

Modern Physics - Solutions

Sol.1. (i) $E_2 = -13.6 / 4 Z^2$, $E_3 = -13.6 / 9 Z^2$ $E_3 - E_2 = -13.6 Z^2 (1 / 9 - 1 / 4) = +13.6 x 5 / 36 Z^2$ But $E_3 - E_2 = 47.2 \text{ eV}$ (Given) \therefore 13.6 x 5 / 36 Z² = 47.2 \therefore Z = $\sqrt{47.2 \times 36}$ / 13.6 x 5 = 5 (ii) $E_4 = -13.6 / 16 Z^2$ \therefore E₄ - E₃ = -13.6 Z² [1/16 - 1/9] = -13.6 Z² [9 - 16/9 x 16] $= +13.6 \times 25 \times 7 / 9 \times 16 = 16.53 \text{ eV}$ (iii) $E_1 = -13.6 / 1 \times 25 = -340 \text{ eV}$ $\therefore E = E_{\infty} - E_1 = 340 \text{ eV} = 340 \text{ x} 1.6 \text{ x} 10^{-19} \text{ J} [E_{\infty} = 0 \text{ eV}]$ But $E = hc / \lambda$ $\therefore \lambda = hc / E = 6.6 \text{ x } 10^{-34} \text{ x } 3 \text{ x } 10^8 / 340 \text{ x } 10^{-19} \text{ x } 1.6 \text{ x } 10^{-19} \text{ m}$ (iv) Total Energy of 1^{st} orbit = - 340 eV We know that – (T.E.) = K.E. [in case of electron revolving around nucleus] And 2T.E. = P.E. \therefore K.E. = 340 eV; P.E. = - 680 eV **KEY CONCEPT :** Angular momentum in 1st orbit: According to Bohr's postulate, $mvr = nh / 2\pi$ For n = 1, mvr = h / 2π = 6.6 x 10⁻³⁴ / 2π = 1.05 x 10⁻³⁴ J-s. (v) Radius of first Bohr orbit $r_1 = 5.3 \ge 10^{-11} / Z = 5.3 \ge 10^{-11} / 5$ $= 1.06 \text{ x} 10^{-11} \text{ m}$

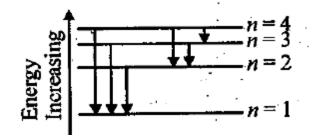


 $E = 12400 / \lambda$ (in Å) eV = 12400 / 975 = 12.75 eV

Also

$$13.6 \left[\begin{array}{c} 1 \ / \ n^2_1 - 1 \ / \ n^2_2 \right] = 12.75 \Rightarrow \left[1 \ / 1 - 1 \ / \ n^2_2 \right] = 12.75 \ / \ 13.6 \Rightarrow n_2 = 4$$

For every possible transition one downward arrow is shown therefore the possibilities are 6.



Note : For longest wavelength, the frequency should be smallest.

This corresponds to the transition from n = 4 to n = 3, the energy will be $E_4 = 13.6 / 4^2$; $E_3 = -13.6 / 3^2$

$$\therefore E_4 - E_3 = 13.6 / 4^2 - (-13.6 / 3^2) = 13.6 [1/9 - 1/16]$$

$$= 0.66 \text{ eV} = 0.66 \text{ x} 1.6 \text{ x} 10^{-19} \text{ J} = 1.056 \text{ x} 10^{-19} \text{ J}$$

Now, E = 12400 / λ (inÅ) eV $\therefore \lambda$ = 18787 Å

<u>Sol.3</u>.

(i) In a nucleus, number of electrons = 0 (:: electrons don't reside in the nucleus atom)

- (ii) number of protons = 11
- (iii) number of neutrons = 24 11 = 13

<u>Sol.4</u>

$${}^{238}_{92} \text{U} \rightarrow {}^{234}_{90} \text{X} + {}^{4}_{2} \text{He} \qquad {}^{234}_{90} \text{X} \rightarrow {}^{234}_{91} \text{Y} + {}^{0}_{-1} \text{e}$$

(i) Atomic number = 91

(ii) Mass number = 234

<u>Sol.5</u>.

 $hc / \lambda_1 - hc / \lambda_0 = K.E._1$ (i)

And hc / λ_2 - hc / λ_0 = K.E.₂(ii)

 \Rightarrow hc / λ_1 - hc / λ_2 = K.E.₁ – K.E₂

<u>Sol.6</u>.

(i) $E_n = -I.E. / n^2$ for Bohr's hydrogen atom. Here, I.E. = $4R \therefore E_n = -4R / n^2$ $\therefore E_2 - E_1 = -4R / 2^2 - (-4R / l^2) = 3 R$...(i)

 $E_2 - E_1 = hv = hc /\lambda \qquad \dots (iii)$

From (i) and (ii)

hc / $\lambda = 3R$

: $\lambda = hc / 3R = 6.6 \text{ x } 10^{-34} \text{ x } 3 \text{ x } 10^8 / 2.2 \text{ x } 10^{-18} \text{ x } 3 = 300 \text{\AA}$

(ii) The radius of the first orbit

Bohr's radius of hydrogen atom = 5×10^{-11} m (given)

$$|E_n| = + 0.22 \text{ x } 10^{-17} \text{ Z}^2 = 4 \text{ R} = 4 \text{ x } 2.2 \text{ x } 10^{-18}$$

 $\therefore Z = 2$

 \therefore r_n = r₀ / Z = 5 x 10⁻¹¹ / Z = 5 x 10⁻¹¹ / 2 = 2.5 x 10⁻¹¹ m

<u>Sol.7</u>.

(i) $E_n = -13.6 / n^2 Z^2 eV / atom$

For Li^{2+} , $Z=3 \div E_n=$ - 13.6 x 9 / n^2 eV / atom

 \therefore E₁ = -13.6 x 9 / 1 and E₃ = -13.6 x 9 / 9 = -13.6

 $\Delta E = E_3 - E_1 = -13.6 - (-13.6 \times 9)$

13.6 x 8 = 108.8 eV /atom

 $\Lambda = 12400 / E (in eV) Å = 12400 / 108.8 = 114 Å$

(ii) The spectral line observed will be three namely 3 \rightarrow 1,

 $3 \rightarrow 2$, $\rightarrow 1$.



