Probability – Solutions

Sol. 1.

To draw 2 black, 4 white and 3 red balls in order is same as arranging two black balls at first 2 places, 4 white at next 4 place, (3^{rd} to 6^{th} place) and 3 red at still next 3 places (7^{th} to 9^{th} place), I.e., B_1 B_2 W_1 W_2 W_3 W_4 R_1 R_2 R_3, which can be done in 2! x 4! x 3! Ways. And total ways of arranging all 2 + 4 + 3 = 9 balls is 9!

∴ Required probability = 2! x 4! x 3!/9! = 1/1260

Sol. 2.

(i) 6 boys and 6 girls sit in a row randomly.

Total ways of their seating = 12!

No. of ways in which all the 6 girls sit together = 6! x 7! (Considering all 6 girls as one person)

∴ Probability of all girls sitting together

= 6! x 7!/12! = 720/12 x 11 x 10 x 9 x 8 = 1/132

(ii) Staring with boy, boys can sit in 6! Ways leaving one place between every two boys and two one a last.

B _ B _ B _ B _ B _ B _

These left over places can be occupied by girls in 6! ways.

∴ If we start, with boys. No. of ways of seating boys and girls alternately = 6! x 6!

In the similar manner, if we start with girl, no. of ways of seating boys and girls alternately

= 6! x 6!

G _ G _ G _ G _ G _ G _

Thus total ways of alternate seating arrangements

= 6! x 6! + 6! x 6!

= 2 x 6! x 6!

∴ Probability of making alternate seating arrangement for 6 boys and 6 girls

= 2 x 6! x 6!/12! = 2 x 720/12 x 11 x 10 x 9 x 8 x 7 = 1/462

Sol. 3.

(a). Let us define the events as:

$E_1 \equiv$ First shot hits the target plane,

$E_2 \equiv$ Second shot hits the target plane,

$E_3 \equiv$ third shot hits the target plane,

$E_4 \equiv$ fourth shot hits the target plane

Then ATQ, $P(E_1) = 0.4$; $P(E_2) = 0.3$;

$P(E_3) = 0.2$; $P(E_4) = 0.1$

$\Rightarrow P(\bar{E}_1) = 1 - 0.4 = 0.6$; $P(\bar{E}_2) = 1 - 0.3 = 0.7$

$P(\bar{E}_3) = 1 - 0.2 = 0.8$; $P(\bar{E}_4) = 1 - 0.1 = 0.9$

(where $\bar{E}_1$ denotes not happening of E_1)

Now the gun hits the plane if at least one of the four shots hit the plane.

Also, P (at least one shot hits the plane).

= 1 - P (none of the shots hits the plane)

$= 1 - P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \bar{E}_4)$

$= 1 - P(\bar{E}_1).\ P(\bar{E}_2).\ P(\bar{E}_3).\ P(\bar{E}_4)$

[Using multiplication thm for independent events] $= 1 - 0.6 \times 0.7 \times 0.8 \times 0.9 = 1 - 0.3024 = 0.6976$

Sol. 4.

Let A denote the event that the candidate A is selected and B the event that B is selected. It is given that

$P(A) = 0.5$ . . . . (1)

$P(A \cap B) \leq 0.3$ . . . . (2)

Now, $P(A) + P(B) - P(A \cap B) = P(A \cup B) \leq 1$

Or $0.5 + P(B) - P(A \cap B) \leq 1$ [Using (1)]

Or $P(B) \leq 0.5 + P(A \cap B) \leq 0.5 + 0.3$ [Using (2)]

Or $P(B) \leq 0.8 \therefore P(B)$ cannot be 0.9

Sol. 5.

We must have one ace in (n – 1) attempts and one ace in the nth attempt. The probability of drawing one ace in first

(n – 1) attempts is ${}^4C_1 \times {}^{48}C_{n-2}/{}^{52}C_{n-1}$ and other one ace in the nth attempt is, ${}^3C_1/[52 - (n - 1)] = 3/53 - n$

Hence the required probability,

$= 4.48! / (n - 2)! (50 - n)! \times (n - 1)! (53 - n)/52! \times 3/53 - n$

$= (n - 1) (52 - n) (51 - n)/50. 49. 17. 13$

Sol. 6.

Given that

$P (A) = 0.3, P (B) = 0.4, P (C) = 0.8$

$P (AB) = 0.08, P (AC) = 0.28, P (ABC) = 0.09$

$P (A \cup B \cup C) \geq 0.75$

To find $P (BC) = x$ (say)

Now we know,

$P (A \cup B \cup C) = P (A) + P (B) + P (C) - P (AB) - P (BC) - P (CA) + P (ABC)$

$\Rightarrow P (A \cup B \cup C) = 0.3 + 0.4 + 0.8 - 0.08 - x - 0.28 + 0.09 = 123 - x$

Also we have,

$P (A \cup B \cup C) \geq 0.75$ and $P (A \cup B \cup C) \leq 1$

$\therefore 0.75 \leq P (A \cup B \cup C) \leq 1$

$\Rightarrow 0.75 \leq 1.23 - x \leq 1$

$\Rightarrow 0.23 \leq x \leq 0.48$

Sol. 7.

