Strictly Confidential: (For Internal and Restricted use only)
Secondary School Examination
March 2019
Marking Scheme – MATHEMATICS (SUBJECT CODE -041)

PAPER CODE: 30/1/1, 30/1/2, 30/1/3

General Instructions: -

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 1-80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.

12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 30/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Let the point A be (x, y)

$\therefore \quad \frac{1+x}{2} = 2$ and $\frac{4+y}{2} = -3$ $\frac{1}{2}$

$\Rightarrow \quad x = 3$ and $y = -10$

$\therefore$ Point A is $(3, -10)$ $\frac{1}{2}$

2. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow \quad 16 - 4k \geq 0$ $\frac{1}{2}$

$\Rightarrow \quad k \leq 4$ $\frac{1}{2}$

OR

Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow$ Product of roots = 1 $\frac{1}{2}$

$\Rightarrow \quad \frac{k}{3} = 1 \Rightarrow k = 3$ $\frac{1}{2}$

3. $\tan 2A = \cot (90° - 2A)$

$\therefore \quad 90° - 2A = A - 24°$ $\frac{1}{2}$

$\Rightarrow \quad A = 38°$ $\frac{1}{2}$

OR

$\sin 33° = \cos 57°$ $\frac{1}{2}$

$\therefore \quad \sin^2 33° + \sin^2 57° = \cos^2 57° + \sin^2 57° = 1$ $\frac{1}{2}$

4. Numbers are 12, 15, 18, ..., 99 $\frac{1}{2}$

$\therefore \quad 99 = 12 + (n - 1) \times 3$

$\Rightarrow \quad n = 30$ $\frac{1}{2}$

5. AB = 1 + 2 = 3 cm $\frac{1}{2}$

$\Delta ABC \sim \Delta ADE$

$\therefore \quad \frac{ar\,(ABC)}{ar\,(ADE)} = \frac{AB^2}{AD^2} = \frac{9}{1}$ $\frac{1}{2}$

$\therefore \quad ar(\Delta ABC) : ar(\Delta ADE) = 9 : 1$

6. Any one rational number between $\sqrt{2}$ (1.41 approx.) and $\sqrt{3}$ (1.73 approx.) 1

e.g., 1.5, 1.6, 1.63 etc.

SECTION B

7. Using Euclid's Algorithm

$$\left.\begin{aligned} 7344 &= 1260 \times 5 + 1044 \\ 1260 &= 1044 \times 1 + 216 \\ 1044 &= 216 \times 4 + 180 \\ 216 &= 180 \times 1 + 36 \\ 180 &= 36 \times 5 + 0 \end{aligned}\right\}$$ $1\frac{1}{2}$

HCF of 1260 and 7344 is 36. $\frac{1}{2}$

OR

Using Euclid's Algorithm

$a = 4q + r, \; 0 \le r < 4$

$\Rightarrow \quad a = 4q, \; a = 4q + 1, \; a = 4q + 2$ and $a = 4q + 3$. 1

Now $a = 4q$ and $a = 4q + 2$ are even numbers. $\frac{1}{2}$

Therefore when a is odd, it is of the form

$a = 4q + 1$ or $a = 4q + 3$ for some integer q. $\frac{1}{2}$

8. $a_n = a_{21} + 120$

$= (3 + 20 \times 12) + 120$

$= 363$ 1

$\therefore \quad 363 = 3 + (n - 1) \times 12$

$\Rightarrow \quad n = 31$ 1

or 31st term is 120 more than a_{21}.

OR

$a_1 = S_1 = 3 - 4 = -1$ $\frac{1}{2}$

$a_2 = S_2 - S_1 = [3(2)^2 - 4(2)] - (-1) = 5$ $\frac{1}{2}$

$\therefore\ d = a_2 - a_1 = 6$ $\frac{1}{2}$

Hence $a_n = -1 + (n - 1) \times 6 = 6n - 7$ $\frac{1}{2}$

Alternate method:

$S_n = 3n^2 - 4n$

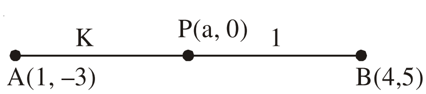
$\therefore \quad S_{n-1} = 3(n - 1)^2 - 4(n - 1) = 3n^2 - 10n + 7$ 1

Hence $a_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - (3n^2 - 10n + 7)$ $\frac{1}{2}$

