ANSWERS XII

CHEMISTRY

1.b	2.a	3.a	4.a	5.c	6.c	7.d	8.d	9.b	10.a	11.c	12.c	13.a
14.a	15.a	16.d	17.a	18.b	19.c	20.b	21.d	22.b	23.c	24.a	25.b	26.c
28.c	29.d	30.d										

PHYSICS

1.a	2.c	3.d	4.a	5.c	6.d	7.c	8.b	9.b	10.a	11.b	12.b	13.c
14.d	15.d	16.d	17.c	18.b	19.c	20.b	21.a	22.d	23.b	24.a	25.a	26.c
27.c	28.b	29.d	30.b									

MATHEMATICS

1.c	2.a	3.b	4.a	5.a	6.b	7.c	8.d	9.d	10.b	11.a	12.c	13.d
14.a	15.b	16.c	17.a	18.b	19.c	20.a	21.b	22.c	23.d	24.a	25.c	26.a
27.b	28.d	29.b	30.a									

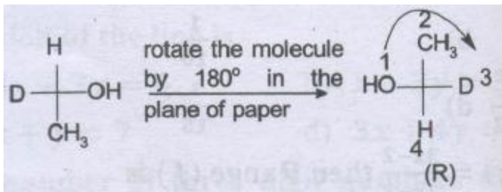
HINTS AND EXPLANATIONS XII
CHEMISTRY

Sol.1

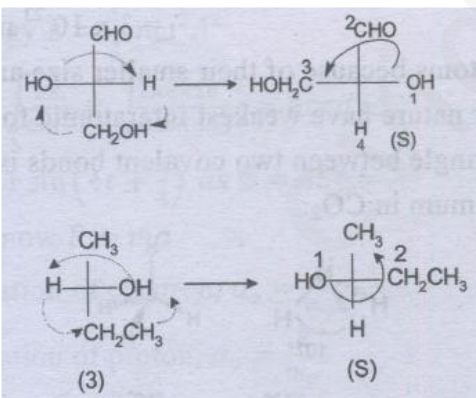
If the atom/group of lowest priority is at the button, simply rotate the eye in the order of descending order of priorities and find the direction, if the arrangement of groups is clockwise the configuration is (R) and if it is in anticlockwise direction then the configuration is (S).

In case of (4) the arrangement of groups (NH_2(1), CHO(2). CH_3(3) is in anticlockwise direction so its configuration us (S).

If the atom/group lowest priority is at the top, rotate the molecule by 180° in the plane of paper so as bring the atoms/group of the lowest priority is at the bottom. Then find the direction of descending priorities as shown below.



If the atom/group of lowest priority is at the left-hand side of horizontal end, then without changing the position of other atoms/groups at the top of vertical end, change the position of other atoms/groups in the anticlockwise or clockwise direction so as to bring the atom/group of the lowest priority at the bottom. Then find the direction of arrangements of atoms/groups in the order of descending priorities.

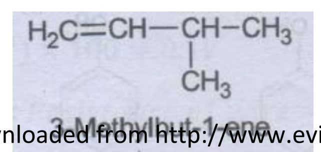


Sol.2

Acetamide (CH_3CONH_2) and benzonitrile (C_6H_5CN) are neutral ; triethylamine $(CH_3CH_2)_3N$ is basic and phenol (C_6H_5OH) is acidic in nature

Sol.3

IUPAC name of the compound is



Sol.4

The organic reaction product from the reaction of methyl magnesium bromide and ethyl alcohol is methane.

$CH_3MgBr + CH_3CH_2OH \rightarrow$

$CH_3CH_2OMgBr \; + \; CH_4$

Sol.5

Ethyl alcohol and methyl alcohol can be distinguished by iodoform test.

$CH_3CH_2OH \; + \; I_2 \; + \; NaOH \rightarrow CHI_3$ (iodoform)

(givesiodoform test)

$CH_3OH \; + \; I_2 \; + \; NaOH \rightarrow$ no iodoform

(does not give iodoform test)

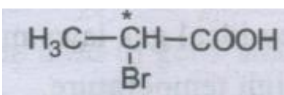
Sol.6

$2\ CrO_4^{2-} + 2H^+ \rightarrow Cr_2O_7^{2-} \; + \; H_2O$

Yellow orange

Sol.7

Optically active compounds contain one or more chiral centre(s).