Given that A and B are independent events

$\therefore P (A \cap B) = P (A). P (B)$ .... (1)

Also given that $P (A \cap B) = 1/6$ .... (2)

And $P (\bar{A} \cap \bar{B}) = 1/3$ .... (3)

Also P ($\bar{A} \cap \bar{B}$) = 1 – P ($\bar{A} \cup \bar{B}$)

⇒ P ($\bar{A} \cap \bar{B}$) = 1 – P (A) - P (B) P ($\bar{A} \cap \bar{B}$)

⇒ 1/3 = 1 - P (A) - P (B) + 1/6

⇒ P (A) + P (B) . . . . (4)

From (1) and (2) we get

P (A). P (B) = 1/6 . . . . (5)

Let P (A) = x and P (B) = y then eq' s (4) and (5) become

x + y = 5/6, xy = 1/6

⇒ x - y = $\pm\sqrt{(x+y)^2 - 4xy}$

= $\pm\sqrt{25/36 - 4/6} = \pm 1/6$

∴ We get x = 1/2 and y = 1/3

Or x = 1/3 and y = 1/2

Thus P (A) = 1/2 and P (B) = 1/3 Or P (A) = 1/3 and P (B) = 1/2 .

Sol. 8.

KEY CONCEPT:

(Total prob. Theorem) If E_1, E_2, E_3 . . . . . . E_n are mutually exclusive and exhaustive events and E is an event which can take place in conjunction with any one of E_1 then

P (E) = P (E_1) P (EI E_1) + P (E_2) P (EI E_2) + ... + P (E_n) P (EI E_n) Let P (A) denote the prob. of people reading newspaper A and P (B) that of people reading newspaper B

Then, P (A) = 25/100 = 0.25

P (B) = 20/100 = 0.20, P (AB) = 8/100 = 0.08

Prob. of people reading the newspaper A but not B = P (AB^c)

= P (A) – P(AB) = 0.25 – 0.08 = 0.17

Similarly,

P ($A^c B$) = P (B) – P (AB) = 0.20 – 0.08 = 0.12

Let E be the event that a person reads an advertisement.

Therefore

ATQ, P (EI AB^c) = 30/100; P (EI A^c B) = 40/100

P (EI AB) = 50/100

∴ By total prob. theorem (as AB^c, A^c B and AB are mutually exclusive)

P (E) = P (EI AB^c) P (AB^c) + P (EI A^c B) P (A^c B) + P (EI AB). P (AB)

= 30/100 x 0.17 + 40/100 x 0.12 + 50/100 x 0.08

= 0.051 + 0.048 + 0.04

Thus the population that reads an advertisements is 13.9%

Sol. 9.

The total number of ways of ticking the answers in any one attempt = 2^4 - 1 = 15.

The student is taking chance at ticking the correct answer, It is reasonable to assume that in order to derive maximum benefit, the three solutions which he submit must be all different.

∴ n = total no. of ways = ${}^{15}C_3$

m = the no. of ways in which the correct solution is excluded ${}^{14}C_3$

Hence the required probability = 1 - ${}^{14}C_3/{}^{15}C_3$ = 1 – 4/5 = 1/5

ALTERNATE SOLUTION:

The candidate may tick one or more of the alternatives. As each alternative may or may not be chosen, the total numbers of exhaustive possibilities are 2^4 - 1 = 15.

Therefore the prob. that the questions are correctly answered by candidate is 1/15.

As such the candidate may be correct on the first, second or third chance. As these events are mutually exclusive, the total probability will be given by

= 1/15 + 14/15 x 1/14 + 14/15 x 13/14 x 1/13 = 1/15 + 1/15 + 1/15 = 3/15 = 1/5

Thus the probability that the candidate gets marks in the question is 1/5.

Sol. 10.

Let A_1 be the event that the lot contains 2 defective articles and A_2 the event that the lot contains 3 defective articles. Also let A be the event that the testing procedure ends at the twelfth testing. Then according to the question :

P (A_1) = 0.4 and P (A_2) = 0.6

Since $0 < P(A_1) < 1$, $0 < P(A_2) < 1$, and $P(A_1) + P(A_2) = 1$

$\therefore$ The events A_1, A_2 form a partition of the sample space. Hence by the theorem of total probability for compound events, we have

NOTE THIS STEP:

$P(A) = P(A_1) P(A| A_1) + P(A_2) P(A| A_2)$ . . . . (1)

Here $P(A| A_1)$ is the probability of the event the testing procedure ends at the twelfth testing when the lot contains 2 defective articles. This is possible when out of 20 articles; first 11 draws must contain 10 non defective and 1 defective articles and 12^{th} draw must give a defective article.

