Subject: **CHEMISTRY, MATHEMATICS & PHYSICS**

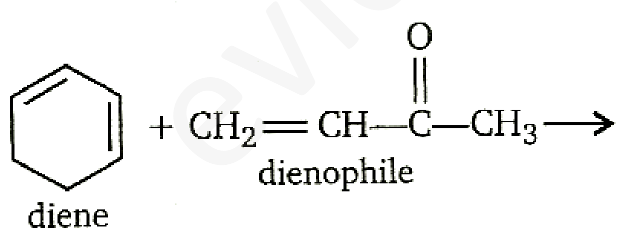
Paper Code: **JEE_ Main_ Sample Paper - II**

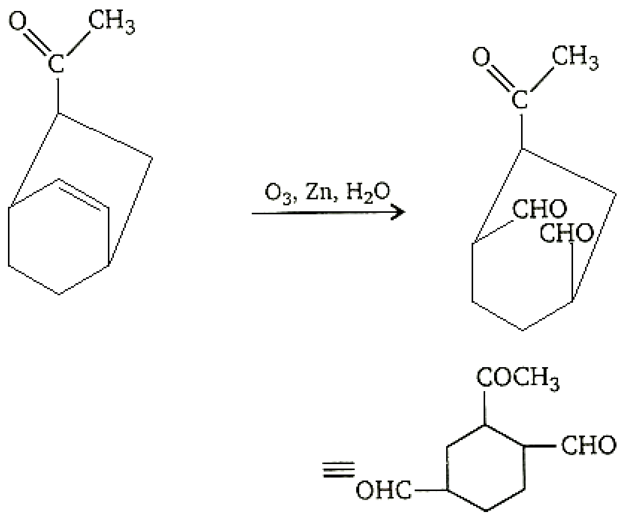
Part – A - Chemistry

1)

Ans: c

Exp: Reaction is Diels-Alder reaction.





2)

Ans: a

Exp: Suppose mole of HCOOH = x

Mole of H_2C_2OH = y

HCOOH + $H_2C_2OH \rightarrow$ CO + H_2O + H_2C_2OH

x x

H_2C_2OH + $H_2SO_4 \rightarrow$ CO + CO_2 + H_2O + H_2SO_4

y y y

Total gaseous products = x + 2y

Only CO_2 is absorbed by KOH, y mole

Hence $\frac{y}{x+2y} = \frac{1}{6}$

$\therefore$ x + 2y = 6y

x = 4y

$\therefore$ $\frac{x}{y} = \frac{4}{1}$

3)

Ans: c

Exp: The nitration of aniline is difficult to carry out with nitrating mixture, since $-NH_2$ group get oxidized which is not required. So, the amino group is first protected by acylation to form acetanilide which is then nitrated to give p-nitro acetanilide as a major product.

4)

Ans: c

Exp: p(H_2) = (1400 Torr) (0.685)

= 959 Torr≡ 959/760 atm = 1.26 atm

According to Henry's law

"Amount of gas absorbed is directly proportional to pressure."

Hence, $\frac{V}{18mL} = \frac{1.26atm}{1atm}$

$V = 23mL$

5)

Ans: c

Exp: $H_2 \rightleftharpoons SO_2$

Initial: 0.5 mol 0.5 mol

After a period of time H_2 being lighter, effuse faster and hence, in larger amount. So, remaining hydrogen must be lesser.

6)

Ans: d

Exp: Weight of CO_2 = 1 g(as absorbed in KOH) Weight of oxygen in oxide

= weight of oxygen in 1 g of

CO_2 = $\frac{32}{44} - \frac{8}{11}$ = 32.7

Weight of metal = 3.7 – $\frac{8}{11} = 32.7$

Equivalent wt. = $\frac{wt.of\ metal}{wt.of\ oxygen}\ \frac{wt.of\ metal}{wt.of\ oxygen}$ X8 = 32.7

According to Dulong-Petit's law: Atomic weight (approx.) = $\frac{6.4}{0.095}$ = 67.37

Valiancy = $\frac{atomic\ weight}{equivalent\ weight}$ = 2 approx.

Exact atomic weight = 32.7 × 2 = 65.4

7)

Ans: d

Exp: Let number of oxides = x

Number of octahedral void = x

Number of tetrahedral void = 2x

Number of A^{2+} ion = $\frac{1}{8}.2x = \frac{x}{4}$

Number of B^{3+} ion = $\frac{x}{2}$

$A^{2+} : B^{3+} : 0^{2-}$

$\frac{x}{4} : \frac{x}{2} : x$

1 2 4

Hence, formula of oxide is AB_2O_4.

8)

Ans: b

Exp: Given,

I. $C(s) + O2 \rightarrow CO_2 \Delta H = -393.3 kJ/mol$

II. $S(s) + O2 \rightarrow SO_2$ Ml $= -293.72$ kJ/mol

III. $CS_2(l) + 3O_2(g) \rightarrow CO_2(g) + 2SO_2(g)$

$\Delta H = -1108.76$ kJ/mol

On putting various enthalpy of formation in equation III

$\Delta H = \Delta H$ (products) – ΔH (reactants)

$-1108.76 = [-393.3 + 2(-293.72)]-[\Delta H_f(CS_2) + 3\times 0]$

$-1108.76 = -393.3 - 2 \times 293.72 - \Delta Hf(CS_2)$

$\Delta H_f(CS_2) = 128.02$ kJ

9)

Ans: b

Exp: On comparing the equation of k with

$k = Ae^{-E_a/RT}$

$E_a/RT = 29000$ k/T

$E_a = (29000k)$ R

$= 241$ kJ mol^{-1}

10)

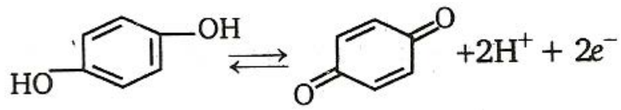
Ans: b

Exp: (a) $\frac{1}{2} H_2(g) \rightarrow H^+ (aq) + e^-$

$E_{cell} = E^o_{cell} - 0.0591 \log [H^+]$

(b) $Ag(s) + Cl(aq) \rightarrow AgCl(s) + e^-$

$E_{cell} = E^o_{cell} - 0.0591 \log 1/[Cl^-]$

(c) 

$E_{cell} = E^{o}_{cell} - 0.0591 \log [H^{+}]$

(d) $Ag(s) \rightarrow Ag+(aq)$

$E_{cell} = E^{o}_{cell} - 0.0591 \log [Ag^{+}]$

11)

Ans: c

Exp: Electrolysis of H_2SO_4 using Pt electrodes:

(at high current density)

At anode: $2HSO_4^- \rightarrow H_2S_2O_8 + 2e$

$At\ cathode : 2H^+ + 2e \rightarrow H_2$

12)

Ans: a

Exp: This is known as Hunsdiecker reaction, silver acetate dissolved in xylene or Cs, yields a halide with one C atom less.

$$CH_3COOAg + Br_2 \xrightarrow{CS_2} CH_3 - Br + AgBr + CO_2$$

13)

Ans: a

Exp: $\lambda_e = h/m_e v_e$, $\lambda_p = h/m_p v_p$

As, $\lambda_e = \lambda_p$, $h/m_e v_e = h/m_p v_p$

or $m_p v_p = m_e v_e$

or $v_p = \frac{m_e}{m_p} v_e = \frac{1}{1840} v_e$.

