ANSWERS KEY

CHEMINSTRY

1.d	2.b	3.c	4.b	5.b	6.b	7.c	8.b	9.a	10.c	11.d	12.d
13.c	14.a	15.b	16.c	17.d	18.d	19.c	20.c	21.b	22.d	23.a	24 c
25.b	26.c	27.b	28.c	29.c	30.c						

PHYSICS

1. A	2.a	3.b	4.a	5.c	6.d	7.d	8.d	9.c	10.d	11.b	12.b
13.a	14.b	15.d	16.a	17.a	18.b	19.b	20.c	21.b	22.a	23.d	24 c
25.a	26.c	27.b	28.d	29.d	30. A						

MATHEMATICS

1.a	2.b	3.b	4.d	5.a	6.a	7.b	8.b	9.a	10.d	11.b	12.d
13.b	14.b	15.c	16.c	17.d	18.b	19.c	20.a	21.a	22.d	23.a	24 b
25.a	26.c	27.b	28.c	29.a	30.c						

CHEMINSTY

Sol 1.

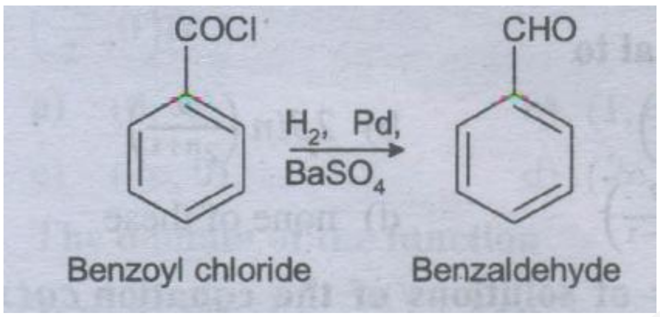
Hot alkaline $KMnO_4$ oxidizes I – bytyne to CH_3CH_2COOH and CO_2.

$H_3C - CH_2 - C \equiv CH \xrightarrow[Heat]{KMnO} H_3C - CH_2 - COOH + CO_2$

Sol 2.

Benzoyl chloride is reduced to benzaldehyde

(Rosenmund's reduction).



Sol 3.

Since rate increases 2 times on increasing the initial concentration of reactant 4 times, rate depends upon ½ power of the concentration, i.e., Rate = k $[A]^{1/2}$, these order of reaction is 1/2.

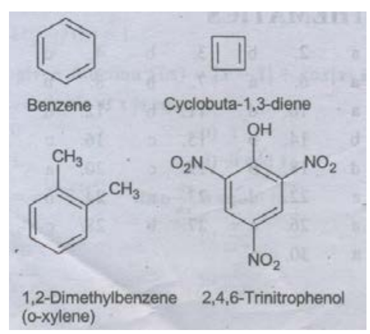
Sol 4.

Q = I x t = 2 x 20 x 60 (s) = 2400 C

$2H_2O \rightarrow O_2 + 4 H^+ + 4 e$

4 x 96500 C are required to $O_2 = \frac{22.4}{4 \times 96500} \times 2400 = 0.1392$ L

Sol 5. Cyclobutadiene is an antiaromatic compound; it contains 4π electrons in a cyclic conjugated system. Benzene, o-xylene and pieric acid are aromatic compounds according to Huckel's rule of aromaticity.



Sol 6.

Cu^{2+}, Fe^{2+} and Fe^{3+} complexes are colored because of presence of unpaired d – electron; Cd^{2+}complex is colored because all electrons in d-subshell (d^{10} case) are paired.

Sol 7.

The enthalpy change depends upon the final and initial state of the reaction; the intermediates do not effect overall enthalpy change of a reaction

Sol 8.

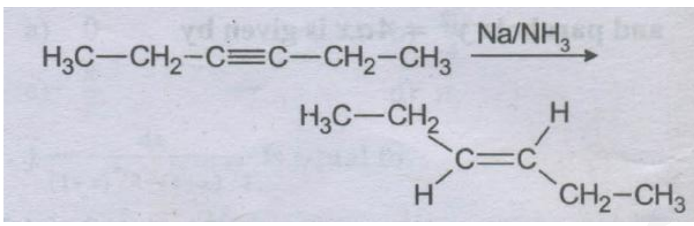
For an ideal gas, compressibility factor, $Z = \frac{pv}{nRT} = 1$

Sol 9.

Calcium can be obtained by electrolysis of molten $CaCI_2$

Sol 10.

Alkynes react with sodium metal In liquid ammonia to form trans alkenes.



Sol 11.

$C_aC_2 \rightarrow C_2H_2 \rightarrow C_2H_2 \rightarrow$ polyethene

64 26 28 n x 28

64 kg 26 kg 28 kg 28 kg

Thus as per balanced equation. Amount of polyethene obtained from 64 kg of clcium carbide is 28 kg.

