I.E.S./I.S.S. Examination ⇒ 2011

E-JTT-L-TUB

STATISTICS – II

Time Allowed : Three Hours | *Maximum Marks : 200*

INSTRUCTIONS

Candidates should attempt FIVE questions in ALL including Questions No. 1 and 5 which are compulsory. The remaining THREE questions should be answered by choosing at least ONE question each from Section A and Section B.

The number of marks carried by each question is indicated against each.

Answers must be written only in ENGLISH.

(Symbols and abbreviations are as usual.)

Any essential data assumed by candidates for answering questions must be clearly stated.

Two graph sheets are attached to this question paper for answering graph-related questions. Candidate is expected to carefully detach these and attach them securely to the answer book.

SECTION A

1. Attempt any *five* parts of the following : *8×5=40*

(a) Obtain the least square estimates of the parameters in a simple linear regression model, where the errors are i.i.d. normal variates. Check whether they are unbiased.

(b) Show that $l'\hat{\beta}$ has minimum variance in the class of linear unbiased estimators of $l'\beta$, if $l'\beta$ is estimable, for the model $(Y, X\beta, \sigma^2 I)$.

(c) For the linear model of one way analysis of variance, derive the test for testing equality of the parameters.

(d) Show that, a statistic T_n for θ is consistent, if $E(T_n) \to \theta$ and $Var(T_n) \to 0$, as $n \to \infty$.

(e) Find the MLE for the parameter θ based on samples from a uniform distribution over $[\theta - Y_2, \theta + Y_2]$.

(f) Obtain the MVB estimator for the parameter θ for the population

$$f(x) = \frac{1}{\theta^p \Gamma(p)} e^{-x/\theta} x^{p-1}; 0 \le x < \infty; \text{ given } p > 0.$$

2. (a) Show that the best fitting linear function for the points $(x_1, y_1), \ldots, (x_n, y_n)$ satisfy the equation

$$\begin{vmatrix} x & y & 1 \\ \sum_i x_i & \sum_i y_i & n \\ \sum_i x_i^2 & \sum_i x_i y_i & \sum_i x_i \end{vmatrix} = 0, \quad i = 1, \ldots, n.$$

10

(b) Considering the sum of the angles is 180°, find the best estimates of the three angles A, B, C for the following data by the method of least squares :

A	:	35°	40°	45°
B	:	60°	62°	58°
C	:	83°	80°	77°

10

(c) Obtain an unbiased estimator of $\frac{1}{\theta}$ for

$$f(x; \theta) = \theta (1 - \theta)^{x-1}; \; x = 1, 2, ...; \; 0 < \theta < 1.$$

Also, what is its distribution ? 10

(d) If T_1 is the most efficient estimator and T_2 is any other estimator of θ with efficiency 'e', show that

$$\text{Var}(T_1 - T_2) = \left(\frac{1}{e} - 1\right) \text{Var}(T_1).$$

10

3. (a) For the linear model $Y = X\beta + \varepsilon$ and A and B matrix of constants, show that,

$$X'XA = X'XB, \text{ iff } XA = XB.$$

10

(b) For the one-way model

$y_{ij} = \mu + \alpha_i + l_{ij}$; $i = 1, 2, 3$; $j = 1, 2, ..., n_i$; $E(l_{ij}) = 0$, check whether $\mu + \alpha_1$ and $\alpha_1 - \alpha_3$ are estimable, when $n_i = 4 - i$. 10

(c) Show that the mean square deviation $E(\hat{\theta} - \theta)^2$ of an estimator $\hat{\theta}$ of θ can never fall below a positive limit, when the range is independent of θ and the second derivative of the likelihood function exists. 10

(d) Find the MLE for N for the hypergeometric distribution

$$f(x) = \begin{cases} \dfrac{\dbinom{M}{x}\dbinom{N-M}{n-x}}{\dbinom{N}{n}}, & \max(0, n-N+M) \le x \le \min(n, M) \\ 0 & , \text{ otherwise.} \end{cases}$$

10

4. (a) For the simple linear model

$$y_i = \beta_0 + \beta_1 x_i + l_i, \quad i = 1, \ldots, N$$

obtain the variance of $\hat{\beta}_1$. *10*

(b) Write a note on Bhattacharya's bounds. Explain how it is a generalization of the Cramer – Rao inequality. *10*

(c) If two most efficient estimators is distributed in the bivariate normal form (in the limit), show that the correlation between them tends to unity. *10*

(d) Let $X_1, \ldots, X_n$ denote a sample from $b(1, \theta)$, $0 < \theta < 1$ where

$$h(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)}\ \theta^{\alpha-1}(1-\theta)^{\beta-1},\ 0 < \theta < 1.$$

Show that, the Bayes decision rule for the quadratic loss function, is a weighted average of the MLE of θ and the mean $\alpha / (\alpha + \beta)$ of the prior distribution. *10*

SECTION B

5. Attempt any ***five*** parts of the following : *8×5=40*

(a) Distinguish between randomized and non-randomized tests. Give an example.

(b) Obtain the MP size α test for testing $H_0 : p = p_0$ against $H_1 : p = p_1; p_1 > p_0$ based on a sample of size n from b(1, p).

(c) Explain the uses of control charts in quality control.

(d) What is double sampling inspection plan ? Suggest the general method of plotting the OC function of such a plan.

(e) Let $X \sim N_p(\mu, \Sigma)$. Find the marginal distribution of any sub vector of X.

(f) What is the Neyman – Pearson fundamental lemma ? Explain its usefulness in Testing of Statistical Hypotheses.

6. (a) Obtain the BCR for testing $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma = \sigma_1$ based on samples from $N(0, \sigma^2)$. *10*

(b) What is an unbiased test ? Give an example. *10*

(c) What do you understand by producer's risk and consumer's risk in single sampling inspection plans ? Explain their importance. *10*

(d) Find the MLE's for μ and Σ for the population $N_p(\mu, \Sigma)$. *10*

7. (a) Obtain the OC for the normal distribution with unit variance while testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$. *10*

(b) What are AOQ and AOQL ? Explain their uses. *10*

(c) Derive the null distribution of sample correlation coefficient based on samples from a bivariate normal population. *10*

(d) How will you test the equality of the components of a mean vector in a multivariate normal population ? *10*

8. (a) Test the hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ based on a sample of size n from a uniform distribution in the interval $0 \leq x \leq \theta$. *10*

(b) What are sequential sampling plans ? Explain one of them. *10*

(c) Obtain the distribution of the sample variance and covariance matrix in samples from a p-variate normal population. *10*

(d) What is dimension reduction technique ? (Use principal components to explain the technique.) *10*

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