14)

Ans: c

Exp: Molecular mass N_2 = 28; CO = 28

Number of molecules of N_2

(V = 0.5L, T = 27^oC, p = 700 mm) = n

Number of molecules of N_2

(V = 1 L, T = 27^oC, p = 700 mm) = 2n

Number of molecules of CO

(V = 1 L, T = 27^{o}C, p = 700 mm) = 2n

15)

Ans: b

Exp: Equivalent of $H_2S_2O_8$ + Equivalent of O_2 (at anode) = Equivalent of H_2 (at cathode)

Eq. of H_2 = $\frac{Volume\ Released}{Equivalent\ Volume}$

= $\frac{9.72}{11.2}$ = 0.87

Eq. of O_2 = $\frac{2.35}{5.6}$ = 0.42

Wt. of $H_2S_2O_8$ = g–eq. × eq. wt.

= 0.45 × $\frac{mol.wt.}{2}$

= $\frac{0.45 \times 194}{2}$ = 43.65

16)

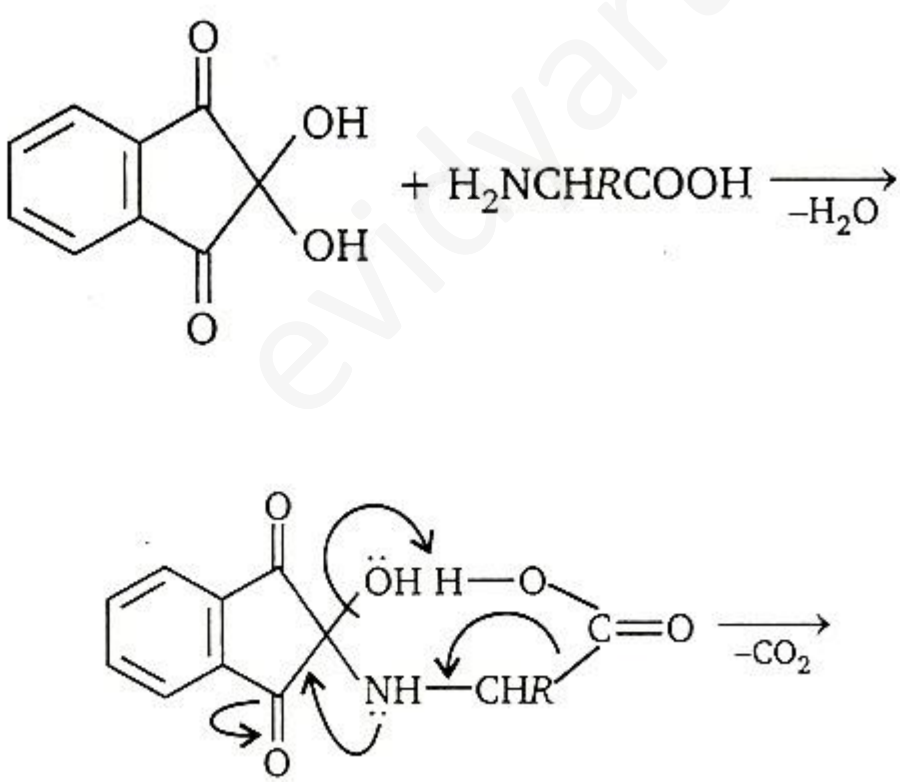
Ans: b

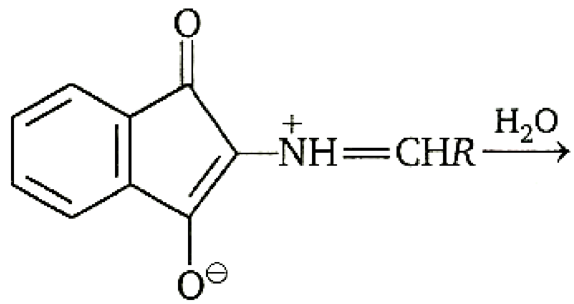
Exp: Benzoic acid, oxalic acid and picric acid are sufficient acidic to evolve CO_2 but (b) is comparatively less acidic that's why doesn't evolve CO_2.

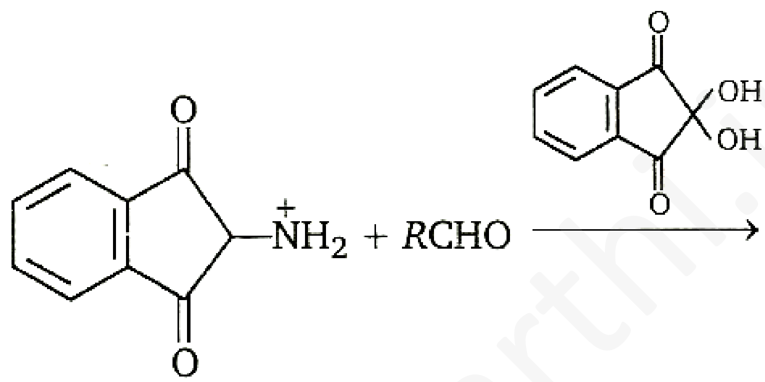
17)

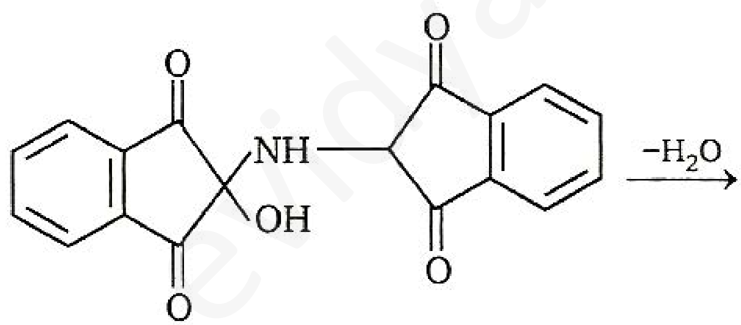
Ans: c

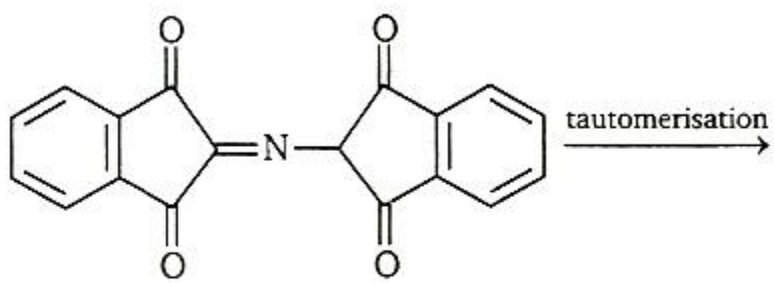
Exp:

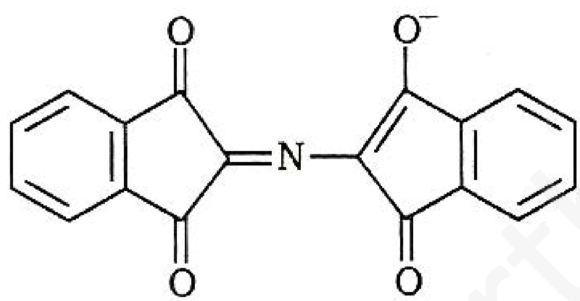












purple violet complex

18)

Ans: b

Exp: The main purpose of using leveling bulb is to assure that pressure within the reaction vessel is same as that in the room.