K.E. = $0.0327 \text{ eV} = 0.327 \text{ x} 1.6 \text{ x} 10^{-19} \text{ J}$

 $\frac{1}{2} m_n v_n^2 = 0.0327 \text{ x } 1.6 \text{ x } 10^{-19}$

 \Rightarrow v_n [2 x 0.0327 x 1.6 x 10⁻¹⁹/ 1.675 x 10⁻²⁷]^{1/2}

 \Rightarrow v_n = 0.25 x 10⁴ m/s

Time taken by the neutron to travel 10 m will be

 $t = d / v_n = 10 / 0.25 \times 10^4 = 4 \times 10^{-3} s$

Let the number of neutron initially be a.

$$\lambda = 0.693 / t_1 / 2 = 0.693 / 700 \text{ s}^{-1}$$

We know that

 $t = 2.303 \ / \ \lambda \log a \ / \ a - x$

 \Rightarrow 4 x 10⁻³ / 2.303 x 0.693 / 700 = 1=log₁₀ a / a - x

 $\Rightarrow \log_{10} a / a - x = 1.72 x 10^{-6} \Rightarrow a / a - x = 1.000004$

 \Rightarrow x / a = 3.96 x 10⁻⁶

<u>Sol.9</u>.

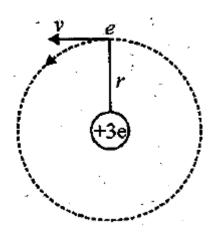
I = 0.125 V − 7.5 ⇒ dI = 0.125 dV or dV / dI = 1 / 0.125 = 8 We know that plate resistance, $r_p = dV / dI = 8mΩ$ The trans conductance , $g_m = [dI / dV_g] v = constt$ At $V_g = -1$ volt , V = 300 volt, the plate current I = [0.125 x 300 − 7.5] mA = 30 mA Also it is given that $V_g = -3V$, V = 300 V and I = 5mA $\therefore g_m = [30 - 5 / -1 - (-3)] = 25 / 2 x 10^{-3} = 12.5 x 10^{-3} s$ The characteristics are given in the form of parallel lines. Amplification factor

 $= r_p g_m = 8 x 10^3 x 12.5 x 10^{-3} = 100$

<u>Sol.10</u>.

(i) Let m be the mass of electron. Then the mass of meson is 208 m. According to Bohr's postulate, the angular momentum of mu-meson should be an integral multiple of $h/2\pi$.





 \therefore (208M) vr = nh/ 2 π

 \therefore v = nh / 2 π x 208 mr = nh / 416 π mr ...(i)

Note : Since mu-meson is moving in a circular path, therefore, it needs centripetal force which is provided by the electrostatic force between the nucleus and mumesion.

 \therefore (208m)v² / r = 1 / 4 $\pi\epsilon_0$ x 3e x e / r²

 \therefore r = 3e² / 4 π ϵ_0 x 208 mv²

Substituting the value of v from (1), we get

 $r = 3e^2 x 416 \pi mr x 416 \pi mr / 4 \pi \epsilon_0 x 208 mn^2 h^2$

$$\Rightarrow$$
 r = n² h² ϵ_0 / 624 π me²

(ii) The radius of the first orbit of the hydrogen atom

 $= \varepsilon_0 h^2 / \pi m e^2 \qquad \dots (ii)$

To find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for hydrogen atom, we equate eq. (i) and (ii)

...(i)

 $N^2 h^2 / 624 \pi me^2 = \epsilon_0 h^2 / \pi me^2 \Rightarrow n = \sqrt{624} \approx 25$

(iii) $1 / \lambda = 208 \text{ R x } \text{Z}^2 [1 / n^2_1 - 1 / n^2_2]$

 $\Rightarrow 1 / \lambda = 208 \text{ x} 1.097 \text{ x} 10^7 \text{ x} 3^2 [1/1^2 - 1/3^2]$

 $\Rightarrow \lambda = 5.478 \text{ x } 10^{-11} \text{ m}$

Sol.11.

 $E_1 = 12400 / 4144 = 2.99 \text{ eV}, E_2 = 12400 / 4972 = 2.49 \text{ eV},$

 $E_3 = 12400 / 6216 = 1.99 \text{ eV}.$

 \Rightarrow Only first two wavelengths are capable of ejecting photoelectrons.

Energy incident per second

 $= 3.6 / 3 \times 10^{-3} \times 10^{-4} = 1.2 \times 10^{-7} \text{ J/s}$



 $n^2 = 1.2 \ge 10^{-7} / 2.99 \ge 1.6 \ge 10^{-19} = 3 \ge 10^{11}$

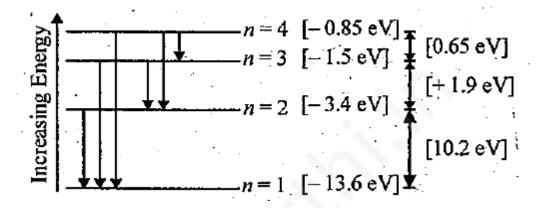
Total number of photons = $2(n_1 + n_2)$

 $= 3.01 \ge 10^{11} + 2.51 \ge 10^{11} = 5.52 \ge 10^{11}$

 \therefore Total number of photoelectrons ejected in two seconds = 11 x 10¹¹

Sol.12

(i) The transition of six different photon energies are shown.



Since after absorbing monochromatic light, some of the emitted photons have energy more and some have less than 2.7 eV, this indicates that the excited level B is n = 2. (This is because if n = 3) is the excited level then energy less than 2.7 eV is not possible)

(ii) For hydrogen like atoms we have

 $E_n = -13.6 / n^2 Z^2 eV / atom$

 $E^4 - E^2 = -13.6 / 16 Z^2 - (-13.6 4) Z^2 = 2.7$

 \Rightarrow Z² x 13.6 [1/4 - 1/16] = 2.7

 \Rightarrow Z² = 2.7 13.6 x 4 x 16 / 12 \Rightarrow I.E. = 13. 6 Z² (1/1² - 1/∞²)

