5 Expansion formulae

Let's recall.

We have studied the following expansion formulae in previous standard.

(i) $(a + b)^2 = a^2 + 2ab + b^2$, (ii) $(a - b)^2 = a^2 - 2ab + b^2$,

(iii) $(a + b)(a - b) = a^2 - b^2$

Use the above formulae to fill proper terms in the following boxes.

(i) $(x + 2y)^2 = x^2 + \square + 4y^2$

(ii) $(2x - 5y)^2 = \square - 20xy + \square$

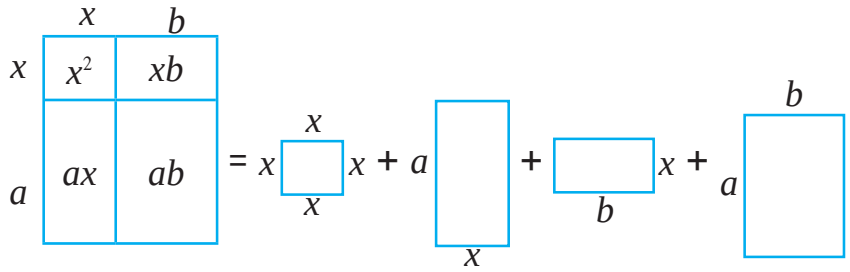
(iii) $(101)^2 = (100 + 1)^2 = \square + \square + 1^2 = \square$

(iv) $(98)^2 = (100 - 2)^2 = 10000 - \square + \square = \square$

(v) $(5m + 3n)(5m - 3n) = \square - \square = \square - \square$

Let's learn.

Activity : Expand $(x + a)(x + b)$ using formulae for areas of a square and a rectangle.



$(x + a)(x + b) = x^2 + ax + bx + ab$

$(x + a)(x + b) = x^2 + (a + b)x + ab$

(I) Expansion of $(x + a)(x + b)$

$(x + a)$ and $(x + b)$ are binomials with one term in common. Let us multiply them.

$(x + a)(x + b) = x(x + b) + a(x + b) = x^2 + bx + ax + ab$

$= x^2 + (a + b)x + ab$

$\therefore$ $\boxed{(x + a)(x + b) = x^2 + (a + b)x + ab}$

Expand

Ex. (1) $(x + 2)(x + 3) = x^2 + (2 + 3)x + (2 \times 3) = x^2 + 5x + 6$

Ex. (2) $(y + 4)(y - 3) = y^2 + (4 - 3)y + (4) \times (-3) = y^2 + y - 12$

Ex. (3) $(2a + 3b)(2a - 3b) = (2a)^2 + [(3b) + (-3b)]2a + [3b \times (-3b)]$

$= 4a^2 + 0 \times 2a - 9b^2 = 4a^2 - 9b^2$

Ex. (4) $\left(m+\frac{3}{2}\right)\left(m+\frac{1}{2}\right) = m^2 + \left(\frac{3}{2}+\frac{1}{2}\right)m + \frac{3}{2} \times \frac{1}{2} = m^2 + 2m + \frac{3}{4}$

Ex. (5) $(x - 3)(x - 7) = x^2 + (-3 - 7)x + (-3)(-7) = x^2 - 10x + 21$

Practice Set 5.1

1. Expand.

(1) $(a + 2)(a - 1)$ (2) $(m - 4)(m + 6)$ (3) $(p + 8)(p - 3)$

(4) $(13 + x)(13 - x)$ (5) $(3x + 4y)(3x + 5y)$ (6) $(9x - 5t)(9x + 3t)$

(7) $\left(m+\frac{2}{3}\right)\left(m-\frac{7}{3}\right)$ (8) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$ (9) $\left(\frac{1}{y}+4\right)\left(\frac{1}{y}-9\right)$

Let's learn.

(II) Expansion of $(a + b)^3$

$(a + b)^3 = (a + b)\,(a + b)\,(a + b) = (a + b)\,(a + b)^2$

$= (a + b)(a^2 + 2ab + b^2)$

$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$

$= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$

$\therefore$ $\boxed{(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3}$

Let us study some examples based on the above expansion formula.