$= 6n - 7$ $\frac{1}{2}$

9. Let the required point be (a, 0) and required ratio AP : PB = k : 1 $\frac{1}{2}$



$\therefore \quad a = \dfrac{4k+1}{k+1}$

$0 = \dfrac{5k-3}{k+1}$

$\Rightarrow \quad k = \dfrac{3}{5}$ or required ratio is 3 : 5 1

Point P is $\left(\dfrac{17}{8}, 0\right)$ $\frac{1}{2}$

10. Total number of outcomes = 8 $\frac{1}{2}$

Favourable number of outcomes (HHH, TTT) = 2 $\frac{1}{2}$

Prob. (getting success) = $\frac{2}{8}$ or $\frac{1}{4}$ $\frac{1}{2}$

$\therefore$ Prob. (losing the game) = $1-\frac{1}{4}=\frac{3}{4}$. $\frac{1}{2}$

11. Total number of outcomes = 6.

(i) Prob. (getting a prime number (2, 3, 5)) = $\frac{3}{6}$ or $\frac{1}{2}$ 1

(ii) Prob. (getting a number between 2 and 6 (3, 4, 5)) = $\frac{3}{6}$ or $\frac{1}{2}$. 1

12. System of equations has infinitely many solutions

$\therefore \quad \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$ $\frac{1}{2}$

$\Rightarrow \quad c^2 = 36 \Rightarrow c = 6$ or $c = -6$ ...(1) $\frac{1}{2}$

Also $-3c = 3c - c^2 \Rightarrow c = 6$ or $c = 0$ ...(2) $\frac{1}{2}$

From equations (1) and (2)

$c = 6$. $\frac{1}{2}$

SECTION C

13. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b are coprime positive integers and $b \neq 0$.

So $\sqrt{2} = \frac{a}{b}$

$\Rightarrow \quad a^2 = 2b^2$ 1

Thus a^2 is a multiple of 2

$\Rightarrow$ a is a multiple of 2. $\frac{1}{2}$

Let a = 2 m for some integer m

$\therefore \quad b^2 = 2m^2$ $\frac{1}{2}$

Thus b^2 is a multiple of 2

$\Rightarrow$ b is a multiple of 2 $\frac{1}{2}$

Hence 2 is a common factor of a and b.

This contradicts the fact that a and b are coprimes

Hence $\sqrt{2}$ is an irrational number. $\frac{1}{2}$

14. Sum of zeroes = k + 6 1

Product of zeroes = 2(2k – 1) 1

Hence $k + 6 = \frac{1}{2} \times 2(2k - 1)$

$\Rightarrow \quad k = 7$ 1

15. Let sum of the ages of two children be x yrs and father's age be y yrs.

$\therefore \quad y = 3x$ ...(1) 1

and $y + 5 = 2(x + 10)$ ...(2) 1

Solving equations (1) and (2)

$x = 15$

and $y = 45$

Father's present age is 45 years. 1

OR

Let the fraction be $\frac{x}{y}$

$\therefore \quad \frac{x-2}{y} = \frac{1}{3}$ ...(1) 1

and $\frac{x}{y-1} = \frac{1}{2}$ ...(2) 1

Solving (1) and (2) to get x = 7, y = 15.

$\therefore$ Required fraction is $\frac{7}{15}$ 1

16. Let the required point on y-axis be (0, b) $\frac{1}{2}$

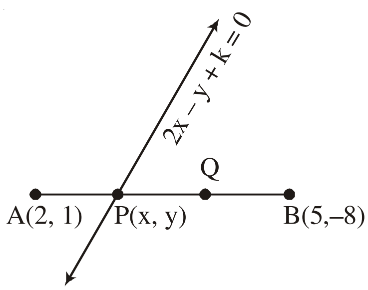
$\therefore \quad (5-0)^2 + (-2-b)^2 = (-3-0)^2 + (2-b)^2$ 1

$\Rightarrow \quad 29 + 4b + b^2 = 13 + b^2 - 4b$

$\Rightarrow \quad b = -2$ 1

$\therefore$ Required point is (0, –2) $\frac{1}{2}$

OR



AP : PB = 1 : 2 $\frac{1}{2}$

$x = \frac{4+5}{3} = 3$ and $y = \frac{2-8}{3} = -2$ $\frac{1}{2}+\frac{1}{2}$

Thus point P is (3, –2). $\frac{1}{2}$

Point (3, –2) lies on $2x - y + k = 0$

$\Rightarrow \quad 6 + 2 + k = 0$

$\Rightarrow \quad k = -8.$ 1

17. LHS $= \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$ 1

$= (\sin^2\theta + \cos^2\theta) + \operatorname{cosec}^2\theta + \sec^2\theta + \frac{2\sin\theta}{\sin\theta} + 2\frac{\cos\theta}{\cos\theta}.$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$ $1\frac{1}{2}$

