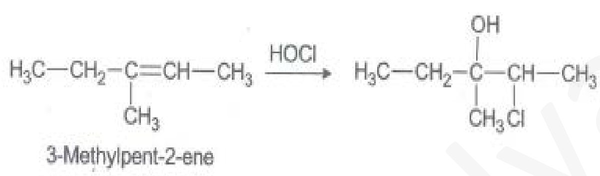
Sol 14.

Caprolactum polymerizes to give Nylon g/



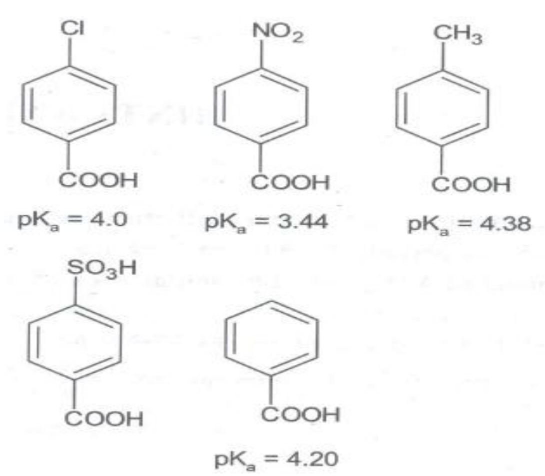
Sol 15.

3 – Methylpent – 2 – ene reacts with HOCI as shown below.



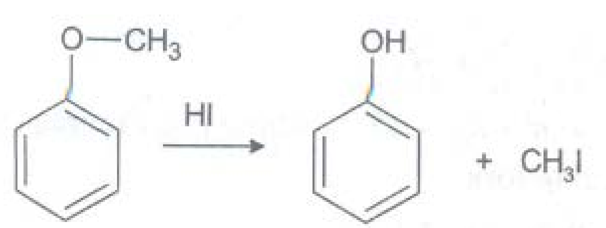
Sol 16.

As compared to benzoic acid (pKa = 4.20), p – CH_3-C_6 C_4 COOH (pKa = 4.38) is a weaker acid.



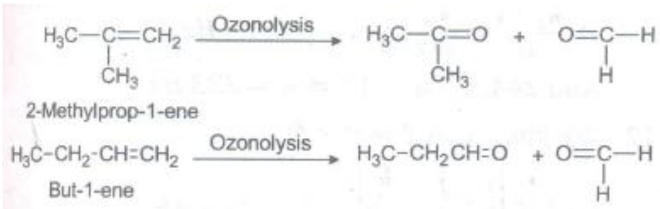
Sol 17.

Due to resonance, the bond between Ar – O is difficult to cleave than the bond between CH_3-O.



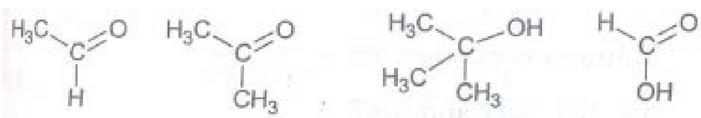
Sol 18.

On ozonolysis, 2 – methylpropene and butane – 1 give different products as shown below.



Sol 19.

Hybridization of carbon atoms is sp^3 in $(CH_3)_3$ COH.



Sol 20.

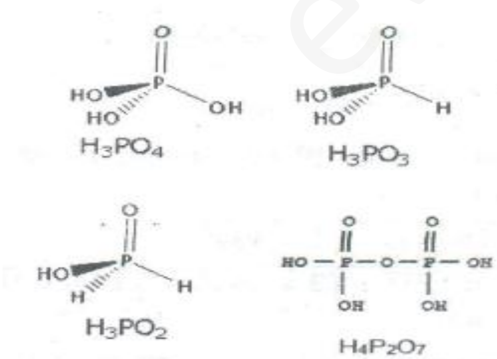
Compounds containing a CH_3 – CO – group or alcohols which can give this group on oxidation are used for the preparation of iodoform. $CH_3CH_2CH_2OH$ on oxidation gives CH_3CH_2CHO which does not contain a CH_3 – CO – group.

Sol 21.

Bile acids act as emulsifier in lipid metabolism.

Sol 22.

H_3PO_2 has maximum numbers of P – H bonds.

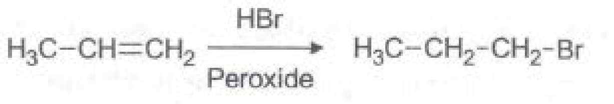


Sol 23.

