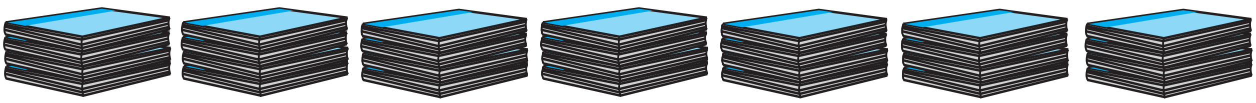
6 Indices

Let's recall.

Each of 7 children was given 4 books.

Total notebooks = 4 + 4 + 4 + 4 + 4 + 4 + 4 = 28 notebooks



Here, addition is the operation that is carried out repeatedly.

Addition of the same number again and again can be shown as a multiplication. Total notebooks = 4 + 4 + 4 + 4 + 4 + 4 + 4 = 4 × 7 = 28

Let's learn.

Base and Index

Let us see how the multiplication of a number by itself several times is expressed in short.

2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 : Here, 2 is multiplied by itself 8 times.

This is written as 2^8 in short. This is the index form of the multiplication.

Here, 2 is called the **base** and 8, the **index** or the **exponent**.

8 ⟵ Index
2 ⟵ Base

Example 5 × 5 × 5 × 5 = 5^4 Here 5^4 is in the index form.

In the number 5^4, 5 is the base and 4 is the index.

This is read as '5 raised to the power 4' or '5 raised to 4', or 'the 4th power of 5'.

Generally, if a is any number, $a \times a \times a \times$.......... ($m$ times) = a^m

Read a^m as 'a raised to the power m' or 'the m^{th} power of a'.

Here m is a natural number.

$\therefore$ $5^4 = 5 \times 5 \times 5 \times 5 = 625$. Or, the value of the number 5^4 = 625.

Similarly, $\left[\frac{-2}{3}\right]^3 = \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} = \frac{-8}{27}$ means that the value of $\left[\frac{-2}{3}\right]^3$ is $\frac{-8}{27}$.

Note that $7^1 = 7$, $10^1 = 10$. **The first power of any number is that number itself.** If the power or index of a number is 1, the convention is not to write it.

Thus $5^1 = 5$, $a^1 = a$.

Practice Set 26

1. Complete the table below.

Sr. No.	Indices (Numbers in index form)	Base	Index	Multiplication form	Value
(i)	3^4	3	4	$3 \times 3 \times 3 \times 3$	81
(ii)	16^3				
(iii)		(-8)	2		
(iv)				$\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}$	$\frac{81}{2401}$
(v)	$(-13)^4$				

2. Find the value.

(i) 2^{10} (ii) 5^3 (iii) $(-7)^4$ (iv) $(-6)^3$ (v) 9^3

(vi) 8^1 (vii) $\left(\frac{4}{5}\right)^3$ (viii) $\left(-\frac{1}{2}\right)^4$

Square and Cube

$3^2 = 3 \times 3$

3^2 is read as '3 raised to 2'

or 3 'squared' or 'the square of 3'

$5^3 = 5 \times 5 \times 5$

5^3 is read as '5 raised to 3'

or '5 cubed' or 'the cube of 5'.

Remember :

The second power of any number is the square of that number.
The third power of any number is the cube of that number.

Let's learn. Multiplication of Indices with the Same Base.

Example $2^4 \times 2^3$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^7$

Therefore, $2^4 \times 2^3 = 2^{4+3} = 2^7$

Example $(-3)^2 \times (-3)^3$

$= (-3) \times (-3) \times (-3) \times (-3) \times (-3)$

$= (-3)^5$

Therefore, $(-3)^2 \times (-3)^3 = (-3)^{2+3} = (-3)^5$

Example $\left(\frac{-2}{5}\right)^2 \times \left(\frac{-2}{5}\right)^3 = \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) = \left(\frac{-2}{5}\right)^5$

Therefore, $\left(\frac{-2}{5}\right)^2 \times \left(\frac{-2}{5}\right)^3 = \left(\frac{-2}{3}\right)^{2+3} = \left(\frac{-2}{5}\right)^5$

Now I know !

If a is a rational number and m and n are positive integers, then $a^m \times a^n = a^{m+n}$

Practice Set 27

(1) Simplify.