$= 7 + \cot^2\theta + \tan^2\theta =$ RHS $\frac{1}{2}$

OR

LHS $= \left(1 + \frac{1}{\tan A} - \operatorname{cosec} A\right)(1 + \tan A + \sec A)$

$= \frac{1}{\tan A}(\tan A + 1 - \sec A)\,(1 + \tan A + \sec A)$ 1

$= \frac{1}{\tan A}[(1 + \tan A)^2 - \sec^2 A]$ 1

$= \frac{1}{\tan A}[1 + \tan^2 A + 2\tan A - 1 - \tan^2 A]$

$= 2 =$ RHS 1

Alternate method

$$\text{LHS} = \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$ 1

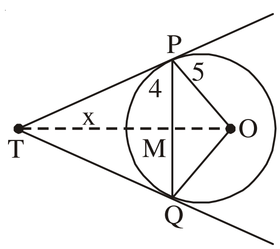
$$= (\sin A+\cos A-1)(\cos A+\sin A+1)\cdot\frac{1}{\cos A\sin A}$$

$$= \left[(\sin A+\cos A)^2-1\right]\times\frac{1}{\sin A\cos A}$$ 1

$$= (1+2\sin A\cos-1)\times\frac{1}{\sin A\cos A}$$ $\frac{1}{2}$

$= 2 = \text{RHS}$ $\frac{1}{2}$

18. Join OT and OQ. $\frac{1}{2}$



TP = TQ

$\therefore$ TM $\perp$ PQ and bisects PQ

Hence PM = 4 cm

Therefore OM = $\sqrt{25-16}=\sqrt{9}=3$ cm. $\frac{1}{2}$

Let TM = x

From ΔPMT, $PT^2 = x^2 + 16$

From ΔPOT, $PT^2 = (x + 3)^2 - 25$

Hence $x^2 + 16 = x^2 + 9 + 6x - 25$

$\Rightarrow 6x = 32 \Rightarrow x = \frac{16}{3}$ 1

Hence $PT^2 = \frac{256}{9}+16=\frac{400}{9}$

$\therefore\ PT=\frac{20}{3}$ cm. 1

19. $\Delta ACB \sim \Delta ADC$ (AA similarity)

$\Rightarrow \quad \dfrac{AC}{BC} = \dfrac{AD}{CD}$ ...(1) 1

Also $\Delta ACB \sim \Delta CDB$ (AA similarity)

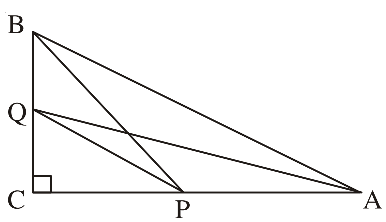
$\Rightarrow \quad \dfrac{AC}{BC} = \dfrac{CD}{BD}$ ...(2) 1

Using equations (1) and (2)

$$\frac{AD}{CD} = \frac{CD}{BD}$$

$\Rightarrow \quad CD^2 = AD \times BD$ 1

OR



Correct Figure $\frac{1}{2}$

$AQ^2 = CQ^2 + AC^2$ 1

$BP^2 = CP^2 + BC^2$ $\frac{1}{2}$

$\therefore\ AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$

$= PQ^2 + AB^2$. 1

20. $AC = \sqrt{64+36} = 10$ cm.

$\therefore$ Radius of the circle (r) = 5 cm. 1

Area of shaded region = Area of circle – Ar(ABCD) $\frac{1}{2}$

$= 3.14 \times 25 - 6 \times 8$ 1

$= 78.5 - 48$

$= 30.5\ cm^2$. $\frac{1}{2}$

21. Length of canal covered in 30 min = 5000 m. $\frac{1}{2}$

$\therefore$ Volume of water flown in 30 min = $6 \times 1.5 \times 5000\ m^3$ 1

If 8 cm standing water is needed

then area irrigated = $\frac{6 \times 1.5 \times 5000}{.08} = 562500\ m^2$. $1+\frac{1}{2}$

22. Modal class is 30-40 $\frac{1}{2}$

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 30 + \left(\frac{16-10}{32-10-12}\right) \times 10$ 2

$= 36.$ $\frac{1}{2}$

SECTION D

23. Let the smaller tap fills the tank in x hrs

$\therefore$ the larger tap fills the tank in (x – 2) hrs.