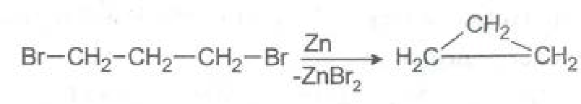
Barfoed's reagent consists of a 0.33 molar solution of neutral copper acetate in 1% acetic acid solution.

Sol 24.

The reaction of propene with HBr in the presence of a peroxide gives 1 – bromopropane.



Sol 25.



Sol 26.

Solubility of sulphates of alkaline earth metals decreases down the group/

Sol 27.

The absolute temperature of an ideal gas proportional to the average kinetic energy of the molecules.

Sol 28.

Setting of plaster of paris takes place due to its hydration.

Sol 29.

Ions containing either all paired or no electrons in d-subshell are colourless.

Sol 30.

As2S3 sol is negatively charged.

PHYSICS

Sol 1.

Hubble's Constant

H = $\frac{velocity}{distance}$

$\therefore$ [H] = $\frac{[LT^{-1}]}{[L]}$ = $[T^{-1}]$

i.e. unit of H is per second

Sol 2.

Velocity before strike, $u = \sqrt{2gh}$

Plane = g sin ∝

And the perpendicular components = g cos∝

Using S = ut + $\frac{1}{2}$ at^2

For vertical direction,

O = v = cos∝ t – $\frac{1}{2}$, 8 ∝ sin t^2

And for horizontal direction

x = u sin ∝ t + $\frac{1}{2}$, 8 sin ∝ t^2

As t = $\frac{2u}{g}$

x = u sin ∝$\left(\frac{2u}{g}\right)$ = $\frac{1}{2}$ g sin ∝$\left(\frac{2u}{g}\right)^2$

= $\frac{2u^2 sin\propto}{g} + \frac{2u^2 \sin\propto}{g} = \frac{4u^2 \sin\propto}{g}$

= $4\ X \frac{2h\ x\ \sin\propto}{g}$ = 8h sin ∝

Sol 3.

Momentum of the pices moving along x and y direction

P$_1\sqrt{(m_1v_1)^2 + (m_2v_2)^2}$

= $\sqrt{(1\ x\ 12)^2 + (2\ x\ 16)^2}$ $= \sqrt{144 + 1024}$ =

34 kg m/s

Momentum of third piece = m_3 v_3 mx 40

Final momentum of shell = (34 – 40m)

Using law of conservation of momentum (34 – 40) m = 0 ⇒ m = 0.8 kg

Total mass of shell = 1 + 2 + 0.8 = 3.8 kg

Sol 4.

The upthrust is more than the weight of the balloon. Therefore, the resultant force does work in lifting. There is a gain in kinetic energy besides potential energy without violating the conservation energy principle.

Sol 5.

Given I = $MR^2 + MR^2 = 2MR^2$ = 2x 3x 1 = 6 gm cm^2

Sol 6.

As g' = $\frac{gR^2}{(R+h)^2}$

i.e.$\frac{1}{4} = \frac{R^2}{(r+h)^2} \Rightarrow \frac{1}{2} = \frac{R}{R+h}$

i.e. (R + h) = 2R

Sol 7.

The fall in pressure will make the air to rush.

Sol 8.

Change in length $\Delta l_1 = l_1 \propto_1$ T

And$\Delta l_2 = l_2 \propto_2 T$

If the stress developed in rods is P_1 and P_2 then Young's modulii

$Y_1 = \frac{p_1}{\Delta l_1/l_1}$ and $Y_2 \frac{P_2}{\Delta l_2/l_2}$

i.e. $Y_1 = \frac{P_1}{l_1 \propto_1 T/l_1}$ and $Y_2 = \frac{P_2}{l_2 \propto_2 T/l_2}$

or $Y_1 = \frac{p_1}{\propto_1 T}$ and $Y_2 \frac{P_2}{\propto_2 T}$

$\Rightarrow P_1 = Y_1 \propto_1$ T and $P_2 = Y_2 \propto_2$ T

As $P_1 = P_2$

$Y_1 \propto_1$ T = $Y_2 \propto_1$ T

$\Rightarrow \frac{Y_1}{Y_2} = \frac{\propto_2}{\propto_1} = \frac{3}{2} \Rightarrow Y_1 : Y_2$ = 3:2

Sol 9.

Both the statements are true.

Sol 10.