19)

Ans: c

Exp: Statement

(i) Suggests order with respect to A is 1.

(ii) Suggests order with respect to B is 0.

(iii) Suggests order with respect to C is 2.

Hence, rate law expression can be written as

$r = k\,[A]^1\,[B]^0\,[C]^2$

Order of reaction = 1 + 0 + 2 = 3

20)

Ans: a

Exp: $H^+ + e^- \rightarrow \frac{1}{2}\,H_2$

$$E_{cell} = E^\circ_{cell} - 0.0591 \log \frac{1}{H^+}$$

$-0.118\text{ V} = 0 + 0.0591 \log [H^+]$

$-0.118\text{ V} = -\,0.0591\,(-\log [H^+])$

$-0.118\text{ V} = -\,0.0591\text{ pH}$

$$pH = \frac{0.118}{0.0591} = 2$$

21)

Ans: b

Exp: Equivalent of acid = Equivalent of base Na_2CO_3

+ Equivalent of base NaOH

milliequivalent $50 \times N = 25 \times 0.5 + 20 \times 0.5$ N = 0.45

22)

Ans: b

Exp: FCC has 4 atoms in a unit cell

BCC has 2 atoms in a unit cell

$$d = \frac{z \times M}{N_0 \times a^3}$$

$$\frac{d_{FCC}}{d_{BCC}} = \frac{4}{2}\frac{(3.0)^3}{(3.5)^3} = 1.26$$

23)

Ans: b

Exp: $-4°F = \frac{5}{9}(-4-32)°\,C = -20°\,C$

$\Delta T_f = 20^{\circ} C = k_f\, m = (1.86^{\circ}\ C/m)\ (m)$

$$m = \frac{20^{\circ}C}{1.86^{\circ}C/m} = 10.70m$$

= (10.7 mol) (46.0 g/mol) = 495 g

24)

Ans: b

Exp: The reaction is the reverse of the ionization reaction of HA. Hence, the equilibrium constant is the reciprocal of K_a.

$$K = [HA]/\lfloor A^- \rfloor \lfloor H_3O^+ \rfloor$$

$$= \frac{1}{K_a} = \frac{1}{1.0\times10^{-6}}$$

25)

Ans: a

Exp: Transport number (T)

$$= \frac{current\ carried\ by\ ion}{total\ currect}$$

Transport number $\propto$ speed of ion

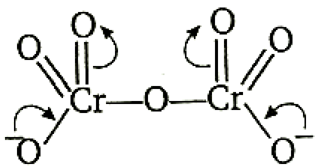
$$T_{Cl^-}\left(HCl\right)=\frac{v_{cl^-}}{v_{Cl^-}+v_{H^+}}$$

$$T_{Cl^-}\left(NaCl\right)=\frac{v_{Cl^-}}{v_{Cl^-}+v_{Na^+}}$$

As $v_{H^+} > v_{Na^+}\, or\, v_{K^+}\, or\, v_{Cs^+}$

26)

Ans: b

Exp: 

6 Cr–O bonds are equivalent due to resonance.

27)

Ans: a

Exp: Equivalent weight of

$FeSO_4 . 7H_2O$ = Mol. wt. = 278

80 mL 0.125 (N) permanganate solution

$\equiv$ (80 × 0.125) N solution

= meq. of $FeSO_4 . 7H_2O$

= 10

Weight = $\frac{10 \times 278}{1000}$ = 2.78 g

Weight of anhydrous $Fe_2(SO_4)_3$

= 5.39 – 2.78 = 2.61 g

28)

Ans: b

Exp: Enthalpy change is a state function, its value doesn't depend on the intermediate conditions or path followed, it is the difference of energy of reactants and products present at 1 atmosphere and 298 K.

29)

Ans: b

Exp: The molality is given by

$$m = \frac{\Delta T_f}{k_f} = \frac{1.16°C}{1.86C/m} = 0.624\,m$$

$$\frac{30.0g}{0.800kg\ solvent} = \frac{37.5g}{kg\ solvent}$$

Hence, 37.5 g is equivalent to 0.624 mol.

$$\frac{37.5g}{0.624mol} = 60.1g / mol$$

The empirical formula weight is 30 g/eq. formula unit. There must be two units per molecule; the formula is $C_2H_2O_2$.

30)

Ans: a

Exp: BrO^-_3 appears at a rate one third that of disappearance

$$BrO^- = \frac{0.056}{3}$$

$= 0.019\ L\ mol^{-1}\ s^{-1}$

Part – C - Physics

31) Ans: c

Exp: When a particle separates from a moving body, it retains the velocity of the body but not its acceleration. At the instant of release, the balloon is 40m above the ground and has an upward velocity of 10m/s. For the motion of the stone from the balloon to the ground, u = 10m/s, s = - 40m, a = - 10m/s^2(g).

32) Ans: c

Exp: For conservation of vertical momentum, the second part must have a vertical downward velocity of 50m/s. For conservation of horizontal momentum, the second part must have a horizontal velocity of 120m/s.

33) Ans: c

Exp: In the horizontal direction, momentum must be conserved, as the floor is frictionless and there is no horizontal force. $\mu\sin\theta = \mu\sin\phi$. In the vertical direction, $v\cos\phi = eu\cos\theta$.

34)

Ans: c

Exp: The centre of mass of the 'block plus wedge' must move with speed mu/(m + ηm) = u/(1+η) = v_{CM}.

$\therefore \frac{1}{2} mu^2 - mgh = \frac{1}{2} (m + \eta m) v^2_{CM}$

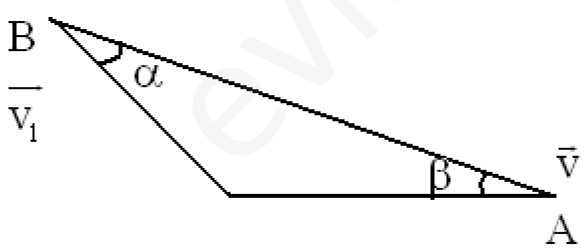
35) Ans: c

Exp: For the first displacement, y = 0. Hence $F_x = 0$ and no work is done. For the second displacement, $F_y = -ka$ and $\Delta y = a$.

Work $F_y \Delta_y = -ka^2$

36) Ans: c

Exp: Along AB velocity components of A and B must be same.