$\therefore P(A| A_1) = {}^{18}C_{10} \times {}^{2}C_1 / {}^{20}C_{11} \times 1/9 = 11/190$

Similarly, $P(A| A_2) = {}^{17}C_9 \times {}^{3}C_1 / {}^{20}C_{11} \times 1/9 = 11/228$

Now substituting the values of $P(A| A_1)$ and $P(A| A_2)$ in eq. (1), we get

$P(A) = 0.4 \times 11/190 + 0.6 \times 11/228 = 11/475 + 11/380 = 99/1900$

Sol. 11.

Since the man is one step away from starting point means that either

(i) man has taken 6 steps forward and 5 steps backward

Or (ii) man has taken 5 steps forward and 6 steps backward.

Taking movement 1 step forward as success and 1 step backward as failure.

$\therefore$ p = Probability of success = 0.4 and q = probability of failure = 0.6

$\therefore$ required probability = $P(X = 6 \text{ or } X = 5)$

$= P(X = 6) + P(X = 5)$

$= {}^{11}C_6 \, p^6 q^5 + {}^{11}C_5 \, p^5 q^6$

$= {}^{11}C_5 \, (p^6 q^5 + p^5 q^6) = {}^{11}C_5 \, (p + q)(p^5 q^5)$

$= 11.\,10.\,9.\,8.\,7/1.\,2.\,3.\,4.\,5 \, (0.4 + 0.6)(0.4 \times 0.6)^5$

$= 462 \times 1 \times (0.24)^5 = 0.37$

Hence the required prob. = 0.37

Sol. 12.

(a) There are following four possible ways of drawing first two balls.

(i) Both the first and the second balls drawn are white.

(ii) The first ball drawn is white and the second ball drawn is black.

(iii) The first ball is black and the second ball drawn is white.

(iv) Both the first and the second balls drawn are black. Let us define events (i), (ii), (iii) and (iv) by E_1, E_2, E_3 and E_4 respectively. Also let E denotes the event that the third ball drawn is black.

Then, $P(E_1) = 2/4 \times 1/3 = 1/6$, $P(E_2) = 2/4 \times 2/3 = 1/3$

$P(E_2) = 2/4 \times 2/5 = 1/5$, $P(E_4) = 2/4 \times 3/5 = 3/10$

Also $P(E \mid E_1) = 1$, since when the event E, has already happened i.e. the first two balls drawn are both white, they are not replaced and so there are left 2 black balls in the urn so that the probability that the third ball drawn in this case is black = 2/2 = 1.

Again $P(E \mid E_2) = ¾$, since when the event E_2 has already happened there are 3 black and one white balls in the urn. So in this case the probability that the third ball drawn is black = ¾.

Similarly, $P(E \mid E_3) = 3/4$ and $P(E \mid E_4) = 2/3$

Now by thm of total prob. for compound events, we have

$P(E) = P(E_1)\, P(E \mid E_1) + P(E_2)\, P(E \mid E_2) + P(E_3)\, P(E \mid E_3) + P(E_4)\, P(E \mid E_4)$

$= 1/6 \times 1 + 1/3 \times 3/4 + 1/5 \times 3/4 + 3/10 \times 2/3 = 1/6 + 1/4 + 3/20 + 1/5 = 23/30$

Sol. 13.

Here the total number of coins is N + 7. Therefore the total number of ways of choosing 5 coins out of N + 7 is ${}^{N+7}C_5$ Let E denotes the event that the sum of the values of the coins is less than one rupee and fifty paisa.

Then E' denotes the event that the total value of the five coins is equal to or more than one rupee and fifty paisa.

NOTE THIS STEP:

The number of cases favorable to E' is

$= {}^2C_1 \times {}^5C_4 \times {}^NC_0 + {}^2C_2 \times {}^5C_3 \times {}^NC_0 + {}^2C_2 \times {}^5C_2 \times {}^NC_1$

$= 2 \times 5 + 10 + 10\,N = 10\,(N + 2)$

$\therefore$ P (E) = 10 (N + 2)/$^{n+1}C_5$

$\Rightarrow$ P (E) = 1 – P (E) = 1 – 10 (N + 2)/$^{N+7}C_5$

Sol. 14.

The probability p_1 (say) of winning the best of three games is = the prob. of winning two games + the prob. of winning three games.

= $^3C_2 (0.6) (0.4)^2 + {^3C_3} (0.4)^3$ [Using Binomial distribution]

Similarly the probability of winning the best five games is p_2 (say) = the prob. of winning three games + the prob. of winning 5 games.