Ex. (1) $(x + 3)^3$

We know that $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

In the given example, $a = x$ and $b = 3$

$\therefore\ (x+3)^3 = (x)^3 + 3\times x^2\times 3 + 3\times x\times (3)^2 + (3)^3$

$= x^3 + 9x^2 + 27x + 27$

Ex. (2) $(3x+4y)^3 = (3x)^3 + 3(3x)^2(4y) + 3(3x)(4y)^2 + (4y)^3$

$= 27x^3 + 3\times 9x^2\times 4y + 3\times 3x\times 16y^2 + 64y^3$

$= 27x^3 + 108x^2y + 144xy^2 + 64y^3$

Ex. (3) $\left(\frac{2m}{n}+\frac{n}{2m}\right)^3 = \left(\frac{2m}{n}\right)^3 + 3\left(\frac{2m}{n}\right)^2\left(\frac{n}{2m}\right) + 3\left(\frac{2m}{n}\right)\left(\frac{n}{2m}\right)^2 + \left(\frac{n}{2m}\right)^3$

$= \frac{8m^3}{n^3} + 3\left(\frac{4m^2}{n^2}\right)\left(\frac{n}{2m}\right) + 3\left(\frac{2m}{n}\right)\left(\frac{n^2}{4m^2}\right) + \frac{n^3}{8m^3}$

$= \frac{8m^3}{n^3} + \frac{6m}{n} + \frac{3n}{2m} + \frac{n^3}{8m^3}$

Ex. (4) $(41)^3 = (40+1)^3 = (40)^3 + 3\times(40)^2\times 1 + 3\times 40\times(1)^2 + (1)^3$

$= 64000 + 4800 + 120 + 1 \quad = 68921$

Practice Set 5.2

1. Expand.

(1) $(k+4)^3$ (2) $(7x+8y)^3$ (3) $(7+m)^3$ (4) $(52)^3$

(5) $(101)^3$ (6) $\left(x+\frac{1}{x}\right)^3$ (7) $\left(2m+\frac{1}{5}\right)^3$ (8) $\left(\frac{5x}{y}+\frac{y}{5x}\right)^3$

Activity : Make two cubes of side a and of side b each. Make six parallelopipeds; three of them measuring $a\times a\times b$ and the remaining three measuring $b\times b\times a$. Arrange all these solid figures properly and make a cube of side $(a+b)$.

Let's learn.

(III) Expansion of $(a-b)^3$

$\therefore\ (a-b)^3 = (a-b)(a-b)(a-b) = (a-b)(a-b)^2$

$= (a-b)(a^2-2ab+b^2)$

$= a(a^2-2ab+b^2) - b(a^2-2ab+b^2)$

$= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3$

$= a^3 - 3a^2b + 3ab^2 - b^3$

$\therefore$ $\boxed{(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3}$

Ex. (1) Expand $(x - 2)^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ Here taking , $a = x$ and $b = 2$,

$(x - 2)^3 = (x)^3 - 3 \times x^2 \times 2 + 3 \times x \times (2)^2 - (2)^3$

$= x^3 - 6x^2 + 12x - 8$

Ex. (2) Expand $(4p - 5q)^3$.

$(4p - 5q)^3 = (4p)^3 - 3(4p)^2(5q) + 3(4p)(5q)^2 - (5q)^3$

$(4p - 5q)^3 = 64p^3 - 240p^2q + 300pq^2 - 125q^3$

Ex. (3) Find cube of 99 using the expansion formula.

$(99)^3 = (100 - 1)^3 = (100)^3 - 3 \times (100)^2 \times 1 + 3 \times 100 \times (1)^2 - 1^3$

$= 1000000 - 30000 + 300 - 1 = 9,70,299$

Ex. (4) Simplify.

(i) $(p + q)^3 + (p - q)^3 = p^3 + 3p^2q + 3pq^2 + q^3 + p^3 - 3p^2q + 3pq^2 - q^3$

$= 2p^3 + 6pq^2$

(ii) $(2x + 3y)^3 - (2x - 3y)^3$

$= [(2x)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2 + (3y)^3]$

$- [(2x)^3 - 3(2x)^2(3y) + 3(2x)(3y)^2 - (3y)^3]$

$= (8x^3 + 36x^2y + 54xy^2 + 27y^3) - (8x^3 - 36x^2y + 54xy^2 - 27y^3)$

$= 8x^3 + 36x^2y + 54xy^2 + 27y^3 - 8x^3 + 36x^2y - 54xy^2 + 27y^3$

$= 72x^2y + 54y^3$

Now I know.

(i) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a + b)$

(ii) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a - b)$

Practice Set 5.3

1. Expand.

(1) $(2m - 5)^3$ (2) $(4 - p)^3$ (3) $(7x - 9y)^3$ (4) $(58)^3$

(5) $(198)^3$ (6) $\left(2p - \frac{1}{2p}\right)^3$ (7) $\left(1 - \frac{1}{a}\right)^3$ (8) $\left(\frac{x}{3} - \frac{3}{x}\right)^3$

2. Simplify.

(1) $(2a + b)^3 - (2a - b)^3$ (2) $(3r - 2k)^3 + (3r + 2k)^3$

(3) $(4a - 3)^3 - (4a + 3)^3$ (4) $(5x - 7y)^3 + (5x + 7y)^3$

Let's learn.