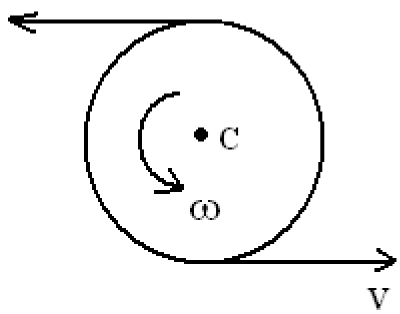
∴ v'cosα = vcosβ

Or v'=vcosβ/cosα

37)

Ans: c

Exp:



$\omega = v/R$ and $v_c = 0$

38) Ans: c

Exp: Work done $W = \frac{1}{2} I\omega^2$

If x is the distance of mass 0.3kg from the center of mass, we will have,

$I = (0.3)x^2 + (0.7)(1.4 - x)^2$

For work to be minimum, the moment of inertia (I) should be minimum,

Or $dI/dx = 0$ or $2(0.3x) - 2(0.7)(1.4-x) = 0$

Or $(0.3)x = (0.7)(1.4-x) \Rightarrow x = ((0.7)(1.4))/(0.3+0.7) = 0.98m$

39) Ans: a

Exp: Total energy of a planet in an elliptical orbit is:

$E = - GMm/2a$ (m = mass of planet)

From conservation of mechanical energy

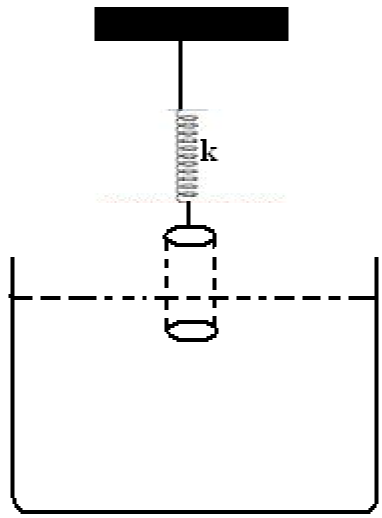
KE + PE = E

Or $\frac{1}{2} Mv^2 - GMm/r = - GMm/2a$

Or $v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$

40) Ans: b

Exp: When cylinder is displaced by an amount x from its mean position, spring force and upthrust both will increase. Hence,



Net restoring force = extra spring force + extra upthrust

Or $F = -(kx + Ax\rho g)$

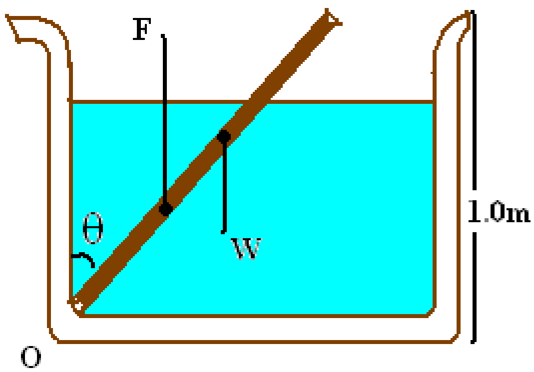
Or $a = -(k + \rho A g/M)x$

Now, $f = 1/2\pi \sqrt{(|a/x|)} = 1/2\pi \sqrt{(k+\rho A g/M)}$

41) Ans: c

Exp: Length of rod inside the water

$= 1.0\sec\theta = \sec\theta$



Upthurst F = (2/2) (secθ) (1/500) (1000)(10)

Or F = 20secθ

Weight of rod W = 2 x 10 = 20N

For rotational equilibrium of rod net torque about O should be zero.

∴ F(Secθ/2) (sinθ) = W (1.0sinθ)

Or 20/2 $sec^2θ$ = 20 or θ = 45^o

∴ F = 20sec45^o

= 20√2 N

For vertical equilibrium of rod, force exerted by the hinge on the rod will be (20√2 – 20)M downwards or 8.28N ≈ 8.3 N downwards.

42) Ans: c

Exp: - Adh/dt = π(hρg)r^4/8ηl

$\int_0^t dt = \int_H^{H/2} - 8\eta lA/\pi\rho gr^4(l/h)$

Or $\quad t = 8\eta lA/\rho\pi gr^4 \ln(2)$

43) Ans: b

Exp: After two seconds both the pulses will move 4 cm towards each other. So, by their superposition, the resultant displacement at every point will be zero. Therefore, total energy will be purely in the form of kinetic. Half of the particles will be moving upwards and half downwards.

44) Ans: c

Exp: Given, $\quad 3(v/4l_C) = 2(v/3l_o)$

$\therefore \quad L_c/l_o = 3/4$

Now, $n(v/4l_C) = m(v/2l_o)$

Or $\quad n/m = 2(l_c/l_o) = 6/4 = 3/2$ or $9/6$

Thus, $n = 9$ and $m = 6$.

45) Ans: c

Exp: $dU = C_v dT = (5/2\ R)dT$

Or $dT = 2(dU)/5R$ ....(1)

From first law of thermodynamics

$dU = dQ - dW$

$= Q - Q/4 = 3Q/4$

Now molar heat capacity

$C = dQ/dT = Q/2(dU/5R)$

$5RQ/2(3Q/4) = 10/3\ R$

46) Ans: a

Exp: NA

47) Ans: a

Exp: $\beta = \lambda D/d$ and $\theta = d/D$ $\therefore \beta = \lambda/\theta$

48) Ans: a

Exp: If charges were placed at all the corners, the field at the centre would be zero. Hence, the field at the centre due to any one charge is equal (and opposite) to the field due to all the other (n – 1) charges.

49) Ans: d

Exp: Potential at ∞ = $V_\infty = 0$.

Potential at the surface of the sphere = $V_S = k\, Q/R$

Potential at the centre of the sphere = $V_C = 3/2\, k\, Q/R$

Let m and –q be the mass and the charge of the particle respectively.

Let V_0 = speed of the particle at the centre of the sphere.

$\frac{1}{2} mv^2 = -q[V_\infty - V_S] = qk\, Q/R$

$\frac{1}{2} mv^2_0 = -q[V_\infty - V_C] = q.\ 3/2\, k\, Q/R$

Dividing, $v^2_0/v^2 = 3/2 = 1.5$ or $v_0 = \sqrt{1.5}v$

50) Ans: c

Exp: Plane conducting surfaces facing each other must have equal and opposite charge densities. Here, as the plate areas are equal, $Q_2 = -Q_3$.

The charge on a capacitor means the charge on the inner surface of the positive plate – in this case, Q_2.

Potential difference between the plates

= charges on the capacitor ÷ capacitance.

∴potential difference = $Q_2/C = 2Q_2/2C = Q_2-(-Q_2)/2C = Q_2-Q_3/2C$

51) Ans: b

Exp: When a capacitor remains connected to a cell, its potential difference remains constant and it's equal to the emf of the cell. Any change in the capacitor may change its capacitance, its charge and the energy stored in it. When the dielectric slab is taken out, the capacitance decreases. Hence charge decreases, and the difference in charge is returned to the cell.