As $V_{rms} = \left(\frac{3P}{\rho}\right)^{1/2}$

$V_{sound} = \left(\frac{\gamma P}{\rho}\right)^{1/2} \Rightarrow \frac{V_{sound}}{V_{rms}} = \left[\frac{\gamma P/\rho}{3P\,/\,\rho}\right]^{1/2} = \sqrt{\frac{\gamma}{3}}$

Sol 11.

As $\frac{n_1}{l_1} = \frac{l_2}{l_1} = \frac{9.2}{10.2}$ Also n n_1 = 12 where n is the frequency of tuning fork And $n_2 - n = 12$

Adding $n_2 - n_1 = 24$ or $n_1 - n = 24$

$\Rightarrow \frac{n_2 - 24}{n_2} = \frac{9.2}{10.2} \Rightarrow n_2 = 244.8$ Hz

And 244.8 – n = 12 ⇒ n = 233 Hz

Sol 12.

We know that $\vec{\tau} = \vec{P}\ x\ \vec{E}$

i.e. $\vec{\tau} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = 3\hat{\imath} - 7\hat{\jmath} + 4\hat{k}$

$\Rightarrow \tau = \sqrt{(3)^2 + (7)^2 + (4)^2} = \sqrt{74} = 8.6$ Nm

∴direction cosines are $\frac{3}{8.6}, \frac{-7}{8.6}, \frac{4}{8.6}$

i.e. 0.5, 0.81 and 0.47

Sol 13.

Let l be the original length of wire and x be its length stretched uniformly such that final length is $1.5l$

$\therefore 4R = \rho\frac{(l-x)}{A} + \rho\frac{(0.5l+x)}{A'}$

Where $A' = \frac{x}{(0.5l+x)}A$

$\Rightarrow 4\rho\frac{l}{A} = \rho\frac{(l-x)}{A} + \rho\frac{(0.5\,l+x)^2}{xA}$

Or $4l = l - x + \frac{l^2}{4x} + \frac{x^2}{x} + \frac{lx}{x} \Rightarrow \frac{x}{l} = \frac{1}{8}$

Sol 14.

As r $\propto \frac{\sqrt{m}}{q}$

$R_H : r_{He} : r_0 = \frac{\sqrt{1}}{1} : \frac{\sqrt{4}}{1} : \frac{\sqrt{16}}{2}$

= 1 : 2 : 2

Obviously H^+ having least radius will have greater deflection.

Sol 15.

As S $(I - I_g) = I_g$ G

$\Rightarrow \frac{I}{I_g} = \frac{G}{S} + 1 = \frac{36}{4} + 1 = 10$

$\Rightarrow \frac{I_g}{I} \; x \; 100 = \frac{1}{10} \; x \; 100 = 10\%$

Sol 16.

As $I = I_0 (1 - e^{t/\tau})$

$= \frac{V}{R}\left(1 - e^{-t\, / \frac{L}{R}}\right)$

$= \frac{12}{6}\left[1 - e^{-t\, / \frac{8.4 \, x \, 10^{-3}}{6}}\right]$

As I = 1A given

$\Rightarrow$ t = 0.97 x 10^{-3} s i.e. t = 1ms

Sol 17.

As P = $\frac{E_{max}}{\sqrt{2}} \; x \frac{I_{max}}{\sqrt{2}} \cos \phi$

$\therefore$ P = $\frac{100}{\sqrt{2}} \; x \frac{1000 \; x \; 10^{-3}}{\sqrt{2}} \; x \cos \frac{\pi}{3}$

= $\frac{100}{2} x \frac{1}{2} = 25 \; w$

Sol 18.

All the first three option are incorrect.

Sol 19.

Given $\frac{I_2}{I_1} = \frac{d_1^2}{1.02\, d_1^2}\left(as\ I\ \propto \frac{1}{d^2}\right)$

= 0.96

⇒ I_2 = 0.96 I,

Decrease = $I_1 - I_2 = I_1 - 0.96\, I_1$ = 0.04 = 4%

Sol 20.

Using ${}^a\mu_w$ x ${}^w\mu_0$ x ${}^0\mu_a$ = 1

⇒${}^w\mu_0$ = $\frac{1}{W_{\mu_0}}\ x\ \frac{1}{0_{\mu_a}} = \frac{a_{\mu_0}}{a_{\mu_w}} = \frac{1.45}{1.33}$ = 1.09

Also ${}^a\mu_w$ = $\frac{1}{w_{\mu_0}} = \frac{1}{1.09} = 0.91$

Sol 21.