Sol 12.

Nesseler's reagent K_2 $[HgI_4]$ is used for detection of ammonium ion. With ammonia it gives a reddish brown ppt or colouration.

Sol 13.

$Cu^{2+} + 2I^- \rightarrow Cu + I_2$

Sol 14.

Molecular orbital energy level diagram for O_2 shows two unpaired electrons in antibonding pi-molecular orbitals so it is paramagnetic.

Sol 15.

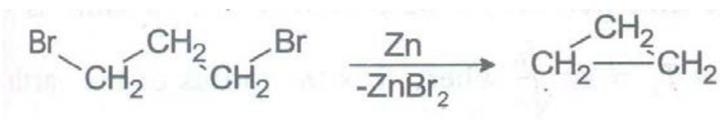
For pH = 1, $[H^+] = 10^{-1}$; for pH = 2, $[H^+] = 10^{-2}$

Total $[H^+] = 0.1 + 0.1 = 0.11$

pH = - log (0.11) = 0.9586

Sol 16.

Dehalogenation of 1,3 – dibromopropane with Zn givscyclopropane.



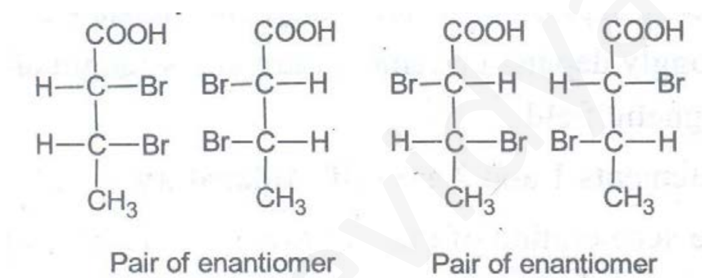
Sol 17.

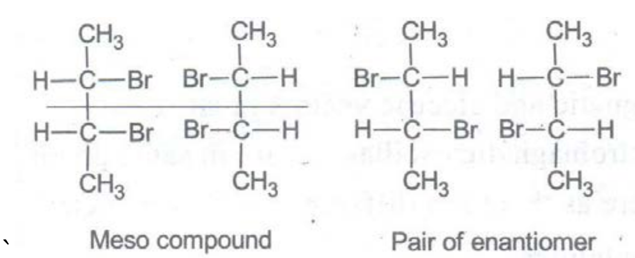
Hydrolysis of CH_3CN gives acetic acid and ammonia.

$CH_3CN \xrightarrow{Hydrolysis} CH_3COOH + NH_3$

Sol 18

Total number of stereo-isomers of $CH_3CHBrCHBrCOOH$ is 4 and for $CH_3CHBrCH$ is 3.

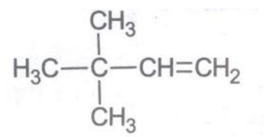




Sol 19

Lithium is less electronegative as compared to hydrogen so hydrogen bonded to lithium in $LiAlH_4$ is transferred as hydride ion.

20/IUPAC name of the given compound is:



3,3-Dimethylbut-1-ene

Sol 21.

$I_2 + I^- \rightarrow I_3^-$, I^- gives electrons to I_2, it acts as a Lewis base.

Sol 22.

On heating in a charcoal cavity Zn^{2+} salt formsZnO which on combination with CoO (from $Co(NO_3)_2$) gives a green mass CoO. ZnO.

Sol 23

R- CH_2CH_2OH can be coverted into R –

CH_2CH_2COOH with the help of PBr_3, KCN, H^+

R – CH_2-CH_2-OH $\xrightarrow{PRr_3}$ R-CH_2-CH_2-Br $\xrightarrow{KCN}$ R-CH_2-CH_2CN

$\xrightarrow{H^+}$ R-CH_2-CH_2-COOH

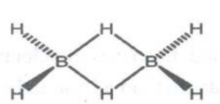
Sol 24.

Al $(OH)_3$ dissolves in excess of NaOH solution to form aluminates.

$Al(OH)_3 + 3NaOH \rightarrow Na_3AlO_3 + 3H_2O$

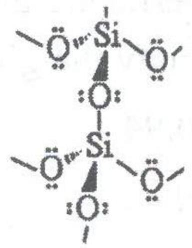
Sol 25.

All six bonds are not similar in diborane.



Sol 26.

In SiO_2, silicon is sp^3 – hybridized.



Sol 27.

PF_3 can undergo rapid hydrolysis

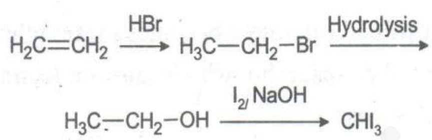
$2PF_3 + 3h_2O \rightarrow 2HF + H_3PO_4 + HPF_4$

Sol 28.

CO is a neutral ligand therefore, oxidation state of Ni in $[Ni(CO)_4]$ is zero.