52) Ans: c

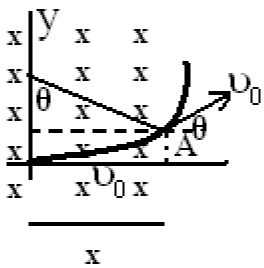
Exp:

$$r = \frac{mv_0}{B_0 q} = \frac{v_0}{B_0 \alpha}$$

$$\frac{x}{r} = \frac{\sqrt{3}}{2} = \sin\theta$$

$$\therefore \qquad \theta = 60^o$$

$$t_{OA} = \frac{T}{6} = \frac{\pi}{3B_0\alpha}$$



Therefore,

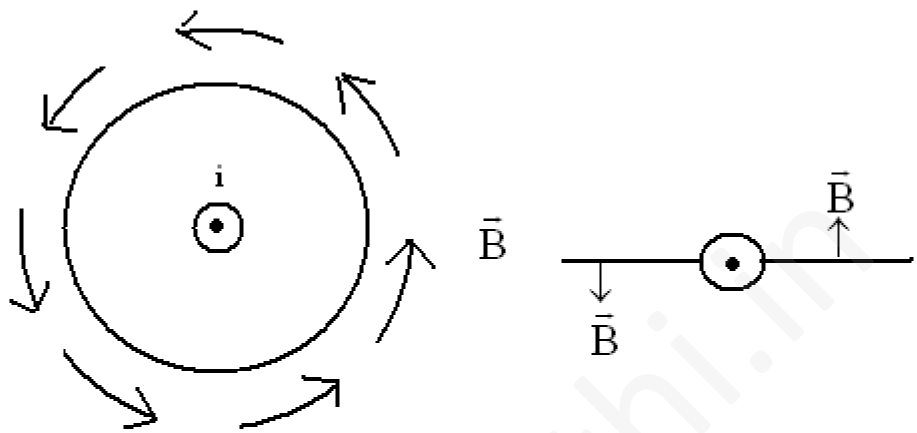
x-co-ordinate of particle at any time $t > \pi/3B_0\alpha$ will be

$$x = \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + v_0\left(t - \frac{\pi}{3B_0\alpha}\right)\cos 60^o$$

$$= \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0}{2}\left(t - \frac{\pi}{3B_0\alpha}\right)$$

53) Ans: b

Exp: If the current flows out of the paper, the magnetic field at points to the right of the wire will be upwards and to the left will be downwards as shown in figure.

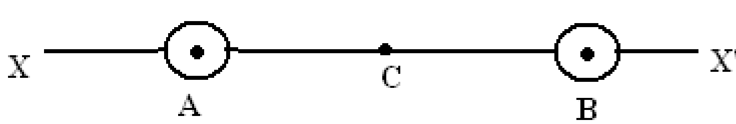


Now, let us come to the problem.

Magnetic field at C = 0

Magnetic field in region BX' will be upwards (+ve) because all points lying in this region are to the right of both the wires.

Magnetic field in region AC will be upwards (+ve), because points are closer to A, compared to B. similarly magnetic field in region BC will be downwards (-ve).



Graph (b) satisfies all these conditions.

54) Ans: b

Exp: $$R = \sqrt{\frac{L}{C}}$$

or $R^2 = \frac{L}{C}$ or $CR = \frac{L}{R}$

Hence, time constant of both the circuits are equal.

$$\tau_C = \tau_L = \tau \qquad \text{(say)}$$

$$i_L = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$

and $$i_C = \frac{V}{R}e^{-t/\tau}$$

$$i_L = i_C$$

$\therefore$ $$1 - e^{-t/\tau} = e^{-t/\tau}$$

or $$2e^{-t/\tau} = 1$$

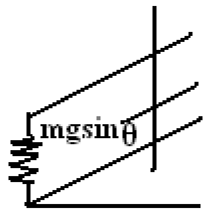
or $$e^{-t/\tau} = \frac{1}{2}$$

or $$\frac{t}{\tau} = \ln(2)$$

or $$t = \tau \ln(2) = CR\ln(2)$$

55) Ans: b

Exp: Terminal velocity is attained when magnetic force is equal to mg sinθ



$$\therefore \quad F_m = mg\sin\theta$$

$$\text{or} \quad ilB = mg\sin\theta$$

$$\text{or} \quad \left(\frac{e}{R}\right)lB = mg\sin\theta$$

$$\text{or} \quad \frac{(Bv_T l)}{R}lB = mg\sin\theta$$

$$\therefore \quad v_T = \frac{mgR\sin\theta}{B^2l^2}$$

56) Ans: d

Exp: Energy released would be:

ΔE = total binding energy of $_2He^4$ – 2(total binding energy of $_1H^2$)

= 4 x 7.0 – 2(1.1)(2) = 23.6 MeV

57) Ans: d

We known that

$$r_n \propto n^2$$

Exp: But $\quad r_{n+1} - r_n = r_{n-1}$

$\therefore \quad (n+1)^2 - n^2 = (n-1)^2$

$$n = 4$$

58) Ans: d

$$B_n = \frac{\mu_0 I_n}{2r_n}$$

or $\quad B_n \propto \frac{I_n}{r_n} \propto \frac{(f_n)}{r_n}$

Exp:

$\therefore \quad B_n \propto \frac{(v_n / r_n)}{r_n} \propto \frac{v_n}{(r_n)^2}$

$$\propto \frac{(Z/n)}{(n^2/Z)^2} \propto \frac{Z^3}{n^5}$$

59) Ans: c

Exp: The given reactions are:

$${}_1H^2 +_1 H^2 \longrightarrow_1 H^3 + p$$

$${}_1H^2 +_1 H^3 \longrightarrow_2 He^4 + n$$

$$3_1H^2 \longrightarrow_2 He^4 + n + p$$

Mass defect

$\Delta m = (3 \times 2.014 – 4.001 – 1.007 – 1.008)amu = 0.026amu$

Energy released = $0.026 \times 931MeV$

$= 0.026 \times 931 \times 1.6 \times 10^{-23}J$

$= 3.87 \times 10^{-12}J$

The average power radiated is $P = 10^{16}W$ or 10^{16} J/s.

Therefore, total time to exhaust all deuterons of the star will be

$t = 1.29x10^{28}/10^{16} = 1.29 \times 10^{12}s = 10^{12}s$

60) Ans: c

$$A_1 = \lambda N_0 e^{-\lambda t_1}$$

or $$t_1 = \frac{1}{\lambda}\ln\left(\frac{\lambda N_0}{A_1}\right) \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

$$A_2 = \lambda N_0 e^{-\lambda t_2}$$

Exp: $\therefore$ $$t_2 = \frac{1}{\lambda}\ln\left(\frac{2\lambda N_0}{A_2}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$t_1 - t_2 = \frac{1}{\lambda}\ln\left(\frac{A_2}{2A_1}\right)$$

$$= \frac{T}{\ln(2)}\ln\left(\frac{A_2}{2A_1}\right)$$

Part – C – Mathematics

61) Ans: A

Exp: Any plane through origin is

Ax + By + Cz = 0 ....(i)

If it is parallel to the line

x-1/2 = y+3/-1 = z+1/-2 then

2A-B-2C=0 ....(ii)

Now,

⊥distance of plane (i) from line

= ⊥ distance of (1, -3, -1) as the line from (i).