= $^5C_3 (0.6)^2 (0.4)^3 + {^5C_3} (0.6) (0.4)^3 + {^5C_5} (0.4)^5$

We have $p_1 = 0.288 + 0.064 = 0.352$

And $p_2 = 0.2304 + 0.0768 + 0.01024 = 0.31744$

As $p_1 > p_2$

$\therefore$ A must choose the first offer i.e. best of three games.

Sol. 15.

Set A has a elements.

$\therefore$ Number of subsets of A = 2”

$\therefore$ Each one of P and Q can be selected in 2” ways.

Hence total no. of ways of selecting P and Q = 2” = 4”.

Let P contains r elements, where r varies from 0 to n, Then, P can be chosen in ^nC_r ways.

Now as $P \cap Q = \phi$, Q can be chosen from the set of all subsets of set consisting of remaining (n – r) elements. This can be done in 2^{n-r} ways.

$\therefore$ P and Q can be chosen in $^nC_r \cdot 2^{n-r}$ ways. But, r can vary from 0 to n

$\therefore$ total number of disjoint sets P and Q are

$$= \sum_{r=0}^{n} {^nC_r} 2^{n-r} = (1+2)^n = 3^n$$

NOTE THIS STEP:

$\therefore$ Required probability $= 3^n/4^n = (3/4)^n$

ALTERNATE SOLUTION:

Let A = {a_1, a_2, a_3, ..., a_n}

For each a_i, $1 \leq i$ n, there Aries 4 cases

(i) $a_1 \in P$ and $a_1 \in Q$

(ii) $a_1 \notin P$ and $a_1 \in Q$

(iii) $a_1 \in P$ and $a_1 \notin Q$

(iv) $a_1 \notin P$ and $a_1 \notin Q$

∴ total no. of ways of choosing P and Q is 4^n. Here case (i) is not favorable as $P \cap Q = \phi$

∴ For each element there are 3 favorable cases and hence total no. of favorable cases 3"

Hence prob. $(P \cap Q) = \phi) = 3^n/4^n = (3/4)^n$

ALTERNATE SOLUTION:

The set P be the empty set, or one element set or two elements set . . . . or n elements set. Then the set Q will be chosen from amongst the remaining n elements or (n – 1). Element for (n – 2) elements . . . . or no elements. Now if P is the empty set then prob. of its choosing is ${}^nC_0/2^n$, if it is one element set the then prob. of its choosing is ${}^nC_1/2^n$, and so on. When the set P consisting of r elements is chosen from A, then the prob. of choosing the set Q from amongst the remaining n - r elements $2^{n-r}/2^n$. Hence the prob. that P and Q have no common elements is given by

$$\sum_{r=0}^{n} {}^nC_r/2^n \,.\, 2^{n-r}/2^n = 1/4^n \sum_{r=0}^{n} {}^nC_r\, 2^{n-r}$$

$= 1/4^n (1 + 2)^n$ (Using Binomial thm.) $= 3^n/4^n = (3/4)^n$

Sol. 16.

KEY CONCEPT :

Baye's theorem: $E_1, E_2, E_3, \ldots\ldots, E_n$ are mutually exclusive and exhaustive events and E is an event which takes place in conjunction with any one of E_1 then the probability of the event E_1 happening when the event E has taken place is given by

$$P(E_1 | E) = \frac{P(E_1)P(E|E_1)}{\sum_{i=1}^{n} P(E_1)P(E|E_1)}$$

Let us define the events :

$A_1 \equiv$ the examinee guesses the answer,

$A_2 \equiv$ the examinee copies the answer

$A_3 \equiv$ the examinee knows the answer,

$A \equiv$ the examinee answers correctly

ATQ, $P(A_1) = 1/3$; $P(A_2) = 1/6$

As any one happens out of A_1, A_2, A_3, these are mutually exclusive and exhaustive events.

$\therefore P(A_1) + P(A_2) + P(A_3) = 1$

$\Rightarrow P(A_3) = 1 - 1/3 - 1/6 = 6 - 2 - 1/6 = 3/6 = 1/2$

Also we have, $P(A| A_1) = 1/4$

[$\because$ out of 4 choices only one is correct.] $P(A| A_2) = 1/8$

(given) $P(A| A_3) = 1$

[If examinee knows the ans., it is correct. i.e. true event]

To find $P(A_3| A)$. By Baye's thm. $P(A_3| A)$

$= P(A_3| A)\, P(A_3)/P(A| A_1)\, P(A_1) + P(A| A_2)\, P(A_2) + P(A| A_3)\, P(A_3)$

$$= \frac{1\frac{1}{2}}{\frac{1}{4}.\frac{1}{3}+\frac{1}{8}.\frac{1}{6}+\frac{1.1}{2}} = 1/2/29/48 = 1/2 \text{ x } 18/29 = 24/29.$$

Sol. 17.