Time taken by both the taps together = $\frac{15}{8}$ hrs.

Therefore $\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$ 2

$\Rightarrow$ $4x^2 - 23x + 15 = 0$ $\frac{1}{2}$

$\Rightarrow$ $(4x - 3)(x - 5) = 0$

$x \neq \frac{3}{4}$ $\therefore x = 5$ 1

Smaller and larger taps can fill the tank seperately in 5 hrs and 3 hrs resp. $\frac{1}{2}$

OR

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Given $\frac{30}{x-y} + \frac{44}{x+y} = 10$ ...(i) 1

and $\frac{40}{x-y} + \frac{55}{x+y} = 13$ ...(ii) 1

Solving (i) and (ii) to get

$x + y = 11$ ..(iii)

and $x - y = 5$ ...(iv)

Solving (iii) and (iv) to get $x = 8$, $y = 3$. 1+1

Speed of boat = 8 km/hr & speed of stream = 3 km/hr.

24. $S_4 = 40 \Rightarrow 2(2a + 3d) = 40 \Rightarrow 2a + 3d = 20$ 1

$S_{14} = 280 \Rightarrow 7(2a + 13d) = 280 \Rightarrow 2a + 13d = 40$ 1

Solving to get $d = 2$ $\frac{1}{2}$

and $a = 7$ $\frac{1}{2}$

$\therefore \quad Sn = \frac{n}{2}[14 + (n-1) \times 2]$

$= n(n + 6)$ or $(n^2 + 6n)$ 1

25. LHS $= \dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$

Dividing num. & deno. by cos A

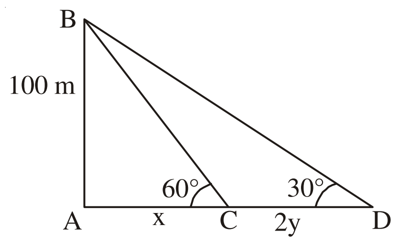
$= \dfrac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$ 1

$= \dfrac{\tan A - 1 + \sec A}{(\tan A - \sec A) + (\sec^2 A - \tan^2 A)}$ 1

$= \dfrac{\tan A - 1 + \sec A}{(\tan A - \sec A)(1 - \sec A - \tan A)}$ 1

$= \dfrac{-1}{\tan A - \sec A} = \dfrac{1}{\sec A - \tan A} =$ RHS 1

26. Correct Figure 1



Let the speed of the boat be y m/min

$\therefore$ CD = 2y

$\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$ 1

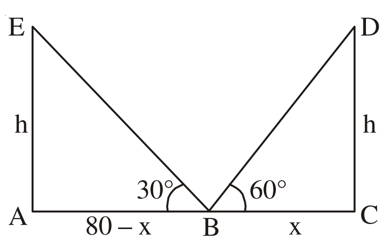
$\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x+2y} \Rightarrow x + 2y = 100\sqrt{3}$ 1

$\therefore\ y = \frac{100\sqrt{3}}{3} = 57.73$ 1

or speed of boat = 57.73 m/min.

OR

Correct Figure 1



Let BC = x so AB = 80 – x

where AC is the road.

$\tan 60° = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ 1

and $\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{80-x} \Rightarrow h\sqrt{3} = 80 - x$ 1

Solving equation to get

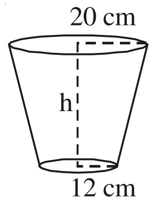
$x = 20,\ h = 20\sqrt{3}$

$\therefore$ AB = 60 m, BC = 20 m and h = $20\sqrt{3}$ m. 1

27. Correct construction of ΔABC. 2

Correct construction of triangle similar to triangle ABC. 2

28.