= 13.6 x 2.7 / 13.6 x 4 x 16 / 12 14.46 eV

(iii) Max. Energy

 $E_4 - E_3 13.6 Z^2 (1 / 4^2 - 1 / 1^2)$

= 13.6 x 2.7 / 13.6 x 4 x 16 / 12 x 15 / 16 = 13.5 eV

Min. Energy

 $E_4 - E_3 = -13.6Z^2 (1/4^2 - 1/3^2)$

= 13.6 x 2.7 / 13.6 x 4 x 16 / 12 x 7 / 9 x 16 = 0.7 eV



For hydrogen like atom energy of the nth orbit is

 $E_n = 13. 6 n^2 Z^2 eV / atom$ For transition from n = 5 to n = 4, Hv = 13.6 x 9 [1 / 16 - 1 / 25] = 13.6 x 9 x 9 / 16 x 25 = 2.754 eVFor transition from n = 4 to n = 3, hv' = 13.6 x 9 [1 / 9 - 1 / 16] = 13.6 x 9 x 7 / 9 x 16 = 5.95 eVFor transition n = 4 to n = 3, the frequency is high and hence wavelength is short. For photoelectric effect, $hv' - W = eV_0$, where W = work function $5.95 x 1.6 x 10^{-19} - W = 1.6 x 10^{-19} x 3.95$ $\Rightarrow W = 2 x 1.6 x 10^{-19} = 2eV$ Again applying hv - W = eV'We get, $2.754 x 1.6 x 10^{-19} - 2 x 1.6 x 10^{-19} = 1.6 x 10^{-19} V'_0$

 \Rightarrow V₀' = 0.754 V

<u>Sol.14</u>.

Energy required per day

 $E = P x t = 200 x 10^{6} x 24 x 60 x 60$

 $= 1.728 \text{ x} 10^{13} \text{ J}$

Energy released per fusion reaction

= [2 (2.0141) - 4.0026] x 931.5 MeV

= 23.85 MeV = 23.85 x 106 x 1.6 x 10⁻¹⁹

 $= 38.15 \text{ x} 10^{-13} \text{ J}$

 \therefore No. of fusion reactions required

 $= 1.728 \times 10^{13} / 38.15 \times 10^{-13} = 0.045 \times 10^{26}$

 \therefore No. of deuterium atoms required

 $= 2 \ge 0.045 \ge 10^{26} = 0.09 \ge 10^{26}$

Number of moles of deuterium atoms

 $= 0.09 \text{ x } 10^{26} / 6.02 \text{ x } 10^{23} = 14.95$

 \therefore Mass in gram of deuterium atoms

= 14.95 x 2 = 29.9 g

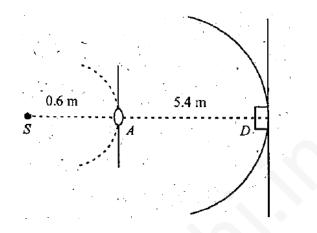


Therefore, the actual mass required = 119.6 g

<u>Sol.15</u>.

Energy of one photon, $E = hc / \lambda = (6.6 \times 10^{-34}) (3.0 \times 10^{8}) / 6000 \times 10^{-10}$

 $= 3.3 \times 10^{-19} \text{ J}$



Power of the source is 2 W or 2 J/s. Therefore, number of photons emitting per second,

$$N_1 = 2 / 3.3 \times 10^{-19} = 6.06 \times 10^{18} / s$$

At distance 0.6 m, number of photons incident unit area per unit time:

$$n_2 = n_1 / 4\pi (0.6)^2 = 1.34 \times 10^{18} / m^2 / s$$

Area of aperture is,

 $S_1 = \pi / 4 d^2 (0.1)^2 = 7.85 \times 10^{-3} m^2$

.. Total number of photons incident per unit time on the aperture,

$$N_3 = n_2 s_1 = (1.34 \text{ xc } 10^{18}) (7.85 \text{ x } 10 + -3) / \text{s}$$

$$= 1.052 \times 10^{16} / s$$

The aperture will become new source of light.

Now these photons are further distributed in all directions. Hence, at the location of detector, photons incident per unit area unit time :

$$N_4 = n_3 / 4 \pi (6 - 0.6)^2 = 1.052 \times 10^{16} / 4 \pi (5.4)^2$$

$$= 2.87 \text{ x} 10^{13} \text{ s}^{-1} \text{ m}^{-2}$$

This is the photon flux at the centre of the screen. Area of detector is 0.5 cm^2 or $0.5 \times 10^{-4} \text{ m}^2$. Therefore, total number of photons incident on the detector per unit time:

 $n_5 = (0.5 \text{ x } 10^{-4}) (2.87 \text{ x } 10^{13} \text{ d}) = 1.435 \text{ x } 10^9 \text{ s}^{-1}$

The efficiency of photoelectron generation is 0.9. Hence, total photoelectrons generated per unit time :

 $n_6 = 0.9 n_5 = 1.2915 \ge 10^9 s^{-1}$ Educational Material Downloaded from http://www.evidyarthi.in/ Get CBSE Notes, Video Tutorials, Test Papers & Sample Papers

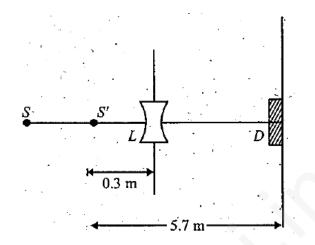


 $i = (e)n_6 = (1.6 \times 10^{-19})(1.2915 \times 10^9) = 2.07 \times 10^{-10} A$

(b) Using the lens formula :

1/v - 1/-0.6 = 1/-0.6 or v = -0.3 m

i.e, image of source (say S', is formed at 0.3 m from the lens,)



Total number of photons incident per unit on the lens are still n_3 or 1.052×10^{16} s. 80% of it transmits to seconds medium Therefore, at a distance of 5.7 m from S' number of photons incident per unit are per unit time will be :

 $N_1 = (80 / 100) (1.05 \times 10^{16}) / (4 \pi) (5.7)^2$

This is the photon flux at the detector.

New value of photocurrent is :

 $i = (2.06 \times 10^{13}) (0.5 \times 10^{-4}) (0.9) (1.6 \times 10^{-19})$

= 1.483 x 10⁻¹⁰ A

(c) For stopping potential

 $hc / \lambda = (E_K)_{max} + W = eV_0 + W$

:. $eV_0 = hc / \lambda - W = 3.315 \text{ x } 10^{-19} / 1.6 \text{ x } 10^{-19} - 1 = 1.07 eV$

$$\therefore V_0 = 1.07 \text{ Volt}$$

Note : The value of stopping potential is not affected by the presence of concave lens as it changes the intensity and not the frequency of photons. The stopping potential depends on the frequency of photons.

Sol.16.

(a)
$${}^{A}_{92}X \rightarrow {}^{228}_{x}Y + {}^{4}_{2}He$$

A = 228 + 4 = 232; 92 = Z + 2 \Rightarrow Z = 90

(b) Let v be the velocity with which α – particle is emitted.