As $\frac{1}{u} + \frac{1}{v} = \frac{1}{fe}$

$\frac{1}{v} + \frac{1}{\infty} = \frac{1}{fe} + \frac{1}{f}$

Here f is the focal length of correcting glass

Subtracting, $f\ =\ -4m$

Power of lens = $\frac{1}{f} =\ \frac{-1}{f} = 0.25D$

Sol 22.

Kinetic energy of emitted photo electron is by

K.E. = hv – φ = 2.07 – 2 = 0.07 eV

= 0.07 x 1.6 x 10^{-19} = 0.112 x 10^{-19} J Wavelength of incident photon

E = $\frac{hc}{\lambda}\ \Rightarrow\ \lambda = \frac{hc}{E} = \frac{6.63\ x\ 10^{-34}\ x\ 3\ x\ 10^8}{2.07\ x\ 1.6\ x\ 10^{-19}}$ = 6 x 10^{-7} m

Sol 23.

De Broglie Wavelength of photo electrons

$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63\ x\ 10^{-34}}{\sqrt{2\ x\ 9.1\ x\ 10^{-31}\ x\ 0.112\ x\ 10^{-19}}}$ = 4.6 x 10^{-9} m

Sol 24.

Here N = $\frac{n(n-1)}{2}$ = 10

i.e. $n^2 - n - 20 = 0$ or n = 5

$\Rightarrow \frac{1}{\lambda} = R\left[\frac{1}{1} - \frac{1}{25}\right]$

or λ = 950 A^0

Sol 25.

Resistance of a semiconductor decreases with increase in temperature

Sol 26.

Modulation index is defined as the ratio of change of amplitude E_m of carrier wave to the amplitude E of original carrier wave

i.e. $m_\alpha = \frac{k_a E_x}{E_x}$

Also $m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} + \frac{a-b}{a+b}$

Sol 27.

Condition of sliding is mg sinθ> μmg Cos θ or

tanθ> μ ⇒tanθ>$\sqrt{3}$

The condition of toppling is

Torque of mg sin θ > torque of mg cosθ

Torque of mg sin θ $\left(\frac{15}{12}\right)$>mg cosθ $\left(\frac{10}{2}\right)$

ortanθ>$\frac{2}{3}$

With increase in value of θ of condition of sliding is satisfied first.

Sol 28.

$l_1 = 2l_2$ and $l_1 = \frac{3}{2}$K

Force constant k $\propto \frac{1}{length\ of\ spring}$

⇒ $k_1 = \frac{3}{2}$ k

Sol 29.

Let δ be the density of material of sphere.

Using the condition of floatation

Weight = Up thrust

V δ g = $\frac{v}{2}$ δ_{oil} g + $\frac{v}{2}$ δ_{Hg} g

Or δ = $\frac{\delta_{oil}}{2} + \frac{\delta H_g}{2} = \frac{0.8}{2} + \frac{13.6}{2} = 7.2\ g/cm^3$

Sol 30.

As $Q_1 = nC_p\ \Delta t$, $Q_2 = nC_v\ \Delta t$

$\frac{Q_2}{Q_1} = \frac{C_v}{C_p} = \frac{1}{\gamma} \Rightarrow Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4}$ = 50 cal

MATHEMATICS

Sol 1.

In (d), the subsets are pair wise disjoint and their union is equal to the set A.

Sol 2.

f(n) = 1 + 4x + $7x^2$ + $10x^3$ + (i)

xf(n) = x + $4x^2$ + $7x^3$ + $10x^4$ + ...∞ (ii)

Subtracting (i) and (ii) we get,

(1 – x) f(n) = 1 + 3x + $3x^2$ + $3x^2$ + $3x^3$ +

⇒ (1 –x) f(n) = 1 + $\frac{3x}{1-x}$

⇒ (1 –x)$\frac{35}{16}$ = $\frac{1-2x}{1-x}$

⇒ 35 $(1 – x)^2$ = 16 + 32x

⇒ $35x^2$ – 102x + 19 = 0

⇒ (7x –19) (5x - 1) = 0

x ≠ $\frac{19}{7}$ (for infinity series common ratio < 1)

∴ x = $\frac{1}{5}$

Sol 4.

$\frac{4}{4x^2+4x+9}$ is greatest when $4x^2 + 4x + 9$ is least.