Volume of the bucket = 12308.8 cm^3

Let r_1 = 20 cm, r_2 = 12 cm

$\therefore \quad V = \dfrac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$\therefore \quad 12308.8 = \dfrac{3.14 \times h}{3}(400 + 144 + 240)$ 1

$\Rightarrow h = \dfrac{12308.8 \times 3}{3.14 \times 784} = 15$ cm 1

Now $l^2 = h^2 + (r_1 - r_2)^2 = 225 + 64 = 289$

$\Rightarrow \quad l = 17$ cm. 1

Surface area of metal sheet used = $\pi r_2^2 + \pi l\,(r_1 + r_2)$

$= 3.14\ (144 + 17 \times 32)$

$= 2160.32\ cm^2$. 1

29. Correct given, to prove, figure and construction $\frac{1}{2} \times 4 = 2$

Correct proof. 2

30.

Class	Frequency	Cumulative freq.		
0-10	f_1	f_1		
10-20	5	$5 + f_1$		
20-30	9	$14 + f_1$		
30-40	12	$26 + f_1$		
40-50	f_2	$26 + f_1 + f_2$		
50-60	3	$29 + f_1 + f_2$		
60-70	2	$31 + f_1 + f_2$	Correct Table	1
	40			

Median = 32.5 $\Rightarrow$ median class is 30-40. $\frac{1}{2}$

Now $32.5 = 30 + \dfrac{10}{12}(20 - 14 - f_1)$ 1

$\Rightarrow f_1 = 3$ 1

Also $31 + f_1 + f_2 = 40$

$\Rightarrow f_2 = 6$ $\frac{1}{2}$

OR

Less than type distribution is as follows

Marks	No. of students		
Less than 5	2		
Less than 10	7		
Less than 15	13		
Less than 20	21		
Less than 25	31		
Less than 30	56		
Less than 35	76		
Less than 40	94		
Less than 45	98		
Less than 50	100	Correct Table	$1\frac{1}{2}$

Plotting of points (5, 2), (10, 7) (15, 13), (20, 21), (25, 31), (30, 56),

(35, 76), (40, 94), (45, 98), (50, 100) $1\frac{1}{2}$

Joining to get the curve $\frac{1}{2}$

Getting median from graph (approx. 29) $\frac{1}{2}$

QUESTION PAPER CODE 30/1/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Let the point A be (x, y)

$\therefore \quad \frac{x+3}{2} = -2$ and $\frac{y+4}{2} = 2$ $\frac{1}{2}$

$\Rightarrow \quad x = -7$ and $y = 0$

Point is (–7, 0) $\frac{1}{2}$

2. Any one rational number between $\sqrt{2}$ (1.41 approx.) and $\sqrt{3}$ (1.73 approx.) 1

e.g., 1.5, 1.6, 1.63 etc.

3. Numbers are 12, 15, 18, ..., 99 $\frac{1}{2}$

$\therefore \quad 99 = 12 + (n - 1) \times 3$

$\Rightarrow \quad n = 30$ $\frac{1}{2}$

4. tan 2 A = cot (90° – 2A)

$\therefore \quad 90° - 2A = A - 24°$ $\frac{1}{2}$

$\Rightarrow \quad A = 38°$ $\frac{1}{2}$

OR

$\sin 33° = \cos 57°$ $\frac{1}{2}$

$\therefore \quad \sin^2 33° + \sin^2 57° = \cos^2 57° + \sin^2 57° = 1$ $\frac{1}{2}$

5. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow \quad 16 - 4k \geq 0$ $\frac{1}{2}$

$\Rightarrow \quad k \leq 4$ $\frac{1}{2}$

OR

Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow$ Product of roots = 1 $\frac{1}{2}$

$\Rightarrow \frac{k}{3} = 1 \Rightarrow k = 3$ $\frac{1}{2}$

6. AB = 1 + 2 = 3 cm $\frac{1}{2}$

$\Delta ABC \sim \Delta ADE$

$\therefore \frac{ar(ABC)}{ar(ADE)} = \frac{AB^2}{AD^2} = \frac{9}{1}$ $\frac{1}{2}$

$\therefore$ ar(ΔABC) : ar(ΔADE) = 9 : 1

SECTION B

7. System of equations has infinitely many solutions.

$\therefore \frac{2}{k+1} = \frac{3}{2k-1} = \frac{7}{4k+1}$ $\frac{1}{2}$

$\Rightarrow 4k - 2 = 3k + 3$ $\frac{1}{2}$

$\Rightarrow k = 5$

Also $12k + 3 = 14k - 7$ $\frac{1}{2}$

$\Rightarrow k = 5$

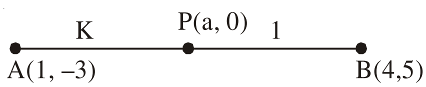
Hence k = 5. $\frac{1}{2}$

8. Total number of outcomes = 6.

(i) Prob. (getting a prime number (2, 3, 5)) = $\frac{3}{6}$ or $\frac{1}{2}$ 1

(ii) Prob. (getting a number between 2 and 6 (3, 4, 5)) = $\frac{3}{6}$ or $\frac{1}{2}$. 1