Sol.8

Alkynes having a terminal ≡ C-H group react with $NaNH_2$ in liquid ammonia to furnish the corresponding sodio-derivatives.

$CH_3C \equiv C - H + NaNH_2 \; \rightarrow CH_3C \equiv C^- \; Na^+ \; + \; NH_3$

Sol.9

In Cotrell smoke precipitator, smoke is allowed to pass through a chamber having a series of plates charged to very high potential (20,00 – 70,000V) when charged particles of smoke get attracted to the charged plates, get precipitated and removed from smoke.

Sol.10

Transition metal oxides in lower oxidation state are basic (VO) whereas in higher oxidation states are acidic (V_2O_5).

Sol.11

Hydrometallurgy is the treatment of metal or the separation of metal from ores and ore concentrates by liquid processes, such as leaching, extraction, and precipitation.

Sol.12

Reduction potential of Mg^{2+} is lower as compared to water so during electrolysis water is reduced in preference to Mg^{2+} . The reduction potential of Cu^{2+}, Ag^{+} and Au^{+} are higher as compared to water so these can be obtained by electrolysis of aqueous solutions of their salts.

Sol.13

Option (a) is correct

Sol.14

Energy of electron = 13.6 x Z^2 eV; where Z is the atomic number. In case of helium ion (atomic number = 2), this comes out to be 13.6 x 4 = 54.4 eV. Thus the ionization potential for He^{+} will be 54.4 ev.

Sol.15

Carbon cannot have valency more than 4 due to absence of d-orbitals.

Sol.16

Species with completely filled or completely empty d-orbitals are colourless due to the absence of d-d transitions. Thus Ti^{4+} (d^0) and Cu^{+}(d^{10}) are colourless.

Sol.17

The oxides of metals like Zn, Cu, Sn, Pb can be reduced by using carbon as reducing agent.

Sol.18

Thermal stabilities of metal carbonates increase down group.

Sol.19

$CuSo_4.5H_2O$ liberates iodine from aqueous solution of KI.

Sol.20

When mercuric iodide is added to an aqueous solution of KI, vapour pressure increases due to which boiling point is lowered and freezing point is raised.

$HgI_2 + 2KI \rightarrow K_2[HgI_4]$

Sol.21

Molecular weight of $(CHCOO)_2Fe$ = 170; atomic weight of Fe = 56

170 g of $(CHCOO)_2Fe$ contains iron = 56 g 100 mg of $(CHCOO)_2$ Fe contains iron = $\frac{56}{170}$ x 100 x 10^{-3} = 32.94 x 10^{-3} g of Fe 400 mg of iron capsule caontains 32.94 mg of Fe % of from = $\frac{32.94}{400}$ x 100 = 8.23%

Sol.22

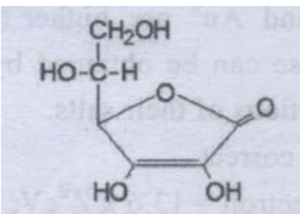
The size of the nucleus of an atom is of the order of 10^{-15} m

Sol.23

Isoelectric point is the *p*H at which a particular molecule carries no net electrical charge. At this point the negative and positive charges are equal.

Sol.24

Chemical name of vitamin C is ascorbic acid



Sol.25

(b)

Sol.26

A large negative ion favourscovalency so NaI has the greatest covalent character.

Sol.27

18g or 18 mL of water = 1 mol

= 6.023×10^{23} molecules

Or volume of 6.023×10^{23} watermolecules

= 18 mL

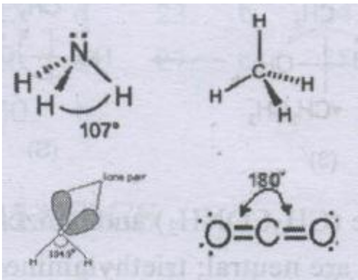
Volume of 1 water molecule = $\frac{18}{6.023 \times 10^{23}}$ = 3×10^{-23} mL

Sol.28

He atoms because of their smaller size and non-polar nature have weakest intratomic forces.

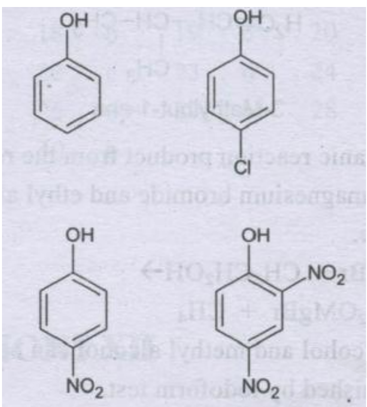
Sol.29

The angle between two covalent bonds is maximum in CO_2.