(i) $7^4 \times 7^2$ (ii) $(-11)^5 \times (-11)^2$ (iii) $\left(\frac{6}{7}\right)^3 \times \left(\frac{6}{7}\right)^5$

(iv) $\left(-\frac{3}{2}\right)^5 \times \left(-\frac{3}{2}\right)^3$ (v) $a^{16} \times a^7$ (vi) $\left(\frac{P}{5}\right)^3 \times \left(\frac{P}{5}\right)^7$

Let's learn. Division of Indices with the Same Base

Example $6^4 \div 6^2 = ?$

$$\frac{6^4}{6^2} = \frac{6\times6\times6\times6}{6\times6}$$

$$= 6 \times 6$$

$$= 6^2$$

$$\therefore\ 6^4 \div 6^2 = 6^{4-2} = 6^2$$

Example $(-2)^5 \div (-2)^3 = ?$

$$\frac{(-2)^5}{(-2)^3} = \frac{(-2)\times(-2)\times(-2)\times(-2)\times(-2)}{(-2)\times(-2)\times(-2)}$$

$$= (-2)^2$$

$$\therefore\ (-2)^5 \div (-2)^3 = (-2)^2$$

Now I know !

If a is a non-zero rational number, m and n are positive integers and $m > n$, then $\frac{a^m}{a^n} = a^{m-n}$

The meaning of a^0

If $a \neq 0$

Then $\frac{a^m}{a^m} = 1$

Also, $\frac{a^m}{a^m} = a^{m-m} = a^0$

$$\therefore\ \boxed{a^0 = 1}$$

The meaning of a^{-m}

$$a^{-m} = a^{-m} \times 1$$

$$= a^{-m} \times \frac{a^m}{a^m}$$

$$= \frac{a^{-m+m}}{a^m}$$

$$= \frac{a^0}{a^m} = \frac{1}{a^m}$$

$$\boxed{a^{-m} = \frac{1}{a^m}}$$

$a^{-m} = \frac{1}{a^m}$ $\therefore$ $a^{-1} = \frac{1}{a}$

$a \times \frac{1}{a} = 1$, $\therefore$ $a \times a^{-1} = 1$

$\therefore$ a^{-1} is the multiplicative inverse of a.

Thus, the multiplicative inverse of $\frac{5}{3}$ is $\frac{3}{5}$.

$$\therefore\ \boxed{\left(\frac{5}{3}\right)^{-1} = \frac{3}{5}}$$

Example Let us consider $\left(\frac{4}{7}\right)^{-3} \cdot \left(\frac{4}{7}\right)^{-3} = \dfrac{1}{\frac{4}{7}\times\frac{4}{7}\times\frac{4}{7}} = \dfrac{1}{\frac{64}{343}} = \dfrac{343}{64} = \left(\frac{7}{4}\right)^{3}$

Now I know!

Hence, we get the rule that if $a \neq 0$, $b \neq 0$ and m is a positive integer, then $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$

Let us see what rule we get by observing the following examples :

Example $(3)^4 \div (3)^6$

$= \dfrac{3^4}{3^6}$

$= \dfrac{3\times3\times3\times3}{3\times3\times3\times3\times3\times3} = \dfrac{1}{3^2}$

$3^4 \div 3^6 = 3^{4-6} = 3^{-2}$

Example $\left(\frac{3}{5}\right)^{2} \div \left(\frac{3}{5}\right)^{5}$

$= \dfrac{\frac{3}{5}\times\frac{3}{5}}{\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}} = \dfrac{1}{\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}} = \dfrac{1}{\left(\frac{3}{5}\right)^{3}}$

$\therefore \left(\frac{3}{5}\right)^{2} \div \left(\frac{3}{5}\right)^{5} = \left(\frac{3}{5}\right)^{2-5} = \left(\frac{3}{5}\right)^{-3}$

Now I know!

If a is a rational number, $a \neq 0$ and m and n are positive integers, then $\dfrac{a^m}{a^n} = a^{m-n}$

Let's learn.

Observe what happens if the base is (– 1) and the index is a whole number.