Let S = defective and Y = non defective. Then all possible outcomes are {XX, XY, YX, YY}

Also P (XX) = 50/100 x 50/100 = 1/4,

P (XY) = 50/100 x 50/100 = 1/4,

P (YX) = 50/100 x 50/100 = 1/4,

P (YY) = 50/100 x 50/100 = 1/4

Here, A =XX ∪ XY; B = XY ∪ YY; C = XX ∪ YY

∴ P (A) = P (XX) + P (XY) = 1/4 + 1/4 = 1/2

∴ P (B) = P (XY) + P (YX) = 1/4 + 1/4 = 1/2

P (C) = P (XX) + P (YY) = 1/4 + 1/4 =1/2

Now, P (AB) = P (XY) = 1/4 = P (A). P (B)
∴ A and B are independent events.

P (BC) = P (YX) = 1/4 = P (B). P (C)

∴ B and C are independent events.

P (CA) = P (XX) = 1/4 = P (C). P (A)

∴ C and A are independent events.

P (ABC) = 0 (impossible event)

≠ P (A) P (B) P (C)

∴ A, B, C are dependent events,

Thus we can conclude that A, B, C are pair wise independent bet A, B, C are dependent events.

Sol. 18.

The given numbers are 00, 01, 02 . . . . 99. These are total 100 numbers, out of which the numbers, the product of whose digits is 18, are 29, 36, 63 and 92.

∴ p = P (E) = 4/100 = 1/25 ⇒ q = 1 – p = 24/25

From Binomial distribution

P (E occurring at least 3 times) = P (E occurring 3 times) + P (E occurring 4 times)

${}^4C_3\, p^3\, q + {}^4C_4\, p^4 = 4 \times (1/25)^3\, (24/25) + (1/25)^4 = 97/(25)^4$

Sol. 19.

E_1 ≡ number noted is 7, E_2 ≡ number notes is 8,

H ≡ getting head on coin, T ≡ getting tail on coin.

Then by total probability theorem,

$P(E_1) = P(H)\, P(E_1 | H) + P(T)\, P(E_1 | T)$

And $P(E_2) = P(H)\, P(E_2 | H) + P(T)\, P(E_2 | T)$

Where P (H) = 1/2 ; P (T) = 1/2

$P(E_1 | H)$ = prob. of getting a sum of 7 on two dice. Here favorable cases are

{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)}

$\therefore$ P (E_1| H) = 6/36 = 1/6

Also P (E_1| T) = prob. if getting '7' numbered card out of 11 cards = 1/11.

P (E_2| H) = Prob. of getting a sum of 8 on two dice. Here favorable cases are

{(2, 6), (6, 2), (4, 4), (5, 3), (3, 5)}

$\therefore$ P (E_2| H) = 5/36

P (E_2| T) = prob. of getting '8' numbered card out of 11 cards = 1/11

$\therefore$ P (E_1) = 1/2 x 1/6 + 1/2 x 1/11 = 1/12 + 1/22 = 11 + 6/132 = 17/132

P (E_2) = 1/2 x 5/36 + 1/2 x 1/11 = 1/2 [55 + 36/396] = 91/792

Now E_1 and E_2 are mutually exclusive events therefore

P (E_1 or E_2) = P (E_1) + P (E_2) = 17/132 + 91/792

= 102 + 91/792 = 193/792 = 0.2436

Sol. 20.

We have 14 seats in two vans. And there are 9 boys and 3 girls. The no. of ways of arranging 12 people on 14 seats without restriction is

$^{14}P_{12}$ = 14!/2! = 7 (13!)

Now the no. of ways of choosing back seats is 2. And the no. of ways of arranging 3 girls on adjacent seats is 2 (3!). and the no. of ways of arranging 9 boys on the remaining 11 seats is $^{11}P_9$

Therefore, the required number of ways

= 2. (2.3!). $^{11}P_9$ = 4.3!. 11! /2! = 12!

Hence, the probability of the required event

= 12! /7.13! = 1/91

Sol. 21.

(a) Prob. of S_1 to among the eight winners = (prob. of S_1 being in a pair) x (prob. of S_1 winning in the group).

= 1 x 1/2 [$\because$ S_1 is definitely in a group i.e. certain event] = 1/2

(b) If S_1 and S_2 are in the same pair then exactly on wins. If S_1 and S_2 are in two pairs separately then exactly one of S_1 and S_2 will be among the eight winners if S_1 win and S_2 loses or S_1 loses and S_2 wins.

Now the prob. of S_1, S_2 being in the same pair and one wins.

= (prob. of S_1, S_2 being in the same pair) x (prob. of any one winning in the pair)

And the prob. of S_1, S_2 being in the same pair

= n (E)/n (S), where n (S) = the no. of ways in which 16 person can be divided in 8 pairs; n (E) = the no. of ways in which S_1, S_2 are in same pair or 14 persons can be divided into 7 pairs.