Sol.30

2,4- Dinitrophenol because of the presence of two electron withdrawing nitro groups is most acidic out of the given options.



PHYSICS

Sol.1

The significant figures of volume must watch with that of given length.

Sol.2

We know that V = rω = r (2πn) As r = l sin θ

$\therefore$ V = $l sin\theta\ (2\pi n)$ Or V = 1 × 0.78 × 2 × π × $\frac{2}{\pi} = 3.12 m/s$

Sol.3

As ma cos α = mg sin α

$\therefore a = g \tan \alpha$

Sol. 4

Given $\frac{E'}{E} = \frac{p'^2}{p^2} = \frac{4}{1} \Rightarrow p' = 2p$

Sol.5

As I = $\frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

Sol.6

By kepler's third law, τ is the time period of revolution along 2a = R, which is the distance of fall when the satellite stops

$\left(\frac{\tau}{T}\right)^2 = \left(\frac{a}{R}\right)^3$ $i.e. \left(\frac{R/2}{R}\right)^3 = \frac{1}{8} \Rightarrow \tau = \frac{T}{\sqrt{8}}$ $Time\ of\ fall\ is\ the\ half\ of\ this\ time$

$\therefore \frac{\tau}{2} = \frac{\sqrt{2}T}{8}$

Sol.7

Express Pressure $P \propto \frac{1}{R} \Rightarrow \frac{P_1}{P_2} = \frac{R_2}{R_1}$

Sol.8

Steel has higher specific heat

Sol.9

Real gases obey ideal gas laws more closely at low pressure and high temperature.

Sol.10

The particle which executed simple harmonic motion has a curve of kinetic energy which is given by $\frac{1}{2}m\omega^2(a^2 - x^2)$ which implies that at $x = \pm a$, kinetic energy is zero. As kinetic energy is always positive, the frequency of oscillation of kinetic energy is 2v

Sol.11

Given y = A sin(ωt + φ) and energy E = $\frac{1}{2}m\omega^2A^2$

$\Rightarrow \ \omega = \sqrt{\frac{2E}{mA^2}} = \sqrt{\frac{2\times8\times10^{-3}}{0.1\times(0.1)^2}} = 4rad/s$

∴ y = 0.1 sin $\left(4t + \frac{\pi}{4}\right)$as $\emptyset = 45° = \frac{\pi}{4}$

Sol.12

As we know F = ma

Acceleration of electron, $a_e = \frac{F}{m_e}$

Acceleration of proton, $a_p = \frac{F}{m_p}$

$\Rightarrow S = \frac{1}{2}a_p t_2^2 = \frac{1}{2}a_e t_1^2$

$\Rightarrow \frac{t_2}{t_1} = \sqrt{\frac{a_e}{d_p}} = \sqrt{\frac{m_p}{m_e}} = \left(\frac{m_p}{m_e}\right)^{1/2}$

Sol.13

Current, I = $\frac{e.m.f}{total\,resistance} = \frac{2}{3+R}$

Potential drop in wire

$V = (1\times10^{-3})\times100 = 0.1V$

Also V = $I⍰ \times Resistance\ of\ wire = \left(\frac{2}{3+R}\right)^3$

Equating the above two equations

0.1 = $\left(\frac{2}{3+R}\right)^3 \Rightarrow R = 57\Omega$

Sol.14

As $B \ \propto \frac{1}{r}$

∴ To double B, r should be halved

Sol.15

At the axis of wire, the intensity of magnetic field will be zero.

Sol.16

As $P = EI\cos\emptyset$ And $\cos\emptyset = \cos\frac{\pi}{2} = 0$

$\therefore$ P=0

Sol.17

Noth statemen the hysteresis loss and heating effect or current are responsible for heating of transformer.

Sol.18

Both statements are simply current.

Sol.19

As r become 30^0

$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60^0}{\sin 30^0} = \frac{\sqrt{3}\times 2}{2\times 1} = 1.732$

Sol.20

Lenses cannot be used to get a sharp image of object.

Sol.21

These degree of diffraction is proportional to wavelength.