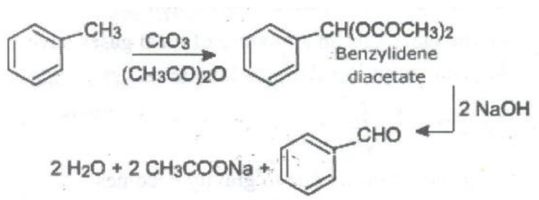
Sol 29.

Iodo form is formed.



Sol 30.

The role of acetic anhydride which is used as a solvent in CrO_3 oxidation of toluene to benzaldehyde is to protect further oxidation of benzaldehyde.



PHYSICS

Sol 1.

Here $[X] = \frac{[ML^2T^{-2}][ML^2T^{-1}]^2}{[M^{-1}L^3T^{-2}]^2[M^5]}$

$= \frac{[M^3L^6T^{-4}]}{[M^3L^6T^{-4}]} = [M^0L^0T^0]$

∴xis a dimension less quantity

⇒xmust be an angle is a dimensionless quantity

Sol 2.

Magnitudes and directions of velocity and accelerationare individual and to be seen in this context also

Sol 3.

D' Alembert and Inertia are the two physical quantities which satisfy the given condition

Sol 4.

For glass balls e = 0.94

For ivary balls e = 0.81

For cork balls e = 0.65

For lead balls e = 0.20

Coefficient of restitution is maximum for glass balls.

Sol 5.

A ball at rest on a horizontal surface has neutral equilibrium

Sol 6.

In an artificial satellite g becomes meaningless due to free fall of spaceship which causes a feeling of weightlessness.

Sol 7.

As viscosity has high values at low temperatures, the cold fluid flows sluggishly

Sol 8.

Neutral temperature is independent of the temperature is independent ofhot junction, temperature of cold junction and temperature of inversion

Sol 9.

All the molecules in a mixture of ideal gases have the same mean translational kinetic energy

Sol 10.

Using $T = 2\pi\sqrt{\frac{l}{g}}$

As the acceleration due to gravity becomes (5 g + g) = 6g, as mentioned in the question, we get new time period as

$T' = 2\pi\sqrt{\frac{l}{6g}} = \frac{T}{6}$

Sol 11.

The time periods for both bodies will be same as $T_1 = T_2 = 2\pi\sqrt{\frac{R}{g}}$ where R is the radius of the earth decreases.

Sol 12.

As the dielectric is introduced, charge on plate decreases.

Sol 13.

The formula The formula for drawing power at a voltage V is $P = \left(\frac{v}{v_0}\right) P_0$

Sol 14.

The ratio of radii

$\frac{r_e}{r_p} = \frac{m_e}{m_p}$

Sol 15.

The susceptibility of ferromagnetic substances strongly depend on temperature and strength of magnetic field

Sol 16.

Statement 1 and 2 are self explanatory

Sol 17.

The acceleration of magnet M will be larger than g when it is below the ring R and moving away from it.

Sol 18.

Magnetic and electric vectors in an electromagnetic oscillations are in same phase where as there is difference of $\pi/2$ in their orientations.

Sol 19.

Using $d = \frac{\lambda}{\sin\theta'}$ we get

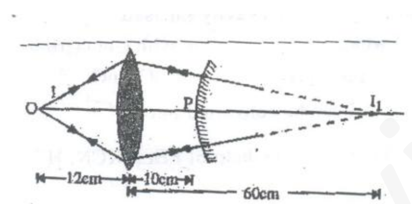
$D = \frac{400 \times 100^{-9}}{\sin 30^0} = \frac{400 \times 10^{-9}}{1/2} = 800 \times 10^{-7}$ m

Again using $d \sin\theta = \frac{3\lambda}{2d}$

$\text{Sin}\,\theta = \frac{3 \times 4 \times 10^{-7}}{2 \times 8 \times 10^{-7}} = \frac{3}{4}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{3}{4}\right)$

Sol 20.



Without mirror:

Using Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Given u = - 12 cm, f = 10 cm $\Rightarrow \frac{1}{v} + \frac{1}{12} = \frac{1}{10}$

$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{12} = \frac{1}{60}$

$\Rightarrow$ v = 60 cm

Which shows that in the absence of mirror, the image is formed at 60 cm away from the lens.

Sol 21.

With Mirror :

Using above figure

PI_1 = radius of curvature of the mirro

i.e. R = 60 – 10 = 50 cm

$\Rightarrow f = \frac{R}{2} = \frac{50}{2} = 25$ cm

Sol 22.

Kinetic energy of electron is greater than kinetic energy of proton due to larger velocities.

Sol 23.

The infrared radiation will be emitted if the transition between the staten = 5 to n = 4 is allowed since the infrared radiation has less energy than that of ultraviolet radiation.