$(-1)^6 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = 1 \times 1 \times 1 = 1$

$(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = 1 \times 1 \times (-1) = -1$

If m is an even number then $(-1)^m = 1$, and if m is an odd number, then $(-1)^m = -1$

Practice Set 28

1. Simplify.

(i) $a^6 \div a^4$ (ii) $m^5 \div m^8$ (iii) $p^3 \div p^{13}$ (iv) $x^{10} \div x^{10}$

2. Find the value.

(i) $(-7)^{12} \div (-7)^{12}$ (ii) $7^5 \div 7^3$ (iii) $\left(\frac{4}{5}\right)^{3} \div \left(\frac{4}{5}\right)^{2}$ (iv) $4^7 \div 4^5$

Let's learn.

The Index of the Product or Quotient of Two Numbers

Let us observe the following examples to see what rule we get.

Example $(2 \times 3)^4$

$= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^4 \times 3^4$

Example $\left(\frac{4}{5}\right)^3$

$= \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

$= \frac{4\times4\times4}{5\times5\times5} = \frac{4^3}{5^3}$

Now I know!

If a and b are non-zero rational numbers and m is an integer, then

(1) $(a \times b)^m = a^m \times b^m$ (2) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$\left(a^m\right)^n$, that is, the Power of a Number in Index Form

Example $\left(5^2\right)^3$

$= 5^2 \times 5^2 \times 5^2$

$= 5^{2+2+2}$

$= 5^{2\times3}$

$= 5^6$

Example $\left(7^{-2}\right)^{-5} = \frac{1}{\left(7^{-2}\right)^5}$

$= \frac{1}{7^{-2}\times7^{-2}\times7^{-2}\times7^{-2}\times7^{-2}}$

$= \frac{1}{7^{(-2)\times5}}$

$= \frac{1}{7^{-10}} = 7^{10}$

$$a^{-m} = \frac{1}{a^m}$$

Example $\left(\left(\frac{2}{5}\right)^{-2}\right)^3$

$= \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} = \left(\frac{2}{5}\right)^{(-2)+(-2)+(-2)} = \left(\frac{2}{5}\right)^{-6}$

$\left(a^m\right)^n = a^m \times a^m \times a^m \times \dots\dots\dots n \text{ times} = a^{m+m+m\dots\dots\dots n \text{ times}} = a^{m\times n}$

From the above examples, we get the following rule.

Now I know!

If a is a non-zero rational number and m and n are integers, then $\left(a^m\right)^n = a^{m\times n} = a^{mn}$

Laws of Indices

Remember :

If a is a non-zero number and m and n are integers, then

$$a^m \times a^n = a^{m+n}, \quad a^m \div a^n = a^{m-n}, \quad a^1 = a, \quad a^0 = 1, \quad a^{-m} = \frac{1}{a^m},$$

$$(ab)^m = a^m \times b^m, \quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \quad (a^m)^n = a^{mn}, \quad \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

Practice Set 29

1. Simplify.

(i) $\left[\left(\frac{15}{12}\right)^3\right]^4$ (ii) $\left(3^4\right)^{-2}$ (iii) $\left(\left(\frac{1}{7}\right)^{-3}\right)^4$ (iv) $\left(\left(\frac{2}{5}\right)^{-2}\right)^{-3}$ (v) $\left(6^5\right)^4$

(vi) $\left[\left(\frac{6}{7}\right)^5\right]^2$ (vii) $\left[\left(\frac{2}{3}\right)^{-4}\right]^5$ (viii) $\left[\left(\frac{5}{8}\right)^3\right]^{-2}$ (ix) $\left[\left(\frac{3}{4}\right)^6\right]^1$ (x) $\left[\left(\frac{2}{5}\right)^{-3}\right]^2$

2. Write the following numbers using positive indices.

(i) $\left(\frac{2}{7}\right)^{-2}$ (ii) $\left(\frac{11}{3}\right)^{-5}$ (iii) $\left(\frac{1}{6}\right)^{-3}$ (iv) $(y)^{-4}$

My friend, Maths : In science, in astronomy.

The powers of 10 are especially useful in writing numbers in the decimal system.

(1) The distance between Earth and Moon is 38,40,00,000 m. It can be expressed using the powers of 10 as follows.

$384000000 = 384 \times 10^6$

$384000000 = 38.4 \times 10^7$

$384000000 = 3.84 \times 10^8$ (Standard form)

When writing a very large or a very small number, it is expressed as the product of a decimal fraction with a one-digit integer and the proper power of 10. This is known as the standard form of the number.

(2) The diameter of an oxygen atom is given below in millimetres.