∴ n (E) = $14!/(2!)^7 .7!$ and n (S) = $16!/(2!)^8 .8!$

∴ Prob. of S_1 and S_2 being in the same pair

$$= \frac{\frac{14!}{(21)^7\ 7!}}{\frac{16!}{(2!)^8\ .8!}} = 21.8/16.15 = 1/15$$

The prob. of any one winning in the pair of S_1, S_2 = P (certain event) = 1

∴ The pair of S_1, S_2 being in two pairs separately and any one of S_1, S_2 wins.

= the prob. of S_1, S_2 being in two pairs separately and S_1 wins, S_2 loses + the prob. of S_1, S_2 being in two pairs separately and S_1 loses, S_2 wins.

$$= \left[1 - \frac{\frac{14!}{(2!)^7 .7.!}}{\frac{16!}{(2!)^8 .8!}}\right] \text{x } 1/2 \text{ x } 1/2 + \left[1 - \frac{\frac{14!}{(2!)^7 .7.!}}{\frac{16!}{(2!)^8 .8!}}\right] \text{x } 1/2 \text{ x } 1/2$$

$$= 2x \frac{\frac{16 - 14!x16}{(2!)^8 .8!}}{\frac{16!}{(2!)^8 .8!}} \text{ x } 1/4 = 1/2 \text{ x } 14 \text{ x } 14!/15 \text{ x } 14! = 7/15$$

∴ Required prob. = 1/15 + 7/15 = 8/15

Sol. 22.

The required probability = 1 – (Probability of the event that the roots of $x^2 + px + q = 0$ are non – real if and only if

$p^2 - 4q < 0$ i.e. if $p^2 < 4q$.

We enumerate the possible values of p and q, for which this can happen in the following table.

q	P	Number of pairs p, q
1	1	1
2	1, 2,	2

3	1, 2, 3	3
4	1, 2, 3	3
5	1, 2, 3, 4	4
6	1, 2, 3, 4	4
7	1, 2, 3, 4, 5	5
8	1, 2, 3, 4, 5	5
9	1, 2, 3, 4, 5	5
10	1, 2, 3, 4, 5, 6	6

Thus, the number of possible pairs = 38. Also, the total number of possible pairs is 10 x 10 = 100.

∴ The required probability

$= 1 - 38/100 = 1 - 0.38 = 0.62$

Sol. 23.

Given that p is the prob. that coin shows a head then $1 - p$ will be the prob. that coin shows a tail.

Now α = P (A gets the 1st head in 1st try)

$\Rightarrow \alpha = P(H) + P(T)\,P(T)\,P(T)\,P(H) + P(T)\,P(T)\,P(T)\,P(T)\,P(T)\,P(T)\,P(H)$

$= p + (1-p)^3 p + (1-p)^6 p + \ldots\ldots\ldots\ldots$

$= p\,[1 + (1-p)^3 (1-p)^6 + \ldots\ldots\ldots\ldots]$

$= p/1 - (1-p)^3$

NOTE THIS STEP . . . (i)

Similarly β = P (B gets the 1st head in 1st try) + P (B gets the 1st head in 2nd try) + ...

$= P(T)\,P(H) + p(T)\,p(T)\,p(T)\,p(T)\,p(T)\,P(H) + \ldots\ldots\ldots\ldots$

$= (1-p)\,p + (1-p)^4 + \ldots\ldots\ldots\ldots$

$= (1-p)\,p/1 - (1-p)^3$ . . . . (ii)

From (i) and (ii) we get $\beta = (1-p)\,\alpha$

Also (i) and (ii) give expression for α and β in terms of p.

Also $\alpha + \beta + \gamma = 1$(exhaustive events and mutually exclusive events)

$\Rightarrow \gamma = 1 - \alpha - \beta = 1 - \alpha - (1-p)\,\alpha$

$= 1 - (2-p)\,\alpha = 1 - (2-p)\,p/1 - (1-p)^3$

$= 1 - (1-p)^3 - (2p - p^2)\,/1 - (1-p)^3$

$= 1 - 1 + p^3 + 3p(1-p) - 2p + p^2/1 - (1-p)^3$

$= p^3 - 2p^2 + p/1 - (1 - p)^3 = p\,(p^2 - 2p + 1)/1 - (1 - p)^3 = p\,(1 - p)^2/1 - (1 - p)^3$

Sol. 24.

The number of ways in which $P_1, P_2 \ldots\ldots\ldots\ldots, P_8$ can be paired in four pairs.

