SAMPLE QUESTION PAPER
CLASS-XII (2016-17)
MATHEMATICS (041)

Time allowed: **3** hours   Maximum Marks: **100**

General Instructions:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

SECTION-A

Questions from 1 to 4 are of 1 mark each.

1. What is the principal value of $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$?

2. A and B are square matrices of order 3 each, $|A| = 2$ and $|B| = 3$. Find $|3AB|$

3. What is the distance of the point (p, q, r) from the x-axis?

4. Let f : R → R be defined by $f(x) = 3x^2 – 5$ and g : R → R be defined by $g(x) = \frac{x}{x^2+1}$. Find gof

SECTION-B

Questions from 5 to 12 are of 2 marks each.

5. How many equivalence relations on the set {1,2,3} containing (1,2) and (2,1) are there in all ? Justify your answer.

6. Let l_i, m_i, n_i ; i = 1, 2, 3 be the direction cosines of three mutually perpendicular vectors in space. Show that $AA' = I_3$, where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.

7. If $e^y (x + 1) = 1$, show that $\frac{dy}{dx} = -e^y$

8. Find the sum of the order and the degree of the following differential equations:

$$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1 + x) = 0$$

9. Find the Cartesian and Vector equations of the line which passes through the point (–2, 4,–5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$

10. Solve the following Linear Programming Problem graphically:

Maximize $Z = 3x + 4y$

subject to

$x + y \leq 4, x \geq 0$ and $y \geq 0$

11. A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older child is a boy.

12. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long.

SECTION-C

Questions from 13 to 23 are of 4 marks each.

13. If A + B + C = π, then find the value of

$$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$

OR

Using properties of determinant, prove that

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

14. It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ Rolle's theorem holds in [1, 3] with c = $2 + \frac{1}{\sqrt{3}}$. Find the values of 'a' and 'b'

15. Determine for what values of x, the function $f(x) = x^3 + \frac{1}{x^3}$ ($x \neq 0$) is strictly increasing or strictly decreasing

OR

Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$

16. Evaluate $\int_0^2 (x^2 + 3)\, dx$ as limit of sums.

17. Find the area of the region bounded by the y-axis, y = cos x and y = sinx, $0 \leq x \leq \frac{\pi}{2}$

18. Can $y = ax + \frac{b}{a}$ be a solution of the following differential equation?

$$y = x\frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \quad ....(*)$$

If no, find the solution of the D.E.(*).

OR

Check whether the following differential equation is homogeneous or not

$$x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$$

Find the general solution of the differential equation using substitution y=vx.

19. If the vectors $\vec{p} = a\hat{i} + \hat{j} + \hat{k}$, $\vec{q} = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + \widehat{CK}$ are coplanar, then for a, b, c ≠ 1 show that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

20. A plane meets the coordinate axes in A, B and C such that the centroid of Δ ABC is the point (α, β,γ) . Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

21. If a 20 year old girl drives her car at 25 km/h, she has to spend Rs 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to Rs 5/km. She has Rs 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem. Write any one value reflected in the problem.

22. The random variable X has a probability distribution P(X) of the following form, where k is some number:

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k (ii) Find P(X <2) (iii) Find P(X ≤ 2) (iv)Find P(X ≥ 2)

23. A bag contains (2n +1) coins. It is known that 'n' of these coins have a head on both its sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, find the value of 'n'.

SECTION-D

Questions from 24 to 29 are of 6 marks each

24. Using properties of integral, evaluate $\int_0^{\pi} \frac{x}{1+\sin x} dx$

OR

Find: $\int \frac{\sin x}{\sin^3 x+\cos^3 x} dx$

25. Does the following trigonometric equation have any solutions? If Yes, obtain the solution(s):

$$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = -\tan^{-1} 7$$

OR

Determine whether the operation ∗ define below on $\mathbb{Q}$ is binary operation or not.

$$a * b = ab+1$$

If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements in $\mathbb{Q}$.

26. Find the value of x, y and z, if A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies A′ = A^{-1}

OR

Verify: A(adj A) = (adj A)A = $|A||\mathrm{I}$ for matrix A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

27. Find $\frac{dy}{dx}$, if y = $e^{\sin^2 x}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$

28. Find the shortest distance between the line x – y + 1 = 0 and the curve $y^2 = x$

29. Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:

$\vec{r} = (8 + 3\lambda)\,\hat{\imath} - (9 + 16\lambda)\,\hat{\jmath} + (10 + 7\lambda)\,\hat{k}$

$\vec{r} = 15\,\hat{\imath} + 29\,\hat{\jmath} + 5\,\hat{k} + \mu\,(3\,\hat{\imath} + 8\,\hat{\jmath} - 5\,\hat{k})$