Then

Prese Evaluation FREE Evaluation FREE Evaluation $m = 2 \times 1.6 \times 10^{-19} \times 0.11 \times 3/4.003 \times 10^{-27}$

 \Rightarrow v = 1.59 x 10⁷ ms⁻¹

Applying law of conservation of linear momentum during α – decay we get

$$M_y v_y = m_\alpha v_\alpha \qquad \dots (1)$$

The total kinetic energy of α – particle and Y is

$$E = K.E._{\alpha} + K.E._{y} = \frac{1}{2} m_{\alpha} v_{\alpha}^{2} + \frac{1}{2} m_{y} v_{y}^{2}$$

 $= \frac{1}{2} m_{\alpha} v_{\alpha}^{2} + \frac{1}{2} m_{y} [m_{\alpha} v_{\alpha} / m_{y}] = \frac{1}{2} m_{\alpha} v_{\alpha}^{2} + m_{\alpha} v_{\alpha}^{2} + m_{\alpha}^{2} v_{\alpha}^{2} / 2m_{y}$

 $= \frac{1}{2} \times 4.033 \times 1.6 \times 10^{-27} \times (1.59 \times 10^{7})^{2} [1 + 4.003 / 228.03]$ J

- = 8.55 x 10⁻¹³ J
- = 5.34 MeV

: Mass equivalent of this energy

= 5.34 / 931.5 = 0.0051 a.m.u.

Also, $m_x + m_\alpha + mass$ equivalent of energy (E)

= 228.03 + 4.003 + 0.0057 = 232.03874 u.

The number of nucleus = 92 protons + 140 neutron.

- ∴ Binding energy of nucleus X
- = [92 x 1.008 + 140 x 1.009] 232 . 0387 = 1.9571 u
- = 1.9571 x 931.5 = 1823 MeV.

<u>Sol.17</u>.

(a) The energy of photon causing photoelectric emission

= work function of sodium metal + KE of the fastest photoelectron

= 1.82 + 0.73 = 2.55 eV

(b) We know that $E_n = -13.6 / n^2 eV / atom for hydrogen atom.$

Let electron jump from n_2 to n_1 then

$$E_{n_2} - E_{n_1} = -13.6 / n_2^2 - (13.6 / n_1^2)$$

 $\Rightarrow 2.55 = 13.6 (1/n^2 - 1/n^2)$

By hit and trial we get $n_2 = 4$ and $n_1 = 2$

[angular momentum mvr = nh/ 2π]

(c) Change in angular momentum

 $= n_1 h / 2 \pi - n_2 h / 2 \pi = h / 2 \pi (2 - 4) = h / 2 \pi x (-2) = - h / x$ Educational Material Downloaded from http://www.evidyarthi.in/

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EVidyarthi FREE (d) The momentum of emitted photon can be found by de Broglie relationship

 $\lambda = h / p \Rightarrow p = h / \lambda = hc / c = E / c$ $\therefore p = 2.55 \times 1.6 \times 10^{-19} / 3 \times 10^{8}$

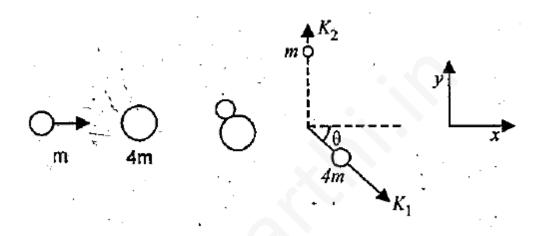
Note : The atom was initially at rest the recoil momentum of the atom will be same as emitted photon (according to the conservation of angular momentum).

Let m be the mass and v be the recoil velocity of hydrogen atom then

 $m \ge v = 2.55 \ge 1.6 \ge 10^{-19} / 3 \ge 10^8$

 \Rightarrow v = 2.55 x 1.6 x 10⁻¹⁹ / 3 x 10⁸ x 1.67 x 10⁻²⁷ = 8.14 m/s

<u>Sol.18</u>.



Applying conservation of linear momentum in horizontal direction

(Initial Momentum)_x = (Final Momentum)_x

$$(\mathbf{P}_1)_{\mathbf{x}} = (\mathbf{P}_f)_{\mathbf{x}}$$

 $\Rightarrow \sqrt{2}Km = \sqrt{2}(4m)K_1 \cos \theta \qquad \dots (i)$

Now applying conservation of linear momentum in Y - direction

 $(P_i)_y = (P_f)_y$ $0 = \sqrt{2K_2 m} - \sqrt{2} (4m) K_1 \sin \theta$ 1

$$\Rightarrow \sqrt{2} \text{ K}_2 \text{ m} = \sqrt{2}(4\text{m})\text{K}_1 \sin \theta \dots \text{ (ii)}$$

Squaring and adding (i) and (ii)

 $2Km + 2Km_2 m = 2 (4m) K_1 + 2 (4m)K_1$

 $K_1 + K_2 = 4K_1 \Rightarrow K = 4 K_1 - K_2 \Rightarrow 4K_1 - K_2 = 65 \dots (iii)$

When collision takes place, the electron gains energy and jumps to higher orbit.

Applying energy conservation

 $K = K_1 + K_2 + \Delta E$



....(iv)

Possible value of ΔE for He^+

Case (1)

 $\Delta E_1 = -13.6 - (-54.4) = 40.8 \text{ eV}$

 $\Rightarrow K_1 + K_2 = 24.2 \text{ eV from (4)}$

Solving with (3), we get

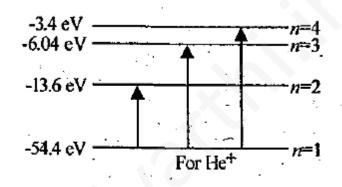
 $K_2=6.36\;\text{eV}$; $K_1=17\;.84\;\text{eV}$

Case (2)

 $\Delta E_2 = -6.04 - (-54.4) = 48.36 \text{ eV}$

 \Rightarrow K₁ + K₂ = 16.64 eV from (4)

Solving with (3), we get $K_2 = 0.312 \text{ eV}$; $K_1 = 16.328 \text{ eV}$



Case (3)

 $\Delta E_3 = -3.4 - (-54.4) = 51.1 \text{ eV}$

 \Rightarrow K₁ + K₂ = 14 eV

Solving with (3), we get

 $K_2=15.8\;eV$; $K_1=\,$ - $1.8\;eV$

But K.E. can never be negative therefore case (3) is not possible.