$= 1/4! \times {}^8C_2 \times {}^6C_2 \times {}^8C_2 \times {}^4C_2 \times {}^2C_2 = 105$

Now, at least two players certainly reach the second round in between P_1, P_2, and P_3 and P_4 can reach in final if exactly two players play against each other in between P_1, P_2, P_3 and remaining player will play against one of the players from P_5, P_6, P_7, P_8 and P_4 plays against one of the remaining three from P_5, P_6, P_7, P_8

This can be possible in ${}^3C_2 \times {}^{4C1} \times {}^3C_1 = 36$ ways

$\therefore$ Prob. that P_4 and exactly one of $P_5 \ldots\ldots\ldots P_8$ reach second round

$= 36/105 = 12/35$

If P_1, P_i, P_4 and P_j where $i = 2$ or 3 and $j = 5$ or 6 or 7 reach the second round then they can be paired in 2 pairs in

$1/2! \times {}^4C_2 \times {}^2C_2 = 3$ ways

But P_4 will reach the final if P_1 plays against P_i and P_4 plays against P_j

Hence the prob. that P_4 reach the final round from the second $= 1/3$.

$\therefore$ prob. that P_4 reach the final is $12/35 \times 1/3 = 4/35$.

Sol. 25.

Given that the probability of showing head by a coin when tossed $= p$

$\therefore$ Prob. of coin showing a tail $= 1 - p$

Now p_n = prob. that no two or more consecutive heads occur when tossed n times.

$\therefore p_1$ = prob. of getting one or more on no head = prob. of H or T = 1

Also p_2 = prob. of getting one H or no H

$= P\,(HT) + P\,(TH) + P\,(TT)$

$= p\,(1 - p) + p(1 - p)\,p + (1 - p)\,(1 - p)$

$= 1 - p^2$, For $n \geq 3$

P_n = prob. that no two or more consecutive heads occur when tossed n times.

= p (last outcome is T) P (no two or more consecutive heads in (n – 1) throw) + P (last outcome H) P ((n – 1)th throw results in a T) P (no two or more consecutive heads in (n – 2) n throws)
$= (1 - p) P_{n-1} + p (1 - p) p_{n-2}$ Hence Proved.

Sol. 26.

Let W_1 (B_1) be the event that a white (a black) ball is drawn in the first draw and let W be the event that a white ball is drawn in the second draw. Then

$P (W) = P (B_1). P (W| B_1) + P (W_1). P (W| W_1)$

$= n/m + n. m/m + n + k + m/m + n. m + k/m + n + k$

$= m(n + m + k)/(m + n) (m + n + k) = m/m + n$

Sol. 27.

The total no. of outcomes = 6^n

We can choose three numbers out of 6 in ${}^6C_3$ ways. By using three numbers out of 6 we can get 3^n sequences of length n. But these sequences of length n which use exactly two numbers and exactly one number.

The number of n – sequences which use exactly two numbers

$= {}^3C_2 [2^n - 1^n - 1^n] = 3(2^n - 2)$ and the number of n sequence which are exactly one number

$= ({}^3C_1) (I^n) = 3$

Thus, the number of sequences, which use exactly three numbers

$= {}^6C_3 [3^n - 3(2^n - 2) - 3] = {}^6C_3 [3^n - 3(2^n) + 3]$

∴ Probability of the required event,

$= {}^6C_3 [3^n - 3(2^n) + 3]/6^n$

Sol. 28.

Let E_1 be the event that the coin drawn is fair and E_2 be the event that the coin drawn is biased.

$\therefore P (E_1) = m/N$ and $P (E_2) = N - m/N$

A is the event that on tossing the coin head appears first and then appears tail.

$\therefore P (A) = P (E_1 \cap A) + P (E_2 \cap A)$

$= P (E_1) P (A| E_1) + P (E_2) P (A| E_2)$

$= m/n (1/2)^2 + (N - m/N) (2/3) (1/3)$ .... (1)

We have to find the probability that A has happened because of E_1

$\therefore$ P (E_1| A) = P (E_1| ∩ A)/P (A)

$$= \frac{\frac{m}{n}\left(\frac{1}{2}\right)^2}{\frac{m}{n}\left(\frac{1}{2}\right)^2 + \frac{N-m}{N}\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)}$$

$$= \frac{m/4}{m/4 + \frac{2(N-m)}{9}} = 9m/m + 8N$$

Sol. 29.

Let us consider

$E_1 \equiv$ event of passing I exam

$E_2 \equiv$ event of passing II exam

$E_3 \equiv$ event of passing III exam

Then a student can qualify in anyone of following ways

1. He passes first and second exam.

2. He passes fires, fails in second but passes third exam.

3. He fails in first, passes second and third exam.

$\therefore$ Required probability

= P (E_1) P (E_2| E_1) + P (E_1) P (E_2| E_1) P (E_3| E_2) + P (E_1) P (E_2| E_1) P (E_3| E_2)

[as an event is dependent on previous one]

= p. p + p. ((1 – p). p/2 + (1 – p) p/2 p

= $p^2 + p^2/2 - p^3/2 + p^2/2 - p^3/2 = 2p^2 - p^3$

Sol. 30.