Sol 24.

After 2 days 1000 atoms become 100.

After another two days 100 atoms will remain $\frac{100}{10}$ i.e.e 10 atoms

∴After 4 days 10 atoms will remain.

Sol 25.

Since pentavalent impurity atom contributes one electron to the intrinsic semiconductor, 10^{24} impurity atoms per unit volume will contribute 10^{24} electrons to the intrinsic semiconductors. Therefore, the free electron density will increase by 10^{24} m^{-3}

Sol 26.

Satellite communication and line of sight (LOS) communications (e.g. MW radio waves) are carried through space waves

Given $d_1 = \sqrt{2rh_1}$ and $d_2 = \sqrt{2Rh_2}$ For maximum range

$D_{max} = d_1 + d_2 = d$ $D_{max} = \sqrt{2Rh_1} + \sqrt{2Rh_2}$

i.e. $d_1 = d_2$ or $\sqrt{2Rh_1} = \sqrt{2Rh_2}$

i.e. $h_1 = h_2$ If $h_1 + h_2 = h$

Then $h_1 = \frac{h}{2}$

Sol 27.

Givne$\frac{mv^2}{R}$∝ R$^{-5/2}$

⇒v∝ R$^{3/4}$

As T = $\frac{2\pi R}{v}$

⇒ T^2∝$\left(\frac{R}{v}\right)^2$

or T^2∝$\left(\frac{R}{R^{-\frac{3}{4}}}\right)^2$

or T^2∝ R$^{7/2}$

Sol 28.

Mass of hanging portion is $\frac{M}{3}$ and centre of mass C is at a distance h = L/6 below the table top.

∴the required workdone, W = mgh

$= \left(\frac{M}{3}\right) g \left(\frac{L}{6}\right) = \frac{mgL}{18}$

Sol 29.

The centre of mass will always remain at rest as net force on centre of mass zero.

Sol 30.

In uniform circular motion, centripetal force acting on the particle. The torque due to this force about the centre is zero. Hence angular momentum about centre remains conserved.

MATHEMATICS

Sol 1. Given Re $\left(\frac{az+b}{cz+d}\right) = 1$

$\Rightarrow \frac{\frac{(az+b)}{cz+c} + \left(\frac{a\bar{z}+b}{cz+d}\right)}{2} = 1$

⇒ $(az + b)\ (c\bar{z} + d) + (a\bar{z} + b)\ (cz + d) = 2\ (cz + c)\ (c\bar{z} + d)$

⇒$bcz + bd = 2\ (c^2\ z\bar{z} + cdz + cd\bar{z} + d^2)$

⇒$2ac\bar{z} + ad\ (z + \bar{z}) + bc\ (z + \bar{z}) + 2bd = 2\ [c^2\ z\bar{z} + cd\ (z + \bar{z}) + d^2]$

⇒ $(2ac - 2c^2)\ (x^2 + y^2) + (ad + bc - 2cd)\ (2x) + 2\ (bd - d^2) = 0$

It is a circle.

Sol 2.

Given that

$\sqrt{x} + \sqrt{x - \sqrt{(1-x)}} = 1$ (i)

$\sqrt{x - \sqrt{(1-x)}} = 1 - \sqrt{x}$ Squaring both sides, we get

$x - \sqrt{(1-x)} = (1-\sqrt{x})^2$ $x - \sqrt{(1-x)} = 1 - 2\sqrt{x} + x$

$x - \sqrt{(1-x)} = 1 - 2\sqrt{x}$ Squaring again on both sides, we get

$1 - x = 1 + 4x - 4\sqrt{x} \Rightarrow 4\sqrt{x} = 5x$

Squaring on both sides again, we get

$16x = 25x^2 \Rightarrow 25x^2 = 16x = 0 \Rightarrow x(25x - 16) = 0$

$\Rightarrow x = 0, x = \frac{16}{25}$ $x = 0$ does not satisfy (i)

Therefore $x = \frac{16}{25}$ So (i) has only one real root.

Sol 3.

Given sequences are 2, 4, 6, 8, 10, 12, 14, 16 and 1, 4, 7, 10, 13, 16

Common terms are 4, 10, 16 . . . .

n^{th}terms of the common sequence

$= 4 + (n - 1)(6) = 6n - 2$

100^{th} term of the first term sequence

$= 2 + (100 - 1) 2 = 200$

100^{th} term of the second sequence

$= 1 + (100 - 1) 3 = 300$

$\Rightarrow t_n \leq 200$

$\Rightarrow 6n - 2 \leq 200$

$\Rightarrow 6n \leq 198$

$\Rightarrow n \leq 33$ $\Rightarrow b = 23$

Sol 4.