Therefore, the allowed values of kinetic energies are only that of case (1) and case (2) and electron can jump upto n = 3 only.

(ii) Thus when electron jumps back there are three possibilities

 $n_3 \longrightarrow n_1 \text{ or } n_3 \longrightarrow n_2 \text{ and } n_2 \longrightarrow n_1$

The frequencies will be

 $y_1 = E_3 - E_2 / h$; $v_2 = E_3 - E_1 / h$; $v_3 = E_2 - E_1 / h$

i.e., 1.82 x 10^{15} Hz; 11.67 x 10^{15} Hz; 9.84 x 10^{15} Hz



 $t_{1/2} = 15$ hours

Activity initially $A_0 = 10^{-6}$ Curie (in small quantity of solution of ${}^{24}Na$) = 3.7 x 10⁴ dps

Observation of blood of volume 1 cm³

After 5 hours, A = 296 dpm

The initial activity can be found by the formula

 $t = 2.303 / \lambda \log_{10} A_0 / A \Rightarrow 5 = 2.303 / 0.693 / 15 x \log_{10} A_0 / 296$

 $\Rightarrow \log_{10} A_0 / 296 = 5 \times 0.693 / 2.303 \times 15 = 0.3010 / 3 = 0.10033$

 \Rightarrow A₀ / 296 = 1.26 \Rightarrow A₀ = 373 dpm = 373 / 60 dps

This is the activity level in 1 cm³. Comparing it with the initial activity level of 3.7×10^4 dps we find the volume of blood.

 $V = 3.7 \times 10^4 / 373 / 60 = 5951.7 \text{ cm}^3 = 5.951 \text{ litre}$

Sol.20.

For hydrogen like atoms

 E_n - 13.6 / $n^2 Z^2 eV$ / atom Given $E_n - E_2 = 10.2 + 17 = 27.2 \text{ eV}$..(i) $E_n - E_3 = 4.24 + 5.95 = 10.2 \text{ eV}$ $\therefore E_3 - E_2 = 17$ But $E_3 - E_2 = -13.6 / 9 Z^2 - (-13.6 / 4 Z^2)$ $= -13.6 Z^{2} [1/9 - 1/4]$ $= -13.6 Z^{2} [4 - 9 / 36] = 13.6 x 5 / 36 Z^{2}$ $\therefore 13.6 \ge 5 / 36 \mathbb{Z}^2 = 17 \Rightarrow \mathbb{Z} = 3$ $E_n - E_2 = 13.6 n_2 \times 3^2 - [-13.6 / 2^2 \times 3^2]$ $= -13.6 [9/n^2 - 9/4] = -13.6 \times 9 [4 - n^2/4n^2]$...(ii) From eq. (i) and (ii), - 13.6 x 9 $[4 - n^2 / 4n^2] = 27.2$ \Rightarrow - 122.4 (4 - n²) = 108.8 n² \Rightarrow n² = 489.6 / 13.6 = 36 \Rightarrow n = 6



(i) $E_n = -3.4 \text{ eV}$

The kinetic energy is equal to the magnitude of total energy in this case.

 \therefore K.E. = + 3.4 eV

(ii) The de Broglie wavelength of electron

 $\Lambda = h / \sqrt{2mK} = 6.64 \times 10^{-34} / \sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19} \text{ eV}}$

 $= 0.66 \text{ x } 10^{-9} \text{ m}$

Sol.22.

(i) From the given information, it is clear that half life of the radioactive nuclei is 10 sec (since half the amount is consumed in 10 second 12.5% i half of 25% pls. note). Mean life $\tau = 1/\lambda = 1/0.693 / t_{1/2} = t_{1/2} / 0.693 = 10 / 0.693 = 14.43$ sec

(ii) $N = N_0 e^{\lambda t}$

 $N/N_0 = 6.25 / 100$

 $\Lambda = 0.0693 \text{ s}^{-1}$

 $6.25 / 100 = e^{-0.0693t}$

 $e^{+0.693t} = 100 / 6.25 = 16$

 $0.0693t = In \ 16 = 2.773$

Or t = 2.733 / 0.0693 = 40 sec.

<u>Sol.23</u>.

Number of atoms of 238 U initially / Numbarof atoms of 238 U finally = 4 / 3 = a / (a - x)

[: Initially one part lead is present with three parts Uranium]

 \therefore t = 2.303 / $\lambda \log \alpha$ / (α - x) = 2.303 x 4.5 x 10⁹ / 0.693 log 4/3

 $= 1.868 \times 10^9$ years.

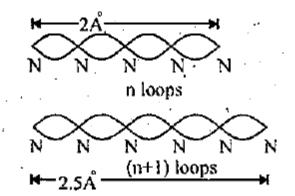
Sol.24.

As nodes are formed at each of the atomic sites, hence

 $2\text{\AA} = n (\lambda / 2)$...(1)

[: Distance between successive nodes = $\lambda / 2$]





and 2.5 Å = $(n + 1) \lambda / 2$

 \div 2.5 /2 = n + 1 / n , 5/ 4 = n + 1 / n or n = 4

Hence, from equation (1),

 $2\text{\AA} = 4 \lambda / 2$ i.e., $\lambda = 1 \text{\AA}$

Now, de Broglie wavelength is given by

 $\lambda = h / \sqrt{2mK}$ or $K = h^2 / \lambda^2 2m$

 $\therefore K = (6.63 \times 10^{-34})^2 / (1 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \text{ eV}$

$$= (6.63)^2 / 8 \times 9.1 \times 1.6 \times 10^2 \text{ eV} = 151 \text{ eV}$$

d will be minimum, when

$$n = 1$$
, $d_{min} = \lambda / 2 = 1$ Å / 2 = 0.5Å

Sol 25.

The reaction involved in α – decay is

$$^{248}_{96}$$
 Cm $\rightarrow ^{244}_{94}$ Pu + 4_2 He

Mass defect

$$\Delta m = Mass of \frac{248}{96} Cm - Mass of \frac{244}{94} Pu - Mass of \frac{4}{2} He$$

= (248.072220 - 244.064100 - 4.002603)u

= 0.005517 u

Therefore, energy released in α – decay will be

$$E_{\alpha} = (0.005517 \text{ x } 931) \text{ MeV} = 5.136 \text{ MeV}$$

Similarly, $E_{fission} = 200 \text{ MeV}$ (given)

Mean life is given ass $t_{mean}=10^{13}\,s=~1/\,\lambda$

: Disintegration constant $\lambda = 10^{-13} \text{ s}^{-1}$

Rate of decay at the moment when number of nuclei are 10^{20} is

 $dN / dt = \lambda N = (10^{-13}) (10^{20}) = 10^7 dps$

EVidyarthi FREE Othese distintegrations, 8% are in fission and 92% are in α – decay.