$0.0000000000000356 = 3.56 \times 10^{-14}$

(3) Try to write the following numbers in the standard form.

The diameter of the sun is 1400000000 m.

The velocity of light is 300000000 m/sec.

(4) The box alongside shows the number called Googol. Try to write it as a power of 10.

Googol

10000000000000000000000000
0000000000000000000000000
0000000000000000000000000
0000000000000000000000000

Let's recall.

Finding the square root of a perfect square

When a number is multiplied by itself the product obtained is the square of the number.

Example $6 \times 6 = 6^2 = 36$

$6^2 = 36$ is read as 'The square of 6 is 36.'

Example $(-5) \times (-5) = (-5)^2 = 25$

$(-5)^2 = 25$ is read as 'The square of (-5) is 25.'

Let's learn.

✸ Finding the square root of a given number

Example $3 \times 3 = 3^2 = 9$ Here, the square of 3 is 9.

Or, we can say that the square root of 9 is 3.

The symbol $\sqrt{\ }$ is used for 'square root'.

$\sqrt{9}$ means the square root of 9. $\therefore \sqrt{9} = 3$

Example $7 \times 7 = 7^2 = 49$ $\therefore \sqrt{49} = 7$

Example $8 \times 8 = 8^2 = 64$. Hence $\sqrt{64} = 8$

$(-8) \times (-8) = (-8)^2 = 64$. Hence, $\sqrt{64} = -8$.

Thus, if x is a positive number, it has two square roots.

Of these, the negative square root is shown as $-\sqrt{x}$ and the positive one as $\sqrt{x}$.

Example Find the square root of 81.

$81 = 9 \times 9 = -9 \times -9$ $\therefore \sqrt{81} = 9$ and $-\sqrt{81} = -9$

Mostly, we consider the positive square root.

✸ Finding the square root by the factors method

Example Find the square root of 144.

Find the prime factors of the given number and put them in pairs of equal numbers.

2	144
2	72
2	36
2	18
3	9
3	3
	1

$144 = 2 \times 72$

$= 2 \times 2 \times 36$

$= 2 \times 2 \times 2 \times 18$

$= \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

Form pairs of equal factors from the prime factors obtained.

Take one factor from each pair and multiply.

$\sqrt{144} = 2 \times 2 \times 3 = 12$ $\therefore \sqrt{144} = 12$

Example Find the square root of 324.

Find the prime factors of the given number and put them in pairs of equal factors.

2	324
2	162
3	81
3	27
3	9
3	3
	1

$324 = 2 \times 162$

$= 2 \times 2 \times 81$

$= 2 \times 2 \times 3 \times 27$

$= 2 \times 2 \times 3 \times 3 \times 9$

$= \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

To find the square root, take one number from each pair and multiply.

$\sqrt{324} = 2 \times 3 \times 3 = 18$ $\quad\quad \therefore \sqrt{324} = 18$

Practice Set 30

⊙ Find the square root. (i) 625 (ii) 1225 (iii) 289 (iv) 4096 (v) 1089

✸ Something more (Square root by the division method)

Find the square root of :

(1) 9801

	99
9	$\overline{98}\,\overline{01}$
+ 9	− 81
189	1701
+ 9	− 1701
198	0000

$\sqrt{9801} = 99$

(2) 19321

	139
1	$\overline{1}\,\overline{93}\,\overline{21}$
+ 1	− 1
23	093
+ 3	− 69
269	2421
+ 9	− 2421
278	0000

(3) 141.61

	11.9
1	$\overline{1}\,\overline{41}.\overline{61}$
+ 1	− 1
21	041
+ 1	− 21
229	2061
+ 9	− 2061
238	0000

This method can be used to find the square root of numbers which have many prime factors and are, therefore, difficult to factorise.

Now let us take $\sqrt{137}$ to see one more use.

	11.7
1	$\overline{1}\,\overline{37}.\overline{00}$
+ 1	− 1
21	037
+ 1	− 21
227	1600
+ 7	− 1589
234	11

$\sqrt{137} > 11.7$

But $(11.8)^2 = 139.24$

$\therefore 11.7 < \sqrt{137} < 11.8$

Thus, we can find the approximate value of $\sqrt{137}$. This method can be used to find the approximate square root of a number whose square root is not a whole number.

❑❑❑