For positive integral solutions

36 = 36 x 1x 1 = 18 x 2 x 1 = 12 x 3x 1 = 9 x 4 x 1 = 9 x 2 = 6x 6 x 1 = 4 x 3 x 3

Number of positive integral solutions

$= \frac{3!}{2!} + \frac{3!}{1!} + \frac{3!}{1!} + \frac{3!}{1!} + \frac{3!}{2!} + \frac{3!}{2!} + \frac{3!}{2!}$

$= 3 + 6 + 6 + 6 + 3 + 3 + 3$

$= 30$

Sol 5.

$7^{20} - 5^{20} = (7 - 5)(7^{19} + 7^{18}.5^{1} + 7^{17}.5^{2} + \ldots\ldots\ldots\ldots\ldots\ldots + 5^{19})$

$\therefore 7^{20} - 5^{20}$ is divisible by 2

Sol 6.

$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a - b)(c - c)(c - a)$

$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ 2a & 2b & 2c \end{vmatrix} \Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$

$= \frac{2}{abc} \begin{vmatrix} a & b & c \\ abc & abc & abc \\ a^2 & b^2 & c^2 \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ a^2 & b^2 & c^2 \end{vmatrix} = -2 \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$\therefore 2\Delta_1 + \Delta_2 = 0$

Sol 7.

If A is orthogonal matrix then $AA^T = I = A^T A$

Sol 8.

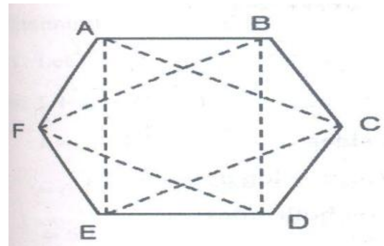
Let $x = \log_{20} 3 = \frac{\log 3}{\log 20} = \frac{1 \log 27}{3 \log 20} > \frac{1}{3}$

$\left[\because \frac{\log 27}{\log 20} > 1\right]$

$\therefore \frac{1}{3} < x < \frac{1}{2}$

$\therefore x \in \left(\frac{1}{3}, \frac{1}{2}\right)$

Sol 9.



Let ABCDEF be a regular hexagon.

Total number of triangles $= 6_{c_3}$

Total form the equilateral the

Vertices should be either A, C, E or B, D, F

$\therefore$ Number of equilateral triangles = 2

The required probability $= \frac{2}{20} = \frac{1}{10}$

Sol 10.

Let E donates the event that sum of eight occurs on the die and A the event that the man reports that it is eight. The events that sum of eight occurs are {(2, 6), (3,5), (4,4), (5,3), (6,2)}

$P(E) = \frac{5}{36}$ $\quad\quad$ $P(E') = \frac{31}{36}$

$P(A/E) = \frac{75}{100} = \frac{3}{4}$ & $P(A/E') = \frac{1}{4}$ $\quad\quad$ By Baye's Theorem

$P(E/A) = \frac{P(E)P(A/E)}{P(E)P(A/E) + P(E')(A/E')}$

$= \frac{\left(\frac{5}{36}\right)\left(\frac{3}{4}\right)}{\left(\frac{5}{36}\right)\left(\frac{3}{4}\right)+\left(\frac{31}{36}\right)\left(\frac{1}{4}\right)}$ $P(E/A) = \frac{\frac{15}{144}}{\frac{15}{144}+\frac{31}{144}}$

$P(E/A) = \frac{15}{46}$

Sol 11.

Given that

$f(x) = 1 + \cos^2 x + \cos^4 + \ldots\ldots\ldots\ldots$

$f(x) = \frac{1}{1-\cos^2 x} = \frac{1}{\sin^2 x}$

$f'(x) = -\frac{\cos x}{\sin^3 x} > 0 \because x \in \left(-\frac{\pi}{2}, 0\right)$

$\Rightarrow f(x)$ is increasing function

$\therefore$ Range $= \left(f\left(-\frac{\pi}{2}\right), f(0)\right)$

$\therefore \left(Lt_{x\to -\frac{\pi}{2}} f(x), Lt_{x\to 0} f(x)\right) = (1, \infty)$

Sol 12.

Let f_1 (x) is defined when – $1 \leq \frac{1-|x|}{3}$

$- 3 \leq 1 - |x| \leq 3$

$- 4 \leq 1 - |x| \leq 2$

$- 2 \leq |x| \leq 2$

$\Rightarrow - 4 \leq x \leq 4_{f2}$

$= [x]$ is defined for every real number

$D_{f2} = R$

We know $D_f = D_{f1} \cap D_{f2}$

$\therefore D_f = [- 4, 4]$

Sol 13.