Therefore, energy released per second

$$= (0.08 \times 10^7 \times 200 + 0.92 \times 10^7) \times 5.136)$$
 MeV

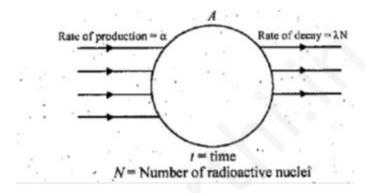
 $= 2.074 \text{ x} 10^8 \text{ MeV}$

 \therefore Power output (in watt) = Energy released per second (J/s)

 $= (2.074 \times 10^8) (1.6 \times 10^{-13})$

 \therefore Power output = 3.32 x 10⁻⁵ watt.

<u>Sol 26</u>.



(a) Let at time 't' number of radioactive are N.

Net rate of formation of nuclei of A.

 $dN / dt = \alpha - \lambda N \text{ or } dN / \alpha - \lambda N = dt$

or
$$\int_{N_0}^N \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

Solving this equation, we get

$$N = 1 / \lambda \left[\alpha - (\alpha - \lambda N_0) e^{-\lambda t} \right] \qquad \dots \dots (1)$$

(b) Substituting $\alpha = 2\lambda N_0$ and

 $t = t_{1/2} = 1n(2) / \lambda$ in equation (1),

we get, $N = 3/2 N_0$

(ii) Substituting $\alpha=2\lambda$ N_0 and $t\to\infty$ in equation (1), we get

 $N=\alpha \ / \ \lambda=2 \ N_0$

<u>Sol 27</u>.

The energy of the incident photon is

 $E_1 = hc / \lambda = (4.14 \text{ x } 10^{-15} \text{ eVs}) (3 \text{ x } 10^8 \text{ m/s}) / (400 \text{ x } 10^{-9} \text{ m}) = 3.1 \text{ eV}$

EVIDYARTHI FREE The maximum kinetic energy of the electrons is $E_{max} = E_1 - W = 3.1 \text{ eV} - 1.9 \text{ eV} = 1.2 \text{ eV}$

It is given that,

$$\begin{pmatrix} Emitted \ electrons \\ of \ maximum \ energy \end{pmatrix} + {}_{2}He^{2+} \longrightarrow in4 \ th \ excited \ state \\ + \ photon \end{pmatrix}$$

The fourth excited state implies that the electron enter I the n = 5 state.

In this state its energy is

 $E_5 = -(13.6 \text{ eV})Z^2 / n^2 = -(13.6 \text{eV})(2)^2 / 5^2$

= - 2.18 eV

This energy of the emitted photon in the above combination reaction is

 $E = E_{max} + (-E_5) = 1.2 \text{ eV} + 2.18 \text{ eV} = 2.4 \text{ eV}$

Note : After the recombination reaction, the electron may undergo transition from a higher level to a lower level thereby emitting photons.

The energies in the electronic levels of He⁺ are

$$E_4 = (-13.6 \text{ eV})(2^2) / 4^2 = -3.4 \text{ eV}$$

 $E_3 = (-13.6 \text{eV})(2^2) / 3^2 = -6.04 \text{ eV}$

$$E_2 = (-13.6 \text{eV}) (2^2) / 2^2 = -13.6 \text{eV}$$

The possible transitions are

 $n = 5 \longrightarrow n = 4$

 $\Delta E = E_5 - E_4 = [-2.18 - (-3.4)] eV = 1.28 eV$

$$n = 5 \rightarrow n = 3$$

 $\Delta E = E_5 - E_3 = [-2.18 - (-6.04)] eV = 3.84 eV$

 $n = 5 \rightarrow n = 2$

 $\Delta E = E_5 - E_2 = [-2.18 - (-13.6)] eV = 11.4 eV$

$$n = 4 \rightarrow n = 3$$

 $\Delta E = E_4 - E_3 = [-3.4 - (-6.04)] eV = 2.64 eV$

<u>Sol 28</u>.

Energy for an orbit of hydrogen like atoms is

 $E_n = -13.6 Z^2 / n^2$

For transition from 2n orbit to 1 orbit

Maximum energy = $13.6 Z^2 (1/1 - 1/(2n)^2)$



Also for transition $2n \rightarrow n$. $40.8 = 13.6 Z^2 (1/n^2 - 1/4n^2) \Rightarrow 40.8 = 13.6 Z^2 (3/4n^2)$ $\Rightarrow 40.8 = 40.8 Z^2 / 4n^2 = Z^2 \text{ or } 2n = Z \dots$ (ii) From (i) and (ii) $204 = 13.6 Z^2 (1 - 1/Z^2) = 13.6 Z^2 - 13.6$ $13.6 Z^2 = 204 + 13.6 = 217.6$ $Z^2 = 217.6 / 13.6 = 16, Z = 4, n = Z / 2 = 4 / 2 = 2$ orbit no. = 2n = 4For minimum energy = Transition from 4 to 3. $E = 13.6 x 4^2 (1/3^2 - 1/4^2) = 13.6 x 4^2 (7/9 x 16)$ = 10.5 eV.Hence n = 2, Z = 4, E_{min} = 10.5 eV

<u>Sol 29</u>.

No. of photons /sec

= Energy incident on platinum surface per second / Energy of on photon

No. of photon incident per second

 $= 2 \times 10 \times 10^{-4} / 10.6 \times 1.6 \times 10^{-19} = 1.18 \times 10^{14}$

As 0.53% of incident photon can eject photoelectrons

: No. of photoelectrons ejected per second

 $= 1.18 \times 10^{14} \times 0.53 / 100 = 6.25 \times 10^{11}$

Minimum energy = 0 eV,

Maximum energy = (10.6 - 5.6) eV = 5eV

<u>Sol 30</u>.