We have $4x^2 + 4x + 9 = (2x + 1)^2 + 8 > 8$

for all x $(\because (2x+1)^2 \geq 0)$

Therefore the min. value of $4x^2 + 4x + 9$ is 8.

Hence the greatest value of $\frac{4}{4x^2+4x+9}$ is $\frac{4}{8} = \frac{1}{2}$

Sol 5.

According to the given condition;

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow 2\cos\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{2}$

Hence $\alpha = \frac{\pi}{3}$

Sol 7.

For infinite number of solutions, the value of delta must be equal to 0

$$\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow a^3 + 1 = 0$

$\Rightarrow a = -1$

Sol 8.

You might solve it as required no. of ways = (5! X 6!)/2

Sol 9.

We have $n(1 + x)^{n-1} = C_1 + 2C_2 x + 3C_3 x^2 + \ldots\ldots\ldots\ldots\ldots + nC_n x^{n-1}$

Put x = -1, we get

$0 = C_1 - 2C_2 + 3C_3 + \ldots\ldots\ldots + (-1)^{n-1} nC_n$

Sol 10.

Given that log 2, log (2^x - 1) and log (2^x + 3) are A.P.

Therefore 2 log (2^x - 1) = log 2 + log (2^x + 3)

⇒ $(2^x - 1)^2 = 2\ (2^x + 3)$

⇒ $2^{2x} – 4.2^x – 5 = 0$

⇒ $(2^x - 5)\ (2^x - 1) = 0$

As 2^x cannot be negative, therefore we get $2^x – 5 = 0 \Rightarrow 2^x = 5$ or $x = \log_2 5$.

Sol 11.

$f'(x) = - 3(3x + 1)^{-2}$

At x = 0, f' (x) = -3 (negative)

Sol 12.

Putting x = tanθ, we get

$y = \cot^{-1}\left(\frac{1-tan\theta}{1+\tan\ \theta}\right)$

$= cot^{-1}\left[\tan\left(\frac{\pi}{4} - \ \theta\right)\right]$

$= \cot^{-1}\left[\cot\left\{\frac{\pi}{2} - \ \left(\frac{\pi}{4} - \ \theta\right)\right\}\right]$

$= \cot^{-1}\left[\cot\left(\frac{\pi}{4} + \ \theta\right)\right]$

$= \frac{\pi}{4} + \ \theta = \frac{\pi}{4} + \ \tan^{-1} x$

$\therefore \frac{dy}{dx} = \frac{1}{(1+x^2)}$

Sol 13.

$\int \sec^2 x\ cosec^2 x\ dx = \int \frac{1}{\sin^2 x \cos^2 x}\ dx$

$= \int \frac{\sin^2 x+\cos^2 x}{\sin^2 x\ \cos^2}\ dx = \int_0^{\frac{\pi}{2}} \frac{\log\ \sec^2\ \theta}{\sec^2\ \theta}.\sec^2\ d\theta$

$= \tan x – \cot x$

Sol 14.

Putting x = tan θ, dx = $\sec^2$ θ dθ, we get,

$I = \int_0^{\pi/2} \frac{\log(1+x^2)}{1+x^2}\, dx = \int_0^{\frac{\pi}{2}} \frac{\log \sec^2\theta}{\sec^2\theta}.\sec^2 d\theta$

$= 2\int_0^{\pi/2} \log \sec\theta \; d\theta = -2\int_0^{\pi/2} \log cos\theta$

= -2 (-π/2 log 2) = π log 2

Sol 15.

Given curve is $y^2 = 2c\left(x + \sqrt{c}\right)$.

Differentiate w.r.t. x, we get

$2y\frac{dy}{dx} = 2c \Rightarrow c = y\frac{dy}{dx}$

Hence, differential equation is

$y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{y\frac{dy}{dx}}\right)$

$\Rightarrow \frac{y}{2\, dy\,/\,dx}\, x = \sqrt{y\frac{dy}{dx}}$

Squaring and multiplying by $\left(\frac{dy}{dx}\right)^2$ we get

$y\left(\frac{dy}{dx}\right)^3 - x^2\left(\frac{dy}{dx}\right)^2 + xy\left(\frac{dy}{dx}\right) - \frac{y^2}{4} = 0$

Hence, order is 1 and degree is 3.

Sol 16.

Here $\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$……. (i)

It is homogeneous equation.