(IV) Expansion of $(a + b + c)^2$

$$(a + b + c)^2 = (a + b + c) \times (a + b + c)$$
$$= a(a + b + c) + b(a + b + c) + c(a + b + c)$$
$$= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$$
$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$\therefore \boxed{(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac.}$$

Ex. (1) Expand: $(p + q + 3)^2$

$= p^2 + q^2 + (3)^2 + 2 \times p \times q + 2 \times q \times 3 + 2 \times p \times 3$

$= p^2 + q^2 + 9 + 2pq + 6q + 6p \quad = p^2 + q^2 + 2pq + 6q + 6p + 9$

Ex. (2) Fill in the boxes with appropriate terms in the steps of expansion.

$(2p + 3m + 4n)^2$

$= (2p)^2 + (3m)^2 + \square + 2 \times 2p \times 3m + 2 \times \square \times 4n + 2 \times 2p \times \square$

$= \square + 9m^2 + \square + 12pm + \square + \square$

Ex. (3) Simplify $(l + 2m + n)^2 + (l - 2m + n)^2$

$= l^2 + 4m^2 + n^2 + 4lm + 4mn + 2ln + l^2 + 4m^2 + n^2 - 4lm - 4mn + 2ln$

$= 2l^2 + 8m^2 + 2n^2 + 4ln$

Practice Set 5.4

1. Expand. (1) $(2p + q + 5)^2$ (2) $(m + 2n + 3r)^2$

(3) $(3x + 4y - 5p)^2$ (4) $(7m - 3n - 4k)^2$

2. Simplify. (1) $(x - 2y + 3)^2 + (x + 2y - 3)^2$

(2) $(3k - 4r - 2m)^2 - (3k + 4r - 2m)^2$ (3) $(7a - 6b + 5c)^2 + (7a + 6b - 5c)^2$

Answers

Practice Set 5.1 (1) $a^2 + a - 2$ (2) $m^2 + 2m - 24$ (3) $p^2 + 5p - 24$

(4) $169 - x^2$ (5) $9x^2 + 27xy + 20y^2$ (6) $81x^2 - 18xt - 15t^2$

(7) $m^2 - \frac{5}{3}m - \frac{14}{9}$ (6) $x^2 - \frac{1}{x^2}$ (9) $\frac{1}{y^2} - \frac{5}{y} - 36$

Practice Set 5.2 (1) $k^3 + 12k^2 + 48k + 64$ (2) $343x^3 + 1176x^2y + 1344xy^2 + 512y^3$

(2) $343 + 147m + 21m^2 + m^3$ (4) 140608 (5) 1030301

(6) $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$ (7) $8m^3 + \frac{12m^2}{5} + \frac{6m}{25} + \frac{1}{125}$

(8) $\frac{125x^3}{y^3} + \frac{15x}{y} + \frac{3y}{5x} + \frac{y^3}{125x^3}$

Practice Set 5.3 **1.** (1) $8m^3 - 60m^2 + 150m - 125$ (2) $64 - 48p + 12p^2 - p^3$

(3) $343x^3 - 1323x^2y + 1701xy^2 - 729y^3$ (4) 1,95,112

(5) 77,62,392 (6) $8p^3 - 6p + \frac{3}{2p} - \frac{1}{8p^3}$

(7) $1 - \frac{3}{a} + \frac{3}{a^2} - \frac{1}{a^3}$ (8) $\frac{x^3}{27} - x + \frac{9}{x} - \frac{27}{x^3}$

2. (1) $24a^2b + 2b^3$ (2) $54r^3 + 72rk^2$

(3) $-288a^2 - 54$ (4) $250x^3 + 1470xy^2$

Practice Set 5.4 **1.** (1) $4p^2 + q^2 + 25 + 4pq + 10q + 20p$

(2) $m^2 + 4n^2 + 9r^2 + 4mn + 12nr + 6mr$

(3) $9x^2 + 16y^2 + 25p^2 + 24xy - 40py - 30px$

(4) $49m^2 + 9n^2 + 16k^2 - 42mn + 24nk - 56km$

2. (1) $2x^2 + 8y^2 + 18 - 24y$ (2) $32rm - 48kr$

(3) $98a^2 + 72b^2 + 50c^2 - 120bc$