The formula for η of power will be

$$\eta = P_{out} / P_{in}$$

 \therefore P_{in} = P_{out} / η = 1000 x 10^{6} / 0.1 = 10^{10} W

Energy required for this power is given by

E = p x t



 $= 3.1536 \times 10^{18} \text{ J}$

 $200\ x\ 1.6\ x\ 10^{\text{-}13}\ J$ of energy is released by 1 fission

 \div 3.1536 x 1018 J of energy is released by

 $3.1536 \: x \: 10^{18} \: / \: 200 \: x \: 1.6 \: x \: 10^{\text{--}13} \: \text{fission}$

 $= 0.9855 \text{ x } 10^{29} \text{ fission}$

 $= 0.985 \text{ x } 10^{29} \text{ of } U^{235} \text{ atoms.}$

 6.023×10^{23} atoms of Uranium has

 $235 \ge 0.9855 \ge 10^{29} / 6.023 \ge 10^{23} = 38451 \ge 0.9855 \ge 0.98555 \ge 0.98555$

<u>Sol 31</u>.

Let the reaction be

$$\stackrel{A}{Z} X \longrightarrow \stackrel{A}{Z} \stackrel{-4}{-2} Y + \stackrel{4}{2} He$$

Here, $m_v = 223.61$ amu and $m_\alpha = 4.002$ amu

We know that

 $\lambda = h / mv \Rightarrow m^2 v^2 = h^2 / \lambda^2 = p^2$

 \Rightarrow But E.K. = p² / 2m . Therefore K.E. = h² / 2m λ^2 ...(i)

Applying eq. (i) for Y and α , we get

K.E._{α} = $(6.6 \times 10^{-34})^2 / 2 \times 4.002 \times 1.67 \times 10^{-27} \times 5.76 \times 10^{-15} \times 5.76 \times 10^{-15}$

 $= 0.0982243 \text{ x } 10^{-11} = 0.982 \text{ x } 10^{12} \text{ J}$

Similarly (E.K.)_y = 0.0178×10^{-12} J

Total energy = 10^{-12}

We know that $E = \Delta mc^2$

 $\therefore \Delta m = E / c^2 = 10^{-12} / (3 \times 10^8)^2 \text{ kg}$

 $1.65 \text{ x } 10^{-27} \text{ kg} = 1 \text{ amu}$

 $:: 10^{-12} / (3 \times 10^8)^2 \text{ kg} = 10^{-12} \text{ amu} / 1.67 \times 10^{-27} \text{ x} (3 \times 10^8)^2$

 $= 10^{-12}$ amu / 1. 67 x 9 x 10^{-27} x $10^{16} = 0.00665$ amu

The mass of the parent nucleus X will be

 $M_x = m_y + m_\alpha + \Delta m$

= 223. 61 + 4.002 + 0.00665 = 227.62 amu



<u>Q 32</u>.

$$\frac{T_{1/2} = 10 \sec}{\lambda_x = 0.1 \mathrm{s}^{-1}} Y \xrightarrow{T_{1/2} = 30 \sec}{\lambda_y = \frac{1}{30} \mathrm{s}^{-1}} Z$$

The rate of equation for the population of X, Y and Z will be

$$dN_x / dt = -\lambda_x N_x \qquad \dots (i)$$

$$dN_y / dt = -\lambda_y N_y + \lambda_x N_x \qquad \dots (ii)$$

$$dN / dt = -\lambda_y N_y \qquad ...(iii)$$

 \Rightarrow On integration, we get

$$N_x = N_0 / \lambda_x - \lambda_y [e^{-\lambda yt} - e^{\lambda xt}]$$

To determine the maximum $N_{\mbox{\tiny y}},$ we find

$$dN_y / dt = 0$$

From (ii)

$$-\,\lambda_y N_y + \lambda_x\,N_x = 0$$

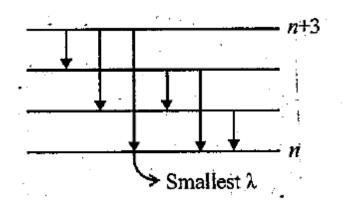
$$\Rightarrow \lambda_x N_x = \lambda_y N_y \qquad \dots \dots (v)$$

$$\Rightarrow \lambda_x (N_0 e^{-\lambda xy}) = \lambda_y [\lambda_x N_0 / \lambda_x - \lambda_y (e^{-\lambda yt} - e^{\lambda xt})]
\Rightarrow \lambda_x - \lambda_y / \lambda_y = e^{-\lambda yt} - e^{-\lambda xt} / e^{-\lambda xt} \Rightarrow \lambda_x / \lambda_y = e^{(\lambda x / \lambda y)t}
\Rightarrow \log_e \lambda_x / \lambda_y = (\lambda_x - \lambda_y)t
\Rightarrow t = \log_e (\lambda_x / \lambda_y) / \lambda_x - \lambda_y = \log_e [0.1 / (1/30)] / 0.1 - 1/30 = 15 \log_e 3
\therefore N_x = N_0 e^{-0.1(15 \log_e 3)} = N_0 e^{\log_e (3-1.5)}
\Rightarrow N_x = N_0 3^{-15} = 10^{20} / 3 \sqrt{3}
Since, dN_y / dt = 0 at t = 15 \log_e 3, \qquad \therefore N_y = \lambda_x N_x / \lambda_y = 10^{20} / \sqrt{3}
And N_2 = N_0 - N_x \cdot N_y
= 10^{20} - (10^{20} / 3\sqrt{3}) - 10^{20} / \sqrt{3} = 10^{20} (3\sqrt{3} - 4/3\sqrt{3})$$

<u>Sol 33</u>.

(a) If x is the difference in quantum number of the states than ${}^{x+1}C_2 = 6 \Rightarrow x = 3$





...(ii)

Now, we have $-z^2 (13.6 \text{eV}) / n^2 = -0.85 \text{eV}$

...(i)

And $-z^2$ (13.6 eV) / (n + 3)² = - 0.544 eV

Solving (i) and (ii) we get n = 12 and z = 3

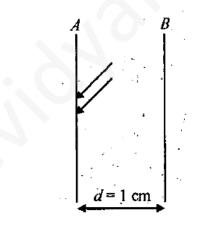
(b) Smallest wavelength λ is given by

hc / $\lambda = (0.85 - 0.544) \text{ eV}$

Solving, we get $\lambda = 4052$ nm.

<u>Sol 34</u>.

