Permutations and Combinations – Solutions

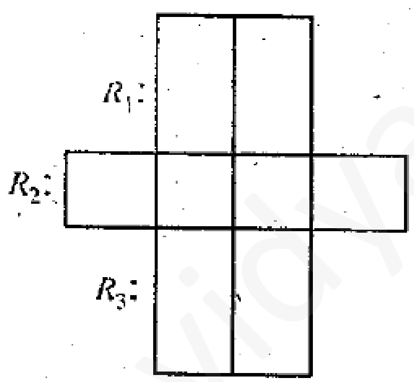
Sol. 1.

As all X's are identical, the question is of selection of 6 squares from 8 squares, so that no row remains empty. Here R_1 has 2 squares, R_2 has 4 squares and R_3 has 2 squares. The selection scheme is as follows:

	R_1	R_2	R_3
	1	4	1
or	1	3	2
or	2	3	1
or	2	2	2

∴ Number of selections are

$^2C_1 \times {}^4C_4 \times {}^2C_1 + {}^2C_1 \times {}^4C_3 \times {}^2C_2 + {}^2C_2 \times {}^4C_3 \times {}^2C_1 + {}^2C_2 \times {}^4C_2 \times {}^2C_2$



Sol. 2.

KEY CONCEPT: If 4n different things are to be equally distributed amongst 4 persons the numbers of ways are $4n! / (n!)^4$ but if 4n things are to form 4 equal groups then the

Numbers of ways are $(4n!) / (n!)^4 \times 4!$

(i) Number of ways of dividing 52 cards equally among 4 players = $52! / (13!)^4$

(ii) Numbers of ways to form 4 groups of 13 cards each = $52! / (13!)^4 \times 4!$

(iii) Number of ways to form 4 sets, three of them having 17

Cards each and fourth just 1 card = $52! / (17!)^3 \times 3! \text{ X } 1!$

Sol. 3.

The various possibilities to put 5 different balls in 3 different size boxes, when no box remains empty :
The balls can be 1, 1 and 3 in different or 2, 2, 1.

Case I : To put 1, 1 and 3 balls in different boxes. Selection of 1, 1 and 3 balls out of 5 balls can be done in $^5C_1 \times {}^4C_1 \times {}^3C_3$ ways and then 1, 1, 3 can permute (as different size boxes) in 3 ! ways.

∴ No. of ways

$= {}^5C_1 \times {}^4C_1 \times {}^3C_3 \times 3! = 5 \times 4 \times 1 \times 6 = 120$

Case II : To put 2, 2 and 1 ball in different boxes. Selection of 2, 2 and 1 balls out of 5 balls can be done in $^5C_2 \times {}^3C_2 \times {}^1C_1$ ways

And then 2, 2, 1 can permute (different boxes) in 3! Ways

∴ No. of ways

$^5C_1 \times {}^3C_2 \times {}^1C_1 \times 3! = 10 \times 3 \times 1 \times 6 = 180$

Combining case I and II, total number of required ways are

= 120 + 180 = 300.

Sol. 4.

m men can be seated in m! ways creating (m + 1) places for ladies to sit.

n ladies out of (m + 1) places (as n < m) can be seated in $^{m+1}P_n$

∴ Total ways = m! x $^{m+1}P_n$

= m! x (m + 1)!/(m + 1 – n)! = (m + 1) 1m! / (m – n + 1)!

Sol. 5.

There four possibilities:

(i) 3 ladies from husband's side and 3 gentlemen from wife's side.

No. of ways in this case

$= {}^4C_3 \times {}^4C_3 = 4 \times 4 = 16$

(ii) 3 gentlemen from husband's side and 3 ladies from wife's side.

No. of ways in this case = $^3C_3 \times {}^3C_3 = 1 \times 1 = 1$

(iii) 2 ladies and one gentleman from husband's side and lady and 2 gentlemen from wife's side.

No. of ways in this case

= ($^4C_4$ x $^3C_1$) x ($^3C_1$ x $^4C_2$) = 6 x 3 x 3 x 6 = 324

(iv) One lady and 2 gentlemen from husband's side and 2 ladies and one gentlemen from wife's side.

No. of ways in this case

= ($^4C_1$ x $^3C_2$) x ($^3C_2$ x $^4C_1$) = 4 x 3 x 3 x 4 = 1444

Hence the total no. of ways are

= 16 + 1 + 324 + 144 = 485.

Sol. 6.

Number of ways drawing at least one black ball = 1 black and 2 other or 2 black and 1 other or 3 black

= $^3C_1$ x $^6C_2$ + $^3C_2$ x $^6C_1$ + $^3C_3$ = 3 x 15 + 3 x 6 + 1

= 45 + 18 + 1 = 64

ALTERNATE SOLUTION:

Number of ways drawing at least one black ball

= Total ways No. of ways of drawing no black ball

= $^9C_3 - {}^6C_3$ = 84 – 20 = 64

Sol. 7.