Since plane and line are parallel

$$\Rightarrow \frac{|A-3B-C|}{\sqrt{A^2+B^2+C^2}} = \frac{5}{3}$$

` The equations (ii) and (iii) are satisfied by values of A, B, C given in choice (a) A = 2, B = 2, C = 1

62) Ans: B

Exp: A = event that a number greater than 4 will appear

P(A) = 2/6 = 1/3

Required Probability

= (2/3).(1/3) +(2/3).(2/3).(2/3).(1/3)+…….∞

= 2/5

63) Ans: A

Exp: $a^2+3a+1/a = a+1/a + 3>5$ ($\because x + 1/x \geq 2$)

$\Rightarrow$Given expression $\geq$ (5) (5) (5) = 125

$\Rightarrow$ Choice (a) is the correct answer.

64) Ans: C

Exp: The locus will be an ellipse $PF_1 + PF_2$ is greater than 2.

The distance between the foci F_1 and F_2 of the ellipse.

Now $F_1F_2 = 2$

⇒Must be greater than 2.

65) Ans: C

Exp: As there are two vowels, they can be arranged either in AE form or EA form

∴ Number of arrangement = 6!/2! = 360

66) Ans: A

Exp:

$$a = 4k, b = 5k, c = 6k, x = \frac{15k}{2}$$
$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} = \sqrt{\frac{1}{8}}$$
$$\sin\frac{B}{2} = \sqrt{\frac{7}{32}}, \sin\frac{C}{2} = \sqrt{\frac{7}{16}}$$
$$\text{Now } \frac{r}{R} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{7}{16}$$
$$\Rightarrow \frac{R}{r} = \frac{16}{7}$$

67) Ans: a

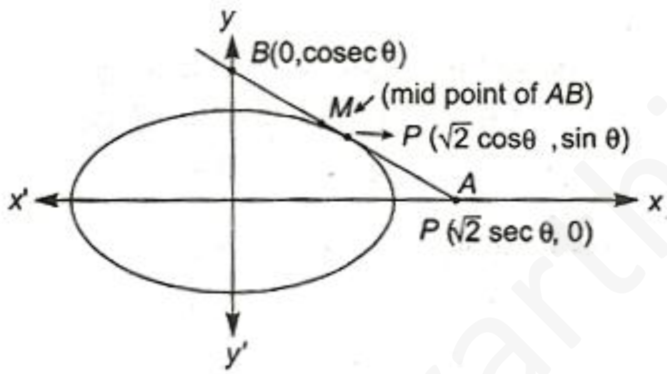
Exp: Let any point be P ($\sqrt{2}\cos\theta$, $\sin\theta$) on

$$\frac{x^2}{2}+\frac{y^2}{1}=1.,$$

Equation of tangent is,

$$\frac{x\sqrt{2}}{2}\cos\theta+\frac{y}{1}\sin\theta=1$$

Whose intercept on coordinate axes are A ($\sqrt{2}\sec\theta$, 0) and B (0, $\operatorname{cosec}\theta$)



∴ Mid-point of its intercept between axes is

$$\left(\frac{\sqrt{2}}{2}\sec\theta,\frac{1}{2}\operatorname{cosec}\theta\right)=(\mathrm{h},\mathrm{k}) \qquad \text{(say)}$$

$\Rightarrow$ $\cos\theta=\frac{1}{\sqrt{2}h}$

And $\sin\theta=\frac{1}{2k}$

Now, $\cos^2\theta+\sin^2\theta=\frac{1}{2h^2}+\frac{1}{4k^2}$

$\Rightarrow$ $\frac{1}{2h^2}+\frac{1}{4k^2}=1$

Thus, locus of mid-point M is

$$\frac{1}{2x^2}+\frac{1}{4y^2}=1$$

68) Ans: D

Exp: Any tangent to the ellipse is

$$y = mx + \sqrt{a^2m^2 + b^2} \text{ or } y - mx - \sqrt{a^2m^2 + b^2} = 0$$

If this touches $x^2 + y^2 = r^2$, then

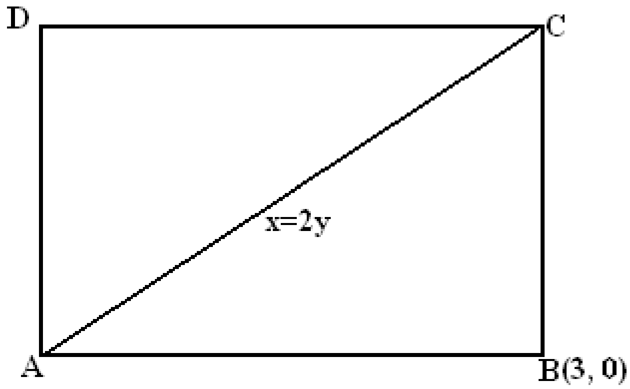
$$\left|\frac{0-0-\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right| = r$$

We easily get $m = \sqrt{\dfrac{r^2-b^2}{a^2-r^2}}$

$\Rightarrow (d)$ is the correct answer

69) Ans: C

Exp: Let the diagonal AC be along the line x = 2y and B be (3, 0).



(Note that (3, 0) does not lie on AC), then slope m of AB or BC is given by

$$\left|\frac{m-\frac{1}{2}}{1+\frac{m}{2}}\right| = \tan 45^\circ \qquad \left(\because \text{Slope } AC = \frac{1}{2}\right)$$

We easily get m = 3, -1/3 whence equation of AB or BC are determined.

70) Ans: A

Exp:

$$\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dx}{dy}=\frac{e^{\tan^{-1}y}-x}{1+y^2}$$

$$\Rightarrow \frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$$

This is linear different equation

$$\text{I.F.} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$$\therefore \text{Solution is } xe^{\tan^{-1} y} = \int \frac{e^{2\tan^{-1} y}}{1+y^2} dy + \frac{c}{2}$$

$$= \frac{e^{2\tan^{-1} y}}{2} + \frac{c}{2}$$

$$\Rightarrow 2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + c$$

$$\Rightarrow \text{Choice } (a) \text{ is correct}$$

71) Ans: C

Exp:

$$\lim_{x\to 0}\frac{f\left(x^2\right)-f\left(x\right)}{f\left(x\right)-f\left(0\right)}$$

Limit is in $\frac{0}{0}$ form, so use L – H rule

$$\lim_{x\to 0}\frac{2xf'\left(x^2\right)-f'\left(x\right)}{f'\left(x\right)} \because f\left(x\right) \text{ is strictly increasing function and hence}$$

$f'\left(x\right)>0$, for all x.

$$\because f'\left(x\right)>0, \forall x\in\left(\text{domain}\right)$$

$$\Rightarrow f'\left(x^2\right)>0, \forall x\in\left(\text{domain}\right)$$

$$\Rightarrow \lim_{x\to 0} 2xf'\left(x^2\right)=0$$

$$\Rightarrow \lim_{x\to 0}\frac{2xf'\left(x^2\right)-f'\left(x\right)}{f'\left(x\right)}=\lim_{x\to 0}\frac{2xf'\left(x^2\right)}{f'\left(x\right)}-\frac{f'\left(x\right)}{f'\left(x\right)}=-1$$