Sol.22

The maximum frequency of x-rays is given by $\frac{eV}{h}$

Sol.23

By using $\lambda_m T = b \Rightarrow T = \frac{b}{\lambda_m} = \frac{2.93\times 10^{-3}}{2.93\times 10^{-10}} = 10^7 k$

Sol.24

From the relation $N = N_0\left(\frac{1}{2}\right)^n$

As $n = \frac{4800}{1600} = 3$

$\Rightarrow N = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Sol.25

Both statements are correct and statement 2 explains statement 1

Sol.26

As $4({}_2He^4) = {}_8O^{16}$

Mass defect, $\Delta m = [4(4.0026) - 15.9994] = 0.011 amV$

$\therefore$ Energy released per nuclei in case of oxygen = (0.011)(931.48) MeV = 10.24 MeV

Sol.27

The electric field at a distance r from the axis is given by $E = \frac{\lambda}{2\pi\epsilon_0 r}$ where λ is the charge per unit length of capacitor = $E \propto \frac{1}{r}$

Sol.28

In Isothermal process $pV = Constant$

$\Rightarrow pdV + Vdp = 0 \Rightarrow \left(\frac{dp}{dV}\right) = -\left(\frac{p}{V}\right)$ and bulk modulus, $B = -\left(\frac{dp}{dv/v}\right) = -\left(\frac{dp}{dv}\right) v$

$\therefore B = -\left[\left(-\frac{p}{v}\right) v\right] = p$

Sol.29

The required heat $Q = (1.1 + 0.22) \times 10^3 \times 1 \times (80 - 15) = 72800$ calories

Mass of steam condensed, $m = \frac{Q}{L} = \frac{72800}{540} \times 10^{-3} = 0.135 kg$

Sol.30

It is essential that the more parallel beam is incident on the curved surface in order that total deviation be equally divided on two surfaces. Thus the spherical aberration will be smaller if the convex surface of lens faces the object.

MATHEMATICS

Sol.1

Since 2^x is positive for every $x \in R$ so $f(x) = 2^x$ is positive real number for every $x \in R$. Also for every $x \in R$ there $\log_2 x \in R$ such that $f(\log_2 x) = 2^{\log_2 x} = x$

Sol.2

We know $\cos^2\theta + sin^2\theta = 1 \Rightarrow cos\theta = \pm\sqrt{1 - sin^2\theta}$

Since θ lies in third quadrant

$\therefore cos\theta = -\sqrt{1 - sin^2\theta} = -\sqrt{1 - \frac{24}{25}} = -\frac{1}{5}$

In third quadrant $tan\theta$ is positive

$\therefore tan\theta = \frac{sin\theta}{cos\theta} \Rightarrow tan\theta = -\frac{2\sqrt{6}}{5} \times \frac{-5}{1} = 2\sqrt{6}$

Sol.3

$(cos\alpha + cos\beta)^2 + (sin\alpha + sin\beta)^2 = \left[2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\right]^2$

$+\left[2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\right]^2 = 4cos^2\left(\frac{\alpha-\beta}{2}\right)cos^2\left(\frac{\alpha-\beta}{2}\right)$

$+4sin^2\left(\frac{\alpha+\beta}{2}\right)cos^2\left(\frac{\alpha-\beta}{2}\right) = 4cos^2\left(\frac{\alpha-b}{2}\right)\left[cos^{2\frac{\alpha+\beta}{2}} + sin^2\frac{\alpha+\beta}{2}\right] = 4cos^2\left(\frac{\propto+\beta}{2}\right)$

Sol.4

Let $\frac{a}{sinA} + \frac{b}{sinB} + \frac{c}{sinC} = k$

$\Rightarrow a = ksinA, b = k\, sinB, c = Ksin\, C$

So $a\, cosA + b\, cosB + c\, cosC$

$= k\, sinA\, cosA + ksinB\, cosB = k\, sinC\, cosC$

$= \frac{k}{2}[sin2A + sin2B + sin2C]$

$= \frac{k}{2}(4sinAsinBsinC)$

$= 2k\, sinAsinBsinC = 2asinBsinC$

Sol.5

Given that $(1+i)y^2 + (6+i) = (2+i)x$

$\Rightarrow (y^2 + 6) + i(y^2 + 1) = 2x + ix$

$y^2 + 6 = 2x$ (i)

$y^2 + 1 = x$ (ii)

From (i) & (ii) we get $y^2 + 6 = 2(y^2 + 1) \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