So now put y = vxand $\frac{dy}{dx} = v + x\frac{dv}{dx}$,

Then, the equation (i) reduces to $\left(\frac{dy}{b} log\ v\right) = \frac{dx}{x}$,

On integrating, we get, Log (log v) = log x + log c

$\Rightarrow \log\left(\frac{y}{x}\right) = cx \Rightarrow y = xe^{ex}$

Sol 17.

On multiplying by 2 on both sides of the first equation, we get

10x + 24y – 2 = 0

Distance between the two lines is

$$\frac{|c-d|}{\sqrt{a^2 + b^2}} = 2$$

$$\Rightarrow \frac{|-2+k|}{\sqrt{100+576}} = 2$$

⇒ |k – 2| = $\sqrt{676}$

Squaring both sides, we get

k = -54, 50

Sol 18.

The slope of the tangent at any point (x, y) is 6/y and the slope of the normal is – 1, hence the product is

(6/y) (-1) = -1

6/y = 1, y = 6

On substituting y = 1, we get x = (k – 6), and on substituting x and y in the parabola, we get k = 9.

Sol 19.

Hence x = -3k -1; y = 2k + 3; z = k -2

Now on substituting it in the options we can say that option (a) is satisfied.

Therefore (a) is the correct option.

Sol 20.

We can see the direction ratios of the line and normal to plane are <1, 2, 3> and <1, -2, 1> respectively.

Hence they are perpendicular, as the line and the normal to the plane are perpendicular, Therefore the line and the plane will be parallel.

Sol 22.

Probability that at least one of A and B will solve the problem = $1-\left(1-\frac{1}{2}\right)\left(1-\frac{3}{4}\right)\ 1-\frac{1}{3}\text{x}\frac{1}{4}=\frac{11}{12}$

Sol 23.

Probability of getting a number less than 5 in each case = $\frac{4\times4\times4}{6\times6\times 6} = \frac{8}{27}$

Sol 25.

26 cards can be chosen out of 52 cards, n $^{52}C_{26}$ ways. There are two ways in which each card can be either from the first pack or from the second.

Total number of ways = $^{52}C_{26}$ x 2^{26}

Sol 26.

$(1+x)^n\left(1+\frac{1^n}{x}\right)$

= $({}^nC_0 + {}^nC_1x + {}^nC_2\,x^2 + \ldots\, {}^nC_nx^n)$

x $({}^nC_0 + {}^nC_1\frac{1}{x} + {}^nC_2\frac{1}{x^2} + \ldots\ldots + {}^nC_n\frac{1}{x^n})$

Term independent of x is

$$(c_0^2 + c_1^2 + c_2^2 + \ldots\ldots + C_n^2)$$

Sol 27.

$$\int_0^{\pi/2} \frac{1}{1+\sin x}\,dx = \int_0^{\pi/2} \frac{1-\sin x}{1+\sin^2 x}\,dx$$

= $\int_0^{\pi/2} \frac{1-\sin x}{\cos^2 x}\,dx = [\tan x - \sec x]_0^{\pi/2}$

= $\left[\frac{\sin x - 1}{\cos x}\right]_0^{\pi/2} = \left[\frac{-\cos x}{1+\sin x}\right]_0^{\pi/2} = 1$

Sol 28.

y = $\tan^{-1}\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)$

$$= \tan^{-1}\left(\frac{\tan x + 1}{1-\tan x}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} + x\right)\right]$$

$\therefore \frac{dy}{dx} = 1$

Sol 29.

Given f (x + y) = f (x) + f (y),

f (1 + 1) = f(2) = f(1) + f(1) = 2f (1) and

f(2 + 1) =f(3) = f(2) + f(1) = 3f (1).

Similarly, f(4) = 4f (1), f(5) = 5f (1) . . . . f(10) = 10f (1).

Given expression is

f(1) + f(2) + f(3) . . . f (10) = 1

⇒ f(1) + 2f(1) + 3f(1) . . . . 10f (1) = 1

⇒ (1 + 2 + 3 + 4 . . . . 10) f(1) = 1

$\Rightarrow f(1) = \frac{1}{55}$

Sol 30.

$$\frac{1}{\sqrt{6-3x}} = \frac{1}{6^{1/2}\left(1-\frac{3x}{6}\right)^{1/2}}$$

$$= \frac{1}{6^{1/2}\left(1-\frac{x}{2}\right)^{1/2}}$$

Expansion is valid if $\left|\frac{x}{2}\right| < 1$

$\Rightarrow |x| < 2$