Given that

$f(x) = \frac{1}{2}\left(f(x-1) + \frac{3}{f(x-)}\right); f(x) > 0 \ \forall \ x \in$

R

Let $Lt_{x\to\infty} f(x) = c$

$\therefore Lt_{x\to\infty} f(x - 1) = c$

and$Lt_{x\to\infty} f(x - 1) = c$

$\therefore c = \frac{1}{2}\left(c + \frac{3}{c}\right)$

$2c^2 = c^2 + 3$

$\Rightarrow c^2 = 3$

$\Rightarrow c = \pm \sqrt{3}$

$\therefore c = \sqrt{3}$

$\therefore Lt_{x\to\infty} f(x) = \sqrt{3}$

Sol 14.

Given that

$$f(x)\{\frac{Cos^{-1}(sinx)}{x-\frac{\pi}{2}} \; x \neq \frac{\pi}{2}$$
$$1 \quad x = \frac{\pi}{2}$$

$$Lt_{x\to\frac{\pi}{2}-}\, f\,(x) = \; Lt_{x\to\frac{\pi}{2}} \frac{\cos^{-1}(sinx)}{x-\frac{\pi}{2}}$$

Put x = $\frac{\pi}{2}$ - h

Then x $\to \frac{\pi}{2} - \; as\; h \to 0$

$$Lt_{x\to\frac{\pi}{2}} - \; f(x) = Lt_{h\to 0}\frac{\cos^{-1}\left[Sin\,\left(\frac{\pi}{2} - \; h\right)\right]}{-h}$$

$$= Lt_{h\to 0}\frac{\cos^{-1}(\cos\, h)}{-h}$$

$$= Lt_{h\to 0}\; \text{-}\frac{h}{h} = \text{- } 1$$

$$Lt_{x\to\frac{\pi}{2}} + \; f\,(x) = \; Lt_{h\to 0}\frac{\cos^{-1}\left[Sin\,\left(\frac{\pi}{2} + \; h\right)\right]}{h}$$

$$= Lt_{h\to 0}\,\frac{\cos^{-1}(\cos\, h)}{h}$$

$$= Lt_{h\to 0}\frac{h}{h} = 1$$

$$\therefore Lt_{x\to\frac{\pi}{2}-}\; f(x) \neq Lt_{x\to\frac{\pi}{2}+} f\,(x)$$

$$\therefore Lt_{x\to\frac{\pi}{2}}\, f\,(x) does\; not\; exist$$

Sol 15.

The given function is f (x) = |x - 1| + cos |x|

$$f\,(x) = \begin{cases} -\,(x-1) + \; cosx, & x < 0 \\ -\,(x-1) + \; cosx, & 0\; \le x < 1 \\ (x-1) + cosx, & 1\; < 1 \end{cases}$$

$$f(x) = \begin{cases} -\,1 - sinx, & x < 0 \\ -\,1 - Sinx, & 0\; \le x < 1 \\ 1 - Sinx, & 1 < x \end{cases}$$ It is clear that f (x) is not differentiable at x = 1

Sol 16.

Given that $e^{x+e^{x+e^{x+\,.....\infty}}}$

$\therefore$ y = e^{x+y}

$\log_e y = (x + y)$

Differentiating both sides w.r.t.x, we get

$\frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}\left(\frac{1}{y} - 1\right)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{y}{1-y}$

Sol 17.

Given that

$\sqrt{x-y} - \sqrt{x+y} = c$ Differentially both sides w.r.t.x, we get

$\frac{1}{2}(x-y)^{-\frac{1}{2}}\left(1 - \frac{dy}{dx}\right) - \frac{1}{2}(x+y)^{-\frac{1}{2}}\left(1 + \frac{dy}{dx}\right) = 0$

$\frac{1}{2\sqrt{x-y}} - \frac{1}{2\sqrt{x+y}} = \left[\frac{1}{2\sqrt{x-y}} + \frac{1}{2\sqrt{x+y}}\right]\frac{dy}{dx}$

$\frac{\sqrt{x+y} - \sqrt{x-y}}{2\sqrt{x+y}\sqrt{x-y}} = \left[\frac{\sqrt{x+y} + \sqrt{x-y}}{2\sqrt{x+y}\sqrt{x-y}}\right]\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{\sqrt{x+y} - x - y}{\sqrt{x+y} + \sqrt{x-y}} = -\frac{c}{\sqrt{x+y} + \sqrt{x-y}}\left(\frac{dy}{dx}\right)_{(a,a)} = \frac{-c}{\sqrt{2a}}$

Sol 18.

Given T = $2\pi\sqrt{\frac{l}{g}}$

Taking log on both sides

logT log2π + $\frac{1}{2}$ (log l – log g)

Taking differential on both sides

$\frac{\delta T}{T} = \frac{1}{2}\left(\frac{\delta l}{l} - \frac{\delta g}{g}\right)$

$\frac{\delta T}{T}$x 100 = $\frac{1}{2}\left(\frac{\delta l}{l}\ x\ 100 - \frac{\delta g}{g}\ x\ 100\right)$

$= \frac{1}{2}(2 - 1.5) = \frac{0.5}{2} = 0.25$

Sol 19.

f (x) = $\int$ e^{2x} (x - 4) (x - 5) dx

f' (x) = e^{2x} (x - 4) (x - 5) < 0

If (x - 4) (x - 5) < 0

If 4 < x < 5

Sol 20.