72) Ans: b

Exp: The given circle $x^2 + y^2 - 4x - 6y - 12 = 0$ has its centre at (2, 3) and radius equal to 5. Let (h, k) be the coordinates of the centre of the required circle. Then the point (h, k) divides the line joining (-1, 1) to (2, 3) in the ratio 3: 2, where 3 is the radius of the required circle. Thus, we have

$$h = \frac{3x2 + 2x - 1}{3+2} = \frac{4}{5} \ and$$

$$k = \frac{3x3 + 2x - 1}{3+2} = \frac{7}{5}$$

Hence, the requied circle is

$$\left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = 3^2$$

$$i.e., 5x^2 + 5y^2 - 8x - 14y - 32 = 0$$

73) Ans: A

Exp:

$$\text{Here,} \left(\vec{r} - \vec{b}\right) x\vec{a} = 0, \text{so} \left(\vec{r} - \vec{b}\right) \parallel \vec{a}$$

$$\therefore \quad \vec{r} - \vec{b} = t\vec{a} \quad \text{or} \quad \vec{r} = \vec{b} + t\vec{a}$$

$$\text{But } \vec{r}.\vec{c} = 0$$

$$\therefore \quad 0 = \vec{b}.\vec{c} + t\vec{a}.\vec{c}$$

$$\therefore \quad t = -\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{c}}$$

$$\therefore \quad \vec{r} = \vec{b} - \frac{\vec{b}.\vec{c}}{\vec{a}.\vec{c}}\vec{a}$$

74) Ans: A

Exp:

$$\frac{S_{kx}}{S_x}=\frac{\frac{Kx}{2}\left[2a+\left(Kx-1\right)\right]}{\frac{K}{2}\left[2a+\left(x-1\right)d\right]}=K\left[\frac{2a-d+Kxd}{2a-d+xd}\right]$$

$$\text{If } 2a-d=0 \text{ then } \frac{S_{Kx}}{S_K}=K\left[\frac{Kxd}{xd}\right]=K^2,$$

Which is possible when a = d/2

75) Ans: A

Exp:

Equations of the given lines are

$$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7} \qquad \ldots(i)$$

$$\text{and } \frac{x-15}{3}=\frac{y-2}{8}=\frac{z-8}{-5} \qquad \ldots\ldots(ii)$$

Let the equations of the line through the point

(1, 2, -4) be x-1/a = y-2/b = z+4/c ……….(iii)

Where a, b, c are its direction rations.

Since (iii) is perpendicular to (i) and (ii), we have

3a-16b + 7c = 0 ……..(iv)

And 3a+8b-5c = 0 …….(v)

Solving (iv) and (v) for a, b, c by the method of cross multiplication, we get

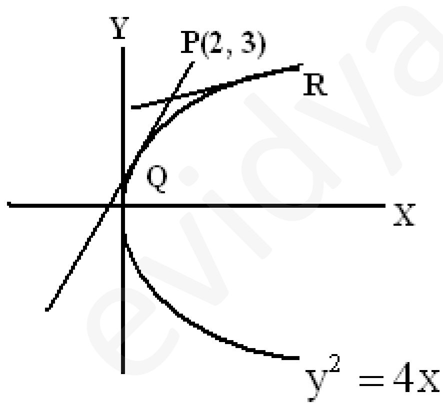
a/(80-56) = b/(21+15) = c/(24+48) ⇒ a/24 = b/36 = c/72

or a/2 = b/3 = c/6 ....(vi)

From (iii) and (vi), we obtain the required line as x-1/2 = y-2/3 = z+4/6

76) Ans: B

Exp: $\left.\begin{array}{r} t_1t_2 = 2 \\ t_1 + t_2 = 3 \end{array}\right] \Rightarrow t_1 = 1 \text{ and } t_2 = 2$



$\text{Hence point } \left(t_1^2, 2t_1\right) \text{and } \left(t_2^2, 2t_2\right)$

$\text{i.e.} \left(1, 2\right) \text{and} \left(4, 4\right)$

77) Ans: A

Exp:

$$\text{Let } L_1(x,y) = x + y + 1 \text{ and } L_2(x,y) = 2x - 3y - 5$$

$$\therefore L_1(10,-20) = 10 - 20 + 1 = -9, \text{ which is } -\text{ve}$$

$$\text{and } L_2(10,-20) = 20 + 60 - 5 = 75, \text{ which is } +\text{ve}$$

$$\therefore \text{Equation of the bisector will be}$$

$$\frac{x+y+1}{\sqrt{2}} = -\left(\frac{2x-3y-5}{\sqrt{13}}\right)$$

$$\Rightarrow x\left(\sqrt{13}+2\sqrt{2}\right) + y\left(\sqrt{13}-3\sqrt{2}\right) + \left(\sqrt{13}-5\sqrt{2}\right) = 0$$

78) Ans: b

Exp: $g(x) = \int_0^x f(t)\,dt \Rightarrow g(2) = \int_0^2 f(t)\,dt$

$\Rightarrow$ $g(2) = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt$ .... (i)

Now, $\frac{1}{2} \le f(t) \le 1 \; for\, t \in [0,1], we\ get$

$$\int_0^1 \frac{1}{2}dt \le \int_0^1 f(t)\,dt \le \int_0^1 1dt$$

(apply line integral inequality)

$\frac{1}{2} \le \int_0^1 f(t)\,dt \le 1$ .... (ii)

Again, $0 \le f(t) \le \frac{1}{2} \; for\ t \in [1,2]$

$$\int_1^2 0\ dt \le \int_1^2 f(t)dt \le \int_1^2 \frac{1}{2} dt$$

(apply line integral inequality)

$\Rightarrow$ $$0 \le \int_1^2 f(t)\ dt \le \frac{1}{2}$$ ………………. (iii)

From Eqs. (ii) and (iii), we get

$$\frac{1}{2} \le \int_0^1 f(t)dt + \int_1^2 f(t)\ dt \le \frac{3}{2}$$

$\Rightarrow$ $$\frac{1}{2} \le g(2) \le \frac{3}{2}$$ [from Eq. (i)]

$\Rightarrow$ $$0 \le g(2) \le \frac{3}{2}$$

79) Ans: A

Exp:

$$e_1^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{12}{4} = 4 \Rightarrow e_1 = 2$$

$$\text{Now } \frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$$

$$\Rightarrow \frac{1}{e_2^2} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow e_2^2 = \frac{4}{3} \Rightarrow e_2 = \frac{2}{\sqrt{3}}$$

80) Ans: B

Exp:

$$V = \frac{4}{3}\pi\left(r^3 - 10^3\right), \text{r being the distance of outer coat}$$

of ice from the centre

$$\therefore \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow 4\pi r^2 \frac{dr}{dt} = 50$$

$$\Rightarrow \frac{dr}{dt} = \frac{50}{4\pi r^2} = \frac{1}{18\pi} \text{cm / min}$$

$$(\because r = 10 + 5)$$

81) Ans: A

Exp:

