C–HENT–N–LBSTB

INDIAN FOREST SERVICE P (EXAM)-2014

# MATHEMATICS

## PAPER—II

| Time Allowed : Three Hours | Maximum Marks : 200 |
|---|---|

### QUESTION PAPER SPECIFIC INSTRUCTIONS

**Please read each of the following instructions carefully before attempting questions**

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in chronological order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in ENGLISH only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

## SECTION—A

1. Answer the following :

   (a) If $G$ is a group in which $(a \cdot b)^4 = a^4 \cdot b^4$, $(a \cdot b)^5 = a^5 \cdot b^5$ and $(a \cdot b)^6 = a^6 \cdot b^6$, for all $a, b \in G$, then prove that $G$ is Abelian. 8

   (b) Let $f$ be defined on $[0, 1]$ as

   $$f(x) = \begin{cases} \sqrt{1-x^2}, & \text{if } x \text{ is rational} \\ 1-x \quad, & \text{if } x \text{ is irrational} \end{cases}$$

   Find the upper and lower Riemann integrals of $f$ over $[0, 1]$. 8

   (c) Using Cauchy integral formula, evaluate

   $$\int_C \frac{z+2}{(z+1)^2(z-2)}\, dz$$

   where $C$ is the circle $|z-i| = 2$. 8

   (d) Obtain the initial basic feasible solution for the transportation problem by North-West corner rule : 8

*Retail Shop*

| | | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | *Supply* |
|---|---|---|---|---|---|---|---|
| | $F_1$ | 1 | 9 | 13 | 36 | 51 | 50 |
| *Factory* | $F_2$ | 24 | 12 | 16 | 20 | 1 | 100 |
| | $F_3$ | 14 | 35 | 1 | 23 | 26 | 150 |
| | | 100 | 70 | 50 | 40 | 40 | |

   (e) Find the constants $a$, $b$, $c$ such that the function

   $$f(z) = 2x^2 - 2xy - y^2 + i(ax^2 - bxy + cy^2)$$

   is analytic for all $z (= x + iy)$ and express $f(z)$ in terms of $z$. 8

2. (a) Let $J_n$ be the set of integers mod $n$. Then prove that $J_n$ is a ring under the operations of addition and multiplication mod $n$. Under what conditions on $n$, $J_n$ is a field? Justify your answer. 10

   (b) Show that the function $f(x) = \sin \frac{1}{x}$ is continuous but not uniformly continuous on $(0, \pi)$. 15

   (c) Evaluate : 15

   $$\int_{|z|=1} \frac{z}{z^4 - 6z^2 + 1}\, dz$$

**3.** *(a)* Let $R$ be an integral domain with unity. Prove that the units of $R$ and $R[x]$ are same. 10

*(b)* Change the order of integration and evaluate $\int_{-2}^{1}\int_{y^2}^{2-y} dx\,dy$. 15

*(c)* Find the bilinear transformations which map the points $-1, \infty, i$ into the points—

*(i)* $i, 1, 1+i$

*(ii)* $\infty, i, 1$

*(iii)* $0, \infty, 1$ 15

**4.** *(a)* Show that the function $f(x) = \sin x$ is Riemann integrable in any interval $[0, t]$ by taking the partition $P = \left\{0, \frac{t}{n}, \frac{2t}{n}, \frac{3t}{n}, \ldots, \frac{nt}{n}\right\}$ and $\int_0^t \sin x\,dx = 1 - \cos t$. 10

*(b)* Find the Laurent series expansion at $z = 0$ for the function

$$f(z) = \frac{1}{z^2(z^2 + 2z - 3)}$$

in the regions *(i)* $1 < |z| < 3$ and *(ii)* $|z| > 3$. 15

*(c)* Solve the following LPP graphically :

$$\begin{aligned} \text{Maximize} \quad & Z = 3x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + x_2 \le 6 \\ & x_1 - x_2 \le 2 \\ & x_2 \le 4 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Write the dual problem of the above and obtain the optimal value of the objective function of the dual without actually solving it. 15

## SECTION—B

**5.** Answer the following :

*(a)* Use Lagrange's formula to find the form of $f(x)$ from the following table : 8

| $x$ | 0 | 2 | 3 | 6 |
|---|---|---|---|---|
| $f(x)$ | 648 | 704 | 729 | 792 |

*(b)* Write a program in BASIC to integrate

$$\int_0^1 e^{-2x} \sin x\,dx$$

by Simpson's $\frac{1}{3}$rd rule with 20 subintervals. 8

*(c)* Show that the general solution of the pde

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$$

is of the form $Z(x, y) = F(x+ct) + G(x-ct)$, where $F$ and $G$ are arbitrary functions. 8

*(d)* Prove that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving in the absence of an external force satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = (\vec{\Omega}\cdot\nabla)\vec{q} + \nu\nabla^2\vec{\Omega}$$

where $\vec{q}$ is the velocity vector with $\vec{\Omega} = \nabla \times \vec{q}$. 8

*(e)* Find the condition that $f(x, y, \lambda) = 0$ should be a possible system of streamlines for steady irrotational motion in two dimensions, where $\lambda$ is a variable parameter. 8

**6.** *(a)* Verify that the differential equation

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$$

is integrable and find its primitive. 10

*(b)* Show that the moment of inertia of a uniform rectangular mass $M$ and sides $2a$ and $2b$ about a diagonal is $\dfrac{2Ma^2b^2}{3(a^2+b^2)}$. 10

*(c)* The values of $f(x)$ for different values of $x$ are given as $f(1) = 4$, $f(2) = 5$, $f(7) = 5$ and $f(8) = 4$. Using Lagrange's interpolation formula, find the value of $f(6)$. Also find the value of $x$ for which $f(x)$ is optimum. 10

*(d)* Write a BASIC program to sum the series $S = 1 + x + x^2 + \ldots + x^n$, for $n = 30, 60$ and $90$ for the values of $x = 0\cdot1\ (0\cdot1)\ 0\cdot3$. 10

**7.** *(a)* Solve : 10

$$(D - 3D' - 2)^2 z = 2e^{2x}\cot(y+3x)$$

*(b)* Solve the following system of equations : 15

$$\begin{aligned} 2x_1 + x_2 + x_3 - 2x_4 &= -10 \\ 4x_1 + 2x_3 + x_4 &= 8 \\ 3x_1 + 2x_2 + 2x_3 &= 7 \\ x_1 + 3x_2 + 2x_3 - x_4 &= -5 \end{aligned}$$

*(c)* A uniform rod $OA$ of length $2a$ is free to turn about its end $O$, revolves with uniform angular velocity $\omega$ about a vertical axis $OZ$ through $O$ and is inclined at a constant angle $\alpha$ to $OZ$. Show that the value of $\alpha$ is either zero or

$$\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$$ 15

**8.** *(a)* Using Runge-Kutta 4th order method, find $y$ from

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with $y(0) = 1$ at $x = 0 \cdot 2,\ 0 \cdot 4$. 10

*(b)* A plank of mass $M$ is initially at rest along a straight line of greatest slope of a smooth plane inclined at an angle $\alpha$ to the horizon and a man of mass $M'$ starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time

$$\sqrt{\frac{2M'a}{(M + M')g\sin\alpha}}$$

where $a$ is the length of the plank. 15

*(c)* Prove that

$$\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$$

is a possible form for the bounding surface of a liquid and find the velocity components. 15

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