(a) Number of electron falling on the metal plate $A = 10^{16} \text{ x} (5 \text{ x} 10^{-4})$



 \therefore Number of photoelectrons emitted from metal plate A upto 10 seconds is

 $N_e = (5 \times 10^4) \times 10^{16} / 10^6 \times 10 = 5 \times 10^7$

(b) Charge on plate B at t = 10 sec

 $Q_b = 33.7 \text{ x } 10^{-12} - 5 \text{ x } 10^7 \text{ x } 1.6 \text{ x } 10^{-19} = 25.7 \text{ x } 10^{-12} \text{ C}$

Also $Q_a = 8 \ge 10^{-12} \text{ C}$

 $E = \sigma_B / 2\epsilon_0 - \sigma A / 2\epsilon_0 = 1 / 2A \epsilon_0 (Q_B - Q_A)$

= 17.7 x 10^{-12} / 5 x 10^{-4} x 8.85 x 10^{-12} = 2000 N/C

(c) K.E. of most energetic particles



Note : $(hv - \phi)$ is energy of photoelectrons due to light e (Ed) is the energy of photoelectrons due to work done by photoelectrons between the plates.

<u>Sol 35</u>.

According to Bohr's model, the energy released during transition from n₂ to n₁ is given by

 $\Delta E = hv = Rhc (Z - b)^2 [1 / n^2_1 - 1 / n^2_2]$

For transition from L shell to K shell

$$B = 1, n_2 = 2, n_1 = 1$$

 \therefore (Z - 1)² Rhc [1/1 - 1 / 4] = hv

On putting the value of $R = 1.1 \times 10^7 \text{ m}^{-1}$ (given),

 $c = 3 \times 10^8 \text{ m/s}$, we get

Z = 42

<u>Sol 36</u>.

 $\lambda = log_{e} \, A_{0} \, / \, A \, / \, t = 1 \, / \, 2 \, log_{e} \, n \, / \, 0.75 \; n$

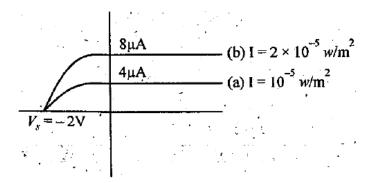
 \Rightarrow Mean Life = 1 / λ = 2 / log_e 4 / 3

<u>Sol 37</u>.

(a) $eV_0 = hv - hv_0 = 5 - 3 = 2 eV$

 \therefore V₀ = 2 volt.

(b) Note : When the intensity is doubled, the saturation current is also doubled,



<u>Sol 38</u>.

a = Initial Uranium atom

(a - x) = Uranium toms left



and n = t / $t_{1/2}$ = 1.5 x 10⁹ / 4.5 x 10⁹ = 13 \therefore a - x = a (1/2)^{1/3} \Rightarrow a / a-x = 1 / (1 / 2)^{1/3} = 2^{1/3} / 1 = 1.26 \Rightarrow x / a - x = 1.26 - 1 = 0.26

<u>Sol 39</u>.

KEY CONCEPT:

The wavelength λ , of photon for different lines of Balmer series is given by

hc $\,$ / λ = 13.6 [1 / 2² – 1 / n²] eV, where n = 3, 4, 5

Using above relation, we get the value of $\lambda = 657$ nm, 487 nm between 450 nm and 700 nm. Since 487 nm, is smaller than 657 nm electron of max. E.K. will be emitted for photon corresponding to wavelength 487 nm with

 $(K.E.) = hc / \lambda - W = (1242 / 487 - 2) = 0.55 eV$

<u>Sol 40</u>.

The de Broglie wave length is given by

$$\lambda = h / mv \Rightarrow \lambda = h / \sqrt{2}mK$$
Case (i) $0 \le x \le 1$
For this, potential energy is E_0 (given)
Total energy = $2E_0$ (given)
 \therefore Kinetic energy = $2E_0 - E_0 = E_0$
 $\lambda_1 = h / \sqrt{2}mE_0$...(i)
Case (ii) $x > 1$
For this, potential energy = 0 (given)
Here also total energy = $2E_0$ (given)
 \therefore Kinetic energy = $2E_0$
 $\therefore \lambda_2 = h / \sqrt{2}m (2E_0)$
Diving (i) and (ii)
 $\lambda_1 / \lambda_2 = \sqrt{2}E_0 / E_0 \Rightarrow \lambda_1 / \lambda_2 = \sqrt{2}$

...(ii)



<u>Sol 41</u>.

(a) KEY CONCEPT : We know that radius of nucleus id given by formula

 $r = r_0 A^{1/3}$ where $r_0 = consft$, and A = mass number.

For the nucleus $r_1 = r_0 4^{1/3}$

For unknown nucleus $r_2 = r_0 (A)^{1/3}$

: $r_2 / r_1 = (A / 4)^{1/3}$, $(14)^{1/3} = (A / 4)^{1/3} \Rightarrow A = 56$

 \therefore No of proton = A – no. of neutrons = 56 – 30 = 26

(b) We know that $v = Rc (Z - b)^2 [1 / n^2_1 - 1 / n^2_2]$

Here, $R=1.1 \ x \ 10^7$, $c=3 \ x \ 10^8$, Z=26

b = 1 (for K_{α}), $n_1 = 1$, $n_2 = 2$

 \therefore v = 1.1 x 10⁷ x 3 x 10⁸ [26 - 1]² [1/1 - 1/4]

= $3.3 \times 10^{15} \times 25 \times 25 \times 34 = 1.546 \times 10^{18} \text{ Hz}$

<u>Sol 42</u>.

Note : nth line of Lyman series means electron jumping from (n + 1) th orbit to 1st orbit.

For an electron to revolve in (n + 1)th orbit.

 $2\pi r = (n+1)\lambda$

 $\Rightarrow \lambda = 2\pi / (n+1) \times r = 2\pi / (n+1) [0.529 \times 10^{-10}] (n+1)^2 / Z$

 $\Rightarrow 1 / \lambda = Z / 2\pi [0.529 \times 10^{-10}] (n+1) \qquad ...(i)$

Also we know that when electron jumps from (n + 1)th orbit to 1st orbit

$$1/\lambda = RZ^{2} [1/1^{2} - 1/(n+1)^{2} = 1.09 \times 10^{7} Z^{2} [1 - 1/(n+1)^{2}]$$

From (i) and (ii)

 $Z / 2\pi (0.529 \text{ x } 10^{-10}) (n + 1) = 1.09 \text{ x } 10^7 Z^2 [1 - 1 / (n + 1)^2]$

On solving, we get n = 24