Now $x = y^2 + 1 \Rightarrow x = 5 \; when \; y = \pm 2$

Sol.6

The given equations is $x^2 - 5ix - 6 = 0$

$\Rightarrow x^2 - 3ix - 2ix + 6i^2 = 0$

$\Rightarrow (x - 3i)(x - 2i) = 0 \Rightarrow x = 3i, 2i$

Sol.7

We have ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11:52$

$\Rightarrow \frac{22!}{(21-r)!} : \frac{20!}{(18-r)!} = 11 : 52$

$\Rightarrow \frac{22!}{(21-r)!} \times \frac{(18-r)}{20!} = \frac{11}{52}$

$\Rightarrow \frac{22\times21\times20!}{(21-r)(20-r)(19-r)(18-r)!} \times \frac{(18-r)!}{20!} = \frac{11}{52}$

$\Rightarrow \frac{22\times21}{(21-r)(20-r)(19-r)} = \frac{11}{52}$

$\Rightarrow (21 - r)(2 - r)(19 - r) = 2 \times 21 \times 52$

$\Rightarrow 2 \times 3 \times 7 \times 4 \times 13 = 12 \times 13 \times 14$

$\Rightarrow (21 - r)(20 - r)(19 - r) = (19 - 7)(21 - 7)(20 - 7)$

$\Rightarrow r = 7$

Sol.8

If F is not included in any word, than we first select 4 letters from the remaining 6 letters. This can be done in $^6C_4$ ways. Now every selection has 4 letters which can be arranged in a row in 4! Ways. Therefore the total number of words

$= {}^6C_4 \times 4! = 360$

Sol.9

Let $(r+1)$th term be independent of x in the given expression

Now $T_{r+1} = {}^{12}C_r x^{12-r}\left(-\frac{1}{x}\right)^r$

$= {}^{12}C_r(-1)^r x^{12-2r}$ (i)

If this term is independent of x, the we must have 12 -- 2r = 0

If this term is independent of x, the we must have 12 -- 2r = 0 $\Rightarrow$ r = 6

So (6 + 1)the i.e. 7th term is independent of x

Putting $r = 6$ in (i) we get

$T_7 = {}^{12}C_6\,(-1)^6 = {}^{12}C_6$

Sol.10

Let a the first term & d be the common difference of a given A.P. Then

$a_3 = 7 and a_7 = 3a_3 + 2$

$a + 2d = 7 and a + 6d = 3(a + 2d) + 2$

$\Rightarrow 2 + 2d = 7 and\ a = -1$

$\Rightarrow a = -1,\ d = 4$ Now $s_n = \frac{x}{2}[2a + (x-1)d]$

$\Rightarrow s_{20} = \frac{20}{2}[2(-1)\ + (20-1)4]$

$\Rightarrow s_{20} = \frac{20}{2}(-2 + 76) = 740$

Sol.11

Let a be the first term and r be the common ratio of the G.P. then

$S = 8$ and$ar = 2$

$\Rightarrow \frac{a}{1-r} = 8andr = \frac{2}{a}$

$\Rightarrow \frac{a}{1-\left(\frac{2}{a}\right)} = 8$

$\Rightarrow a^2 - 8a + 16 = 0$

$\Rightarrow (a-4)^2 = 0 \Rightarrow a = 4$

Sol.12

Let equation of line be

$\frac{x}{2} + \frac{x}{b} = 1$ (i)

It passes through (3, 4). Therefore

$\frac{3}{2} + \frac{4}{b} = 1$ (ii)

Given that $a + b = 14$

$\Rightarrow b = 14 - a$

Put $b = 14 - a$ in (ii) we get

$\frac{3}{2} + \frac{4}{14-a} = 1$

$\Rightarrow a^2 - 13a + 42 = 0$

$\Rightarrow (a-7)(a-6) = 0 \Rightarrow a = 7,6$

For $a = 7, b = 7 and\ f\ or\ a = 6, b = 8$

Put the value of a, b in (i) we get

$\frac{x}{7} + \frac{y}{7} = 1 \Rightarrow$ $a + b = 7$ and $\frac{x}{6} + \frac{y}{8} = 1 \Rightarrow 4x + 3y = 24$

Sol.13

We have to determine the total number of three digit numbers formed by using the digits 1,7,8,9. Clearly , the repetition of digits is allowed. A three digit number has three places vizunit's, ten's and hundred's. Units place can be filled by any of the digits 1,7,8,9. So, units place can be filled in 4 ways. Similarily each one of the ten's and hundred's place can be filled in 4 wyas.