Given that f (x) = e^xCosx

f' (x) = e^xcosx – e^xsinx

f' (x) = $\sqrt{2}e^x\left[\frac{1}{\sqrt{2}}\ Cosx - \frac{1}{\sqrt{2}}\ Sinx\right]$

$= \sqrt{2}e^x\left[\sin\frac{\pi}{4}\ cosx - \cos\frac{\pi}{4}\ sinx\right]$

$= \sqrt{2}e^x \sin\left(\frac{\pi}{4} - x\right)$

f" (x) = $\sqrt{2}e^x\ Sin\ \left(\frac{\pi}{4} - x\right) - \sqrt{2}e^x\ cos\ \left(\frac{\pi}{4} - x\right)$

= 2$e^x$$\left[\frac{1}{\sqrt{2}}\ Sin\ \left(\frac{\pi}{4} - x\right) - \frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{4} - x\right)\right]$

= 2$e^x$$\left[\left(\frac{\pi}{4} - x\right)\cos\frac{\pi}{4} - \ \cos\left(\frac{\pi}{4} - x\right)\sin\frac{\pi}{4}\right]$

= 2 e^x sin $\left(\frac{\pi}{4} - x - \frac{\pi}{4}\right)$

= 2e^x sin (- x) = -2e^xSinx

For maximum slope f" (x) = 0

⇒ sinx = 0 ⇒ x = 0, π

f" (x) = 2 e^x sin $\left(\frac{\pi}{2} - x\right)$ + 2e^x Cos $\left(\frac{\pi}{2} - x\right)$

f" (x) = - 2e^xSinx – 2 e^xcosx

= - 2$\sqrt{2}e^x\left(sinx\frac{1}{\sqrt{2}} + \ cosx\frac{1}{\sqrt{2}}\right)$

= - 2 $\sqrt{2}e^x$ sin $\left(x + \frac{\pi}{4}\right)$

f" (0) = - 2 $\sqrt{2}$ sin $\frac{\pi}{4}$ = - 2 < 0 Maximum slope at x = 0

Sol 21.

Let I $= \int \frac{dx}{(1+x)^{1/2} - (1+x)^{1/3}}$

Put $1 + x = t^6 \Rightarrow dx = 6t^5 dt$

$I = \int \frac{6t^5\, dt}{(t^3 - t^2)} = 6 \int \frac{t^3}{t-1} dt$

$= 6 \int \frac{(t^3 - 1) + 1}{(t-1)} dt$

$= 6 \int \left(t^2 + \ t + 1 + \frac{1}{t-1}\right)\ dt$

$= 6 \left\{\frac{t^3}{3} + \frac{t^2}{2} + \ t + 1n\,|t - 1|\right\} + c$

$I = 2\,(1 + x\,)^{1/2} + 3\,(1 + x\,)^{1/3} + 6\,(1 + x)^{1/6}$

$+ 6\ln\left|(1 + x)^{\frac{1}{6}}\ - 1\right| + c$

Sol 22.

$I = \int_0^1 cos\left(\ 2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)\ dx$

Put x = cos θ ⇒ dx = - sin θ do

$I = \int_{\pi/2}^{0} cos\left(2\cot^{-1}\sqrt{\frac{1-\cos\,\theta}{1+\cos\,\theta}}\right)(-\sin\theta)d\theta$

$= \int_0^{\frac{\pi}{2}} \cos(2\cot^{-1}\tan\theta/2)\ \sin\theta d\theta$

$= \int_0^{\pi/2} \cos\left(2\ \left(\frac{\pi}{2} - \frac{\theta}{2}\right)\right) \sin\theta\ d\ \theta$

$= \int_0^{\pi/2}\ \cos(\pi - \theta)\ \sin\theta\ d\ \theta$

$= \int_0^{\pi/2}\ -cos\ \sin\theta\ d\ \theta$

$= \left[\frac{\cos^2\theta}{2}\right]_0^{\pi/2}$

$= \frac{1}{2}\left(\cos^2\frac{\pi}{2} - \cos^2 0\right)$

$I = \frac{1}{2}(0 - 1)\ =\ -\frac{1}{2}$

Sol 23.