Let us consider the events

$E_1 \equiv$ A hits B   Then P (E_1) = 2/3

$E_2 \equiv$ B hits A   P (E_2) = 1/2

$E_3 \equiv$ C hits A   P (E_3 = 1/3

E ≡ A is hit

P (E) = P ($E_2 \cup E_3$) = 1 – P ($\bar{E}_2 \cap \bar{E}_3$)

= 1 P ($\bar{E}_2$) P ($\bar{E}_3$) = 1 – 1/2. 2/3 = 2/3

To find P ($E_2 \cap \bar{E}_2$ / E)

= P ($E_2 \cap \bar{E}_3$)/P (E) [∵ P ($E_2 \cap \bar{E}_3 \cap E$) = P ($E_2 \cap \bar{E}_3$) i.e., B hits A is hit = B hits A].

= P (E_2). P ($\bar{E}_3$)/P (E) = 1/2 X 2/3 / 2/3 = 1/2

Sol. 31.

Given that A and B are two independent events. C is the event in which exactly of A or B occurs.

Let P (A) = x, P (B) = y

Then P (C) = P (A ∩ $\bar{B}$) + P ($\bar{A}$ ∩ B)

P (A) P ($\bar{B}$) + P ($\bar{A}$) P (B)

[∵ If A and B are independent so are 'A and $\bar{B}$' and '$\bar{A}$ and B'.]

⇒ P (C) = x (1 – y) + y (1 – x) . . . . (1)

Now consider, P (A ∪ B) P ($\bar{A} \cap \bar{B}$)

= [P (A) + P (B) – P (A) P (B)] [P ($\bar{A}$) P ($\bar{B}$)]

= (x + y – xy) (1 –x) (1 – y)

= (x + y) (1 – x) (1 – y) – xy (1 – x) (1 – y) ≤ (x + y) (1 – x) (1 – x) [∵ x, y ∈ (0, 1)]

= x (1 – x) (1 – y) + y (1 – x) (1 – y)

= x (1 – y) + y (1 – x) – x^2 (1 – y) – y^2 (1 – x) ≤ x (1 – y) + y (1 – x) 3 = P (C) [Using eq^n (1)]

Thus P (C) ≥ P (A ∪ B) P ($\bar{A} \cap \bar{B}$) is proved.

Sol. 32.

Let us define the following events

A ≡ 4 white balls are drawn in first six draws

B ≡ 5 white balls are drawn in first six draws

C ≡ 6 white balls are drawn in first six draws

E ≡ exactly one white ball is drawn in next two draws (i.e. one white and one red)

Then P (E) = P (E| A) P (A) + P (E| B) P (B) + P (E| C) P (C)

But P (E |C) = 0 [As there are only 6 white balls in the bag.]

P (E) = P (E| A) P (A) + P (E| B) P (B)

$$= \frac{{}^{10}C_1 x^2C_1}{{}^{12}C_2} \frac{{}^{12}C_2 x^6C_4}{{}^{18}C_6} + \frac{{}^{11}C_1 x^1C_1}{{}^{12}C_2} \frac{{}^{12}C_1 x^6C_5}{{}^{18}C_6}$$

Sol. 33.

Let us define the following events

C ≡ person goes by car,

S ≡ person goes by scooter,

B ≡ person goes by bus,

T ≡ person goes by train,

L ≡ person reaches late

Then we are given in the question

P (C) = 1/7; P (S) = 3/7; P (B) = 2/7 P (T) = 1/7

P (L|C) = 2/9; P (L| S) = 1/9; P (L| B) = 4/9; P (L| T) = 1/9

To find the prob. P (C|$\bar{L}$) [∵ reaches in time ≡ not late] Using Baye's theorem

P (C |$\bar{L}$) = P ($\bar{L}$| C) P (C)/ P ($\bar{L}$| C) P (C) + P ($\bar{L}$| S) P (S) .... (i)

+ P ($\bar{L}$| B) P (B) + A ($\bar{L}$| T) P (T)

Now, P ($\bar{L}$| C) = 1 – 2/9 = 7/9; P ($\bar{L}$| S) = 1 – 1/9 = 8/9

P ($\bar{L}$| B) = 1 – 4/9 = 5/9; P ($\bar{L}$| T) = 1 – 1/9 = 8/9

Substituting these values in eqn. (i) we get

P (C| $\bar{L}$) = 7/9 x 1/7 / 7/9 x 1/7 + 8/9 x 3/7 + 5/9 x 2/7 + 8/9 x 1/7

= 7/7 + 24 + 10 + 8 = 7/49 = 1/7.