$\therefore$ Total number of required numbers = $4\times4\times4=64$

Sol.14

3 consonants out of 7 and 2 vowels out of 4 can be chosen in ${}^7C_3 \times {}^4C_2$ ways. Thus there are ${}^7C_3 \times {}^4C_2$ groups each containing 3 consonants and 2 vowels. Since each group contain 5 letters, which can be arranged among themselves in 5 ! ways. Hence, the required number of words

$= ({}^7C_3 \times {}^4C_2) \times 5! = 25200$

Sol.15

$(99)^5 = (100-1)^5$

$= {}^5c_0 \times (100)^5 - {}^5c_1 \times (100)^4 + {}^5c_2 \times (100)^3$

$= {}^5C_3\ (100)^2 + {}^5c_4\ (100)^1 - {}^5c_5 \times (100)^0$

$= (100)^5 - 5(100)^4 + 10(100)^3$

$-10(100)^2 + 5(100) - 1$

$= (10^{10} + 10^7 + 5 \times 10^2)$

$-(5 \times 10^8 + 10^5 + 1)$

$= 10010000500 - 500100001 \ = 9509900499$

Sol.16

We have $5 + 55 + 555 + \ldots$to n terms $= 5[1 + 11 + 111 + \ldots$to n terms$] = \frac{5}{9}[9 + 99 + 999 + \ldots$to n terms$] =$ $\frac{5}{9}\left\{\begin{matrix}(10+10^2+10^3+\ \ldots+10n)\\ -(1+1+1\ldots\ldots\ldots \text{n times})\end{matrix}\right\} = \frac{5}{9}\left[10 \times \frac{(10^n-1)}{(10-1)} - n\right] = \frac{5}{9}\left[\frac{10}{9}(10^n-1) - n\right] = \frac{5}{81}[10^{n+1} - 10 - 9n]$

Sol.17

Equation of line parallel to

$3x-4y-5=0$is

$3x-4y+\lambda=0$ (i)

Put $x=-1$ in $3x-4y-5=0$

We get $y=-2$

$\therefore$ $(-1,-2)$ is a point on $3x-4y-5=0$ Since the distance between the two lines is one unit.

$\therefore$ the length of perpendicular from $(-1,-2)$ to $3x-4y+\lambda=0$ is 1

i.e. $\frac{|(3)(-1)+(-4)(-2)+\lambda|}{\sqrt{3^2+(-4)^2}}=1 \;=\frac{|5+\lambda|}{5}=1 \Rightarrow |5+\lambda|=5$

$\Rightarrow 5+\lambda=\pm 5 \Rightarrow \lambda=0 \; or \; \lambda=-10$

Put $\lambda=-10$ in (i) we get $3x-4y-10=0$

Sol.18

Let the equation of circle be

$x^2+y^2+2gx+2fy+c=0$ (i)

It passes through (1,0), (−1,0) and (0,1)

$\therefore 1+2g+c=0$ (ii)

$1-2g+c=0$ (iii)

$1+2f+c=0$ (iv)

Subtracting (iii) from (ii), we get

$4g=0 \Rightarrow g=0$

Put $g=0$ in (ii), we get $\mathrm{c}=-1$

Putting c = -1 in (iv), we get $f=0$

Substituting the values of g, f,c in (i),

We get $x^2+y^2=1$

Sol.19

$Lt_{x\to 4}\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$

$= Lt_{x\to 4}\frac{(3-\sqrt{5+x})(3+\sqrt{5+x})(1+\sqrt{5-x})}{(1-\sqrt{5-x})(1+\sqrt{5-x})(3+\sqrt{5+x})}$

$= Lt_{x\to 4}\frac{(9-5-x)}{(1-5+x)}\left(\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right)$

$= Lt_{x\to 4}\frac{-(x-4)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}$

$= Lt_{x\to 4}\frac{-(1+\sqrt{5-x})}{(3+\sqrt{5+x})}$

$= -\frac{(1+1)}{3+3} = -\frac{1}{3}$

Sol.20

The total number of words which can be formed by permuting the letters of the word

'UNIVERSITY' is $\frac{10!}{2!}$ Regarding $2I''$ s as one letter, the number of ways of arrangement in which both I' s are together = 9! Hence required probability = $\frac{9!}{10!/2!} = \frac{1}{5}$