Given $\frac{dy}{dx} = \frac{y\,(x - y\ln y\,)}{x\,(x\ln x\,- y)}$

$\Rightarrow x^2\ln x dy - xydy = xy\,dx - y^2\ln y\,dx$

Dividing both sides by $x^2\,y^2$, we get

$\frac{\ln x}{y^2}dy - \frac{1}{dy}\,dy = \frac{1}{xy}\,dx - \frac{\ln y}{x^2}\,dx$

$\left(\ln x\,\left(-\frac{1}{y^2}dy\right) + \frac{1}{xy}\,dx\right) + \left(\ln y\,\left(-\frac{1}{x^2}\,dx\right) + \frac{1}{xy}\,dy\right) = 0$

D $\left(\frac{\ln x}{y}\right) + \;d\;\left(\frac{\ln y}{x}\right) = 0$

Integrating both sides, we get

$\frac{\ln x}{y} + \frac{\ln y}{x}\; = c$

$\Rightarrow \frac{x\,\ln x\,+ y\,\ln y}{xy} = c$

Sol 24.

Given bisector is $x + 2y - 5 = 0$

Since the bisectors are perpendicular to each other

$\therefore$ other bisector is $2x - y = 0$

Sol 25.

Given equation of circle is

$X^2 + y^2 - 2x - 4y - 4 = 0$ having centre (1,2) and radius = 3

If $x = \lambda$ is parallel to y axis, then $\frac{2 - \lambda}{\sqrt{1}} = \pm\,3$

$\Rightarrow 2 - \lambda = \pm\,3$

$\Rightarrow \lambda = 2 \pm 3$

$\Rightarrow \lambda = 5, - 1$

Pair of lines $(x - 5)\;(x - 1) = 0$

$\Rightarrow x^2 - 4x - 5 = 0$

Sol 26.

Let P $(2t^2, 4t)$ be any point in the parasols $y^2 = 8x$

Normal at point p on $y^2 = 8x$

$y + tx = 4t + 2t^3$

It must passes through centre of circle (0, - 6) for minimum distance between the thro curves.

$-6 + 0 = 4t + 2t^3$

$t^3 + 2t + 3 = 0$

$t = -1$

∴ Point is P (2, 4)

Sol 27.

Equation of tangent of $y^2 = 4ax$ in terms of slope is $y = mx + \frac{a}{m'}$ where m is slope of tangent.

This is also tangent of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore \left(\frac{a}{m}\right)^2 = a^2 m^2 + b^2$

$\Rightarrow a^2\left(\frac{1}{m^2} - m^2\right) = b^2$

$\Rightarrow \frac{1-m^4}{m^2} = \frac{b^2}{a^2}$

$\Rightarrow \frac{(1+m^2)(1-m^2)}{m^2} = \frac{b^2}{a^2}$

$\Rightarrow \frac{1-m^2}{m^2} = \frac{b^2}{a^2\,(1+m^2)} > 0$

$\Rightarrow \frac{1-m^2}{m^2} > 0$

$\Rightarrow \frac{m^2-1}{m^2} < 0$

$\Rightarrow 0 < m^2 < 1$

$\Rightarrow m \in (-1, 0) \cup (0, 1)$

$\Rightarrow m \in (0, 1)$ for positive values of m

Sol. 28

Equation of plane through (1, 2, 3) is

a (x - 1) + b (y - 2) + c (z - 3) = 0 (i)

Which is perpendicular to

x + 2y + 4z = 1andx – 3y – 5z = 2

∴ a + 2b + 4c = 1 (ii)

A – 3b – 5c = 2 (iii)

Eliminating a, b, c from (i), (ii) and (iii) we get

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 2 & 4 \\ 1 & -3 & -5 \end{vmatrix} = 0$$

i.e. 2 (x - 1) + 9 (y - 2) – 5 (z - 3) = 0

2x + 9y – 5z – 5 = 0

Sol 29.

$$\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\sqrt{2(1-cos\theta)}}}}$$ (n number of 2's)

$$\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\sqrt{2+2\sin\frac{\theta}{2}}}}}$$ ((n -1)number of 2's)

- -

$$= \sqrt{2+2\sin\left(\frac{\theta}{2^{n-1}}\right)}$$

$$= \sqrt{2\left\{1+2\sin^2\left(\frac{\theta}{2^n}\right)-1\right\}}$$

$$= 2\sin\left(\frac{\theta}{2^n}\right)$$

Sol 30.

Given cotx + cosecx = 2 sin x

Multiply both sides by sinx $\neq$ 0, we get

cosx + 1 = 2 $\sin^2$ x

cosx + 1 = 2 (1 – $\cos^2$ x)

cosx + 1 = 2 (1 - cosx) (1 + cosx)

(1 + cosx) – 2 (1 - cosx) (1 + cosx) = 0

(1 + cosx) [1 – 2 (1 - cosx)] = 0

(1 + cosx) (- 1 + 2cosx) = 0

cosx = -1, cosx = $\frac{1}{2}$

But cosx $\neq$ - 1 as sinx $\neq$ 0

$\therefore$ cosx = $\frac{1}{2}$

$\Rightarrow$ x = $\frac{\pi}{3}, \frac{5\pi}{3}$

Number of solution in the interval [0,2π] are 2