Given circles $S = x^2 + y^2 + 3x + 7y + 2p - 5 = 0$

$S' = x^2 + y^2 + 2x + 2y - p^2 = 0$

Equation of required circles is $S + \lambda S' = 0$

As it passes through $(1,1)$ the value of $\lambda = \dfrac{-(7+2p)}{\left(6 - p^2\right)}$

If $7 + 2p = 0$, it becomes the second

circle $\therefore$ it is true for all values of p Except for which $7 + 2p = 0$

82) Ans: B

Exp:

$$8^{2n}-(62)^{2n+1}=(1+63)^{n}-(63-1)^{2n+1}$$
$$=(1+63)^{n}+(1-63)^{2n+1}=\left(1+^{n}C_{1}63+^{n}C_{2}(63)^{2}+.....+(63)^{n}\right)+$$
$$\left(1-^{(2n+1)}C_{1}63+^{(2n+1)}C_{2}(63)^{2}+....+(-1)(63)^{(2n+1)}\right)$$
$$=2+63\left(^{n}C_{1}+^{n}C_{2}(63)+....+(63)^{n-1-(2n-1)}C_{1}+^{(2n+1)}C_{2}(63)+....-(63)^{(2n)}\right)$$
$$\therefore \text{Re min der is } 2$$

83) B

$P(x) = x^4 + ax^3 + bx^2 + cx + d$

$P'(x) = 4x^3 + 3ax^2 + 2bx + c$

$Q x \equiv 0$ is a solution for $P'(x) = 0 \Rightarrow c = 0$

$\therefore P(x) = x^4 + ax^3 + bx^2 + d \qquad ....(1)$

Also, we have $P(-1) < P(1)$

$\Rightarrow 1 - a + b + d < 1 + a + b + d \Rightarrow a > 0$

Q, $P'(x) = 0$, only when $x = 0$ and $P(x)$ is differentiable in $(-1, 1)$,

we should have the maximum and minimum at the points $x = -1$

0 and 1

only also, we have $P(-1) < P(1)$

$\therefore$ Max, of $P(x) = \text{Max.}\{P(0), P(1)\}$ & Min. of $P(x) = \text{Min.}\{P(-1), P(0)\}$

in the interval $[0, 1]$

$P'(x) = 4x^3 + 3ax^2 + 2bx = x(4x^2 + 3ax + 2b)$

$Q P'(x)$ has only one root $x = 0$, $4x^2 + 3ax + 2b = 0$ has no real roots.

$\therefore (3a)^2 - 32b < 0 \Rightarrow \frac{9}{32} < \frac{a^2}{b}$

$\therefore b > 0$

Thus, we have $a > 0$ and $b > 0$

$\therefore P'(x) = 4x^3 + 3ax^2 + 2bx > 0, \forall x \in (0, 1)$

Hence $P(x)$ is increasing in $[0, 1]$

$\therefore$ Max. of $P(x) = P(1)$

Similarly, $P(x)$ is decreasing in $[-1, 0]$

Therefore Min. $P(x)$ does not occur at $x = -1$

84) Ans: b

Exp: Given,

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Applying R1→R1 - R3, R2→R2 - R3

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Applying $C^3 \to C^3 + C^1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ sin^2\theta & cos^2\theta & 1+sin^2\theta+4\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow$ $1 + \sin 2\theta + 4 \sin 4\theta + \cos 2\theta = 0$

$$\Rightarrow 2+4\sin 4\theta = 0 \Rightarrow \sin 4\theta = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$$

$$\Rightarrow 4\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$$

$\Rightarrow \quad \theta = \frac{n\pi}{4} - (-1)^n \frac{\pi}{24}$

For $\quad 0 \le \theta \frac{\pi}{2}$, we have $\theta = \frac{7\pi}{24}, \frac{11\pi}{24}$.

85) Ans: a

Exp: Here, sin C $= \frac{1 - \cos A \cos B}{\sin A \sin B} \le 1$ .... (i)

$\Rightarrow \quad \frac{1 - \cos A \cos B}{\sin A \sin B} - 1 \le 0$

$\Rightarrow$ 1 - cos A cos B - sin A sin B $\le$ 0

$\Rightarrow$ 1 - cos (A - B) $\le$ 0

$\Rightarrow$ cos (A - B) $\le$ 1

But cos (A - B) cannot be greater than 1

$\Rightarrow$ cos (A - B) = 1

$\Rightarrow$ A – B = 0

$\Rightarrow$ A = B

$\Rightarrow \quad \sin C = \frac{1 - \cos^2 A}{\sin^2 A} [\text{from Eq.(i)}]$

$$= \frac{sin^2 A}{sin^2 A} = 1$$

$\Rightarrow$ C = 900

$\Rightarrow$ A = B = 450

Using sine rule,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$\Rightarrow$ $\frac{1}{\sqrt{2a}} = \frac{1}{\sqrt{2b}}. = \frac{1}{c}$

$\Rightarrow$ a: b: c = 1:1: $\sqrt{2}$.

86) Ans: c

Exp: $\frac{3}{7} = 0.428, \quad \frac{5}{9} = 0.555, \frac{7}{11} = 0.636$

87) Ans: c

Exp:

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix} = \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c-1 & c+1 & c \end{vmatrix}$$

$$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+1} \begin{vmatrix} a+1 & a & a-1 \\ b+1 & -b & b-1 \\ c-1 & c & c+1 \end{vmatrix} = \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

This is equal to zero only if n+2 is odd i.e. n is odd integer.

88) Ans: C

Exp:

$$\text{Mean}(\bar{x}) = \frac{\text{sum of quantities}}{n} = \frac{\frac{n}{2}(a+1)}{n} = \frac{1}{2}[1+1+100d] = 1+50d$$

$$\text{M.D} = \frac{1}{n}\sum |x_i - \bar{x}| \Rightarrow 265 = \frac{1}{101}[50d+49d+48d+......d+0+d+d....+50d] = \frac{2/d}{101}\frac{50x51}{2}$$

$$\Rightarrow d = \frac{255x101}{50x51} = 10.1$$

89) Ans: D

Exp: 4 novels can be selected from 6 novels in ${}^6C_4$ ways. 1 can be selected from 3 dictionaries in ${}^3C_1$ ways. As the dictionary selected is fixed in the middle, the remaining 4 novels can be arranged in 4! Ways.

∴The required number of ways of arrangement = ${}^6C_4$ x ${}^3C_1$ x 4! = 1080

90) Ans: B

Exp:

Given $f(x) = x^3 + 5x + 1$

Now $f'(x) = 3x^2 + 5 > 0, \forall x \in R$

$\therefore f(x)$ is strictly increasing function

$\therefore$ It is one – one

Clearly, $f(x)$ is a continuous function and also increasing on R,

$\underset{x \to -\infty}{Lt} f(x) = -\infty$ and $\underset{x \to -\infty}{Lt} f(x) = \infty$

$\therefore f(x)$ takes every value between $-\infty$ and ∞

Thus, $f(x)$ is onto function