Sol.21

Let $y = f(x)$ then $y = \frac{3x-2}{2x-3}$

$\Rightarrow 2xy - 3y = 3x - 2$

$\Rightarrow x = \frac{3y-2}{2y-3}$

$\therefore Range\ (f) = R - \left\{\frac{3}{2}\right\}$

Sol.22

$\sin^{-1}\left(\frac{sinx+cosx}{\sqrt{2}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}sinx + \frac{1}{\sqrt{2}}cosx\right)$

$= \sin^{-1}\left(sinx\ cosx\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right)$

$= \sin^{-1}\sin\left(x + \frac{\pi}{4}\right) = x + \frac{\pi}{4}$

Sol.23

Let $\Delta = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$

$\Rightarrow \Delta = a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$\Rightarrow \Delta = a^2b^2c^2 \begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix}$

$[Applying\ c_2 \to c_2 + c_1\ and\ c_3 \to c_3 + \ c_1]$

$\Delta = a^2b^2c^2(-1) \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix}$

$\Rightarrow \Delta = a^2b^2c^2(-1)(-4) = 4a^2b^2c^2$

Sol.24

We know $\sin^{-1} x + \sin^{-1} y \ = \sin^{-1}\left[x\sqrt{1-y^2} \ + y\sqrt{1-x^2}\right]$

$\therefore \sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{7} = \sin^{-1}\left[\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2} + \frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}\right]$

$= \sin^{-1}\left[\frac{3}{5} \times \frac{15}{17} \times \frac{8}{17} \times \frac{4}{5}\right] \ = \sin^{-1}\frac{77}{85}$

Sol.25

$f(x) = \frac{4+x^2}{4x-x^3}$

$= \frac{4+x^2}{x(4-x^2)}$

$f(x) = \frac{4+x^2}{x(2-x)(2+x)}$

$f(x)$ is not defined at $x = 0, -2, 2$

$\therefore f(x)$ is discontinuous at three points

Sol.26

$Lt_{x\to a} f(x) = f(a) \ \forall \ x \in R \ and$

$$Lt_{x\to a} g(x) = g(a) \ \forall \ x \in R$$

$\therefore f(x) \& g(x)$ both are continuous at $\mathrm{x} = 0$

Sol.27

The equation of curve is $x^2 + 3y + y^2 = 5$

Differentiating both sides w.r.t. x, we get $2x + 3\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{2x}{2y+3}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = -\frac{2}{2+3} = -\frac{2}{5}$

Slope of normal at (1, 1) = $-\frac{1}{\left(\frac{dy}{dx}\right)_{(1,1)}} = -\frac{1}{-\frac{2}{5}} = \frac{5}{2}$

Sol.28

We have $f(x) = \frac{x}{(x^2+1)^2}$

$f'(x) = \frac{(x^2+1).1-x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$

For $f(x)$ to be increasing we must have

$f'(x) > 0 \Rightarrow \frac{1-x^2}{(x^2+1)^2} > 0 \Rightarrow 1 - x^2 > 0 \Rightarrow x^2 - 1 < 0$

$\Rightarrow (x-1)(x+1) < 0 \Rightarrow -1 < x < 1 \Rightarrow x \in (-1,1)$

Sol.29

$I = \int \tan^{-1}\left\{\sqrt{\frac{1-\sin x}{1+\sin x}}\right\} dx$

$I = \int \tan^{-1}\left\{\sqrt{\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}}\right\} dx$

$I = \int \tan^{-1}\left\{\sqrt{\frac{2sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\right\} dx$

$I = \int \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}$

$= \int \left(\frac{\pi}{4}-\frac{x}{2}\right) dx = \frac{\pi}{4}x - \frac{x^2}{2} + c$

Sol.30

$$I = \int_0^e \frac{e^{-x}}{1+x} dx$$

Put $e^x = t \Rightarrow e^x dx = dt \Rightarrow dx = \frac{1}{t} dt$

$I = \int_0^e \frac{dt}{t^2(1+t)}$

$= \int_1^e \left(-\frac{1}{t} + \frac{1}{t^2} + \frac{1}{1+t}\right) dt$

$= \left[-\log|\mathrm{t}| + \frac{t^{-1}}{-1} + \log|1+t|\right]_1^e$

$= \left[-\log e - \frac{1}{e} + \log(1+e)\right]$

$-[-log 1 - 1 + \log 2]$

$= -1 - \frac{1}{e} + \log(1+e) + 0 + 1 - \log 2$

$= \log\frac{1+e}{2} - \frac{1}{e}$