Number of ways in which a student can select at least one and almost n books out of (2n + 1) books is equal to

$= {}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots\ldots\ldots + {}^{2n+1}C_n$

$= 1/2\ [2.\ {}^{2n+1}C_1 + 2.\ {}^{2n+1}C_2 + 2.\ {}^{2n+1}C_3 + \ldots\ldots\ldots\ldots + 2.\ {}^{2n+1}C_n]$

$= 1/2\ [({}^{2n+1}C_1 + {}^{2n+1}C_{2n}) + ({}^{2n+1}C_2 + {}^{2n+1}C_{2n-1}) + ({}^{2n+1}C_3 + {}^{2n+1}C_{2n-2}) + \ldots\ldots\ldots + ({}^{2n+1}C_n + {}^{2n+1}C_{n+1})]$

[Using $^nC_r = {}^nC_{n-r}$]

$= 1/2\ [{}^{2n+1}C_1 + {}_{2n}{}^{+1}C_2\ {}^{2n+1}C_3 + \ldots\ldots\ldots + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \ldots\ldots\ldots + {}^{2n+1}C_{2n}]$

$= 1/2\ [{}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ldots\ldots + {}^{2n+1}C_{2n+1} - 1 - 1]$

$= 1/2\ [2^{2n+1} - 2] = 2^{2n} - 1$

ATQ, $2^{2n} - 1 = 63 \Rightarrow 2^{2n} = 64 = 2^6 \Rightarrow 2n = 6 \Rightarrow n = 3$

Sol. 8.

Out of guests half i.e. to be seated on side A and rest 9 on side B. Now out of 18 guests, 4 particular guests desire to sit – on one particular side say side A and other 3 on other side B. Out of rest 18 – 4 – 3 = 11 guests we can select 5 more for side A and rest 6 can be seated on side B. Selection of 5 out of 11 can be done in $^{11}C_5$ ways and 9 guests on each sides of table can be seated in 9! x 9! Ways. Thus there are total $^{11}C_5$ x 9! x 9! Arrangements.

Sol. 9.

Given that there are 9 women and 8 men. A committee of 12 is to be formed including at least 5 women. This can be done in the following ways.

= 5W and 7M or 6W and 6M

Or 7W and 5M

Or 8W and 4M

Or 9W and 3M

No. of ways of forming committee is

$= {}^9C_5 \times {}^8C_7 + {}^9C_6 \times {}^8C_6 + {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$

= 9. 8. 7. 6/4 .3. 2. 1 x 8 + 9. 8. 7/3. 2. 1 x 8. 7/2. 1 + 9. 8/2. 1 x 8. 7. 6/3. 2. 1

+ 9 x 8. 7. 6. 5/4. 3. 2. 1 + 1 x 8. 7. 6/3. 2. 1

= 126 x 8 + 24 x 28 + 36 x 56 + 9 x 70 + 56 = 6062 ways.

(a) The women are in majority in 2016 + 630 + 56 = 2702 ways

(b) The men are in majority in 1008 ways.

Sol. 10.

Let there be n sets of different objects each set containing n identical objects

[eg (1, 1, 1 . . . . (n times)), (2, 2, 2 . . . . 2 (n times)) . . . . (n, n, n . . . . n) n times))]

Then the no. of ways in which these n x n = n^2 objects can be arranged in a row

= $(n^2)! / n!\, n! \ldots\ldots. n! = (n^2)! / (n!)^n$

But this number of ways should be a natural number

Hence $(n^2)! / (n!)^n$ is an integer. $(n \in I)$

Sol. 11.

Given that

Runs scored in kth match = $k.2^{n+1-k}$; $1 \le k \le n$

And runs scored in n matches

$= n + 1/4\ (2^{n+1} - n - 2)$

$$\therefore \sum_{k=1}^{n} k.2^{n+1-k} = n + 1/4\ (2^{n+1} - n - 2)$$

$$\Rightarrow 2^{n+1}\left[\sum_{k=1}^{n}\frac{k}{2^k}\right] = n + 1/4\ (2^{n+1} - n - 2)$$

$\Rightarrow 2^{n+1}\ [1/2 + 2/2^2 + 3/2^3 + \dots\dots\dots\dots + n/2^n]$

$= n + 1/4\ (2^{n+1} - n - 2)$ ....(i)

Let $S = 1/2 + 2/2^2 + 3/2^3 + \dots\dots\dots\dots\dots n/2^n$

$1/2\ S = 1/2^2 + 2/2^3 + \dots\dots\dots\dots\dots\dots + n - 1/2^n + n/2^{n+1}$

Subtracting the above two, we get

$1/2\ S = 1/2 + 1/2^2 + 1/2^3 + \dots\dots\dots\dots\dots\dots + 1/2^n - n/2^{n+1}$

$$\Rightarrow 1/2\ S = \frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} - n/2^{n+1}$$

$\Rightarrow S = 2\ [1 - 1/2^n - n/2^{n+1}]$

$\therefore$ Equation (i) becomes

$2.2^{n+1}\ [1 - 1/2^n - n/2^{n+1}] = n + 1/4\ [2^{n+1} - n - 2]$

$\Rightarrow 2\ [2^{n+1} - 2 - n] = n + 1/4\ [2^{n+1} - 2 - n]$

$\Rightarrow n + 1/4 = 2 \Rightarrow n = 7.$