9. Let the required point be (a, 0) and required ratio AP : PB = k : 1 $\frac{1}{2}$



$$\therefore \quad a = \frac{4k+1}{k+1}$$

$$0 = \frac{5k-3}{k+1}$$

$\Rightarrow \quad k = \frac{3}{5}$ or required ratio is 3 : 5 1

Point P is $\left(\frac{17}{8}, 0\right)$ $\frac{1}{2}$

10. Total number of outcomes = 8 $\frac{1}{2}$

Favourable number of outcomes (HHH, TTT) = 2 $\frac{1}{2}$

Prob. (getting success) = $\frac{2}{8}$ or $\frac{1}{4}$ $\frac{1}{2}$

$\therefore$ Prob. (losing the game) = $1-\frac{1}{4}=\frac{3}{4}$. $\frac{1}{2}$

11. $a_n = a_{21} + 120$

$= (3 + 20 \times 12) + 120$

$= 363$ 1

$\therefore \quad 363 = 3 + (n - 1) \times 12$

$\Rightarrow \quad n = 31$ 1

or 31st term is 120 more than a_{21}.

OR

$a_1 = S_1 = 3 - 4 = -1$ $\frac{1}{2}$

$a_2 = S_2 - S_1 = [3(2)^2 - 4(2)] - (-1) = 5$ $\frac{1}{2}$

$\therefore d = a_2 - a_1 = 6$ $\frac{1}{2}$

Hence $a_n = -1 + (n - 1) \times 6 = 6n - 7$ $\frac{1}{2}$

Alternate method:

$S_n = 3n^2 - 4n$

$\therefore \quad S_{n-1} = 3(n-1)^2 - 4(n-1) = 3n^2 - 10n + 7$ 1

Hence $a_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - (3n^2 - 10n + 7)$ $\frac{1}{2}$

$= 6n - 7$ $\frac{1}{2}$

12. Using Euclid's Algorithm

$$\left.\begin{aligned} 7344 &= 1260 \times 5 + 1044 \\ 1260 &= 1044 \times 1 + 216 \\ 1044 &= 216 \times 4 + 180 \\ 216 &= 180 \times 1 + 36 \\ 180 &= 36 \times 5 + 0 \end{aligned}\right\}$$ $1\frac{1}{2}$

HCF of 1260 and 7344 is 36. $\frac{1}{2}$

OR

Using Euclid's Algorithm

$a = 4q + r,\ 0 \le r < 4$

$\Rightarrow \quad a = 4q,\ a = 4q + 1,\ a = 4q + 2$ and $a = 4q + 3.$ 1

Now $a = 4q$ and $a = 4q + 2$ are even numbers. $\frac{1}{2}$

Therefore when a is odd, it is of the form

$a = 4q + 1$ or $a = 4q + 3$ for some integer q. $\frac{1}{2}$

SECTION C

13.

Class	x	Freq (f)	$u = \frac{x-50}{20}$	fu
0-20	10	12	–2	–24
20-40	30	15	–1	–15
40-60	50	32	0	0
60-80	70	k	1	k
80-100	90	13	2	26
		72 + k		–13 + k

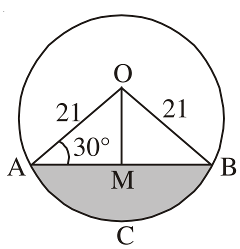
Correct Table 2

$$\bar{x} = 53 = 50 + 20 \times \frac{-13+k}{72+k}$$

$\Rightarrow$ 3k + 216 = 20k – 260

$\Rightarrow$ k = 28 1

14. Draw OM $\perp$ AB



$\angle OAB = \angle OBA = 30°$ $\frac{1}{2}$

$$\sin 30° = \frac{1}{2} = \frac{OM}{21} \Rightarrow OM = \frac{21}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2} = \frac{AM}{21} \Rightarrow AM = \frac{21}{2}\sqrt{3}$$

$$\text{Area of } \Delta OAB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$$

$$= \frac{441}{4}\sqrt{3}\ \text{cm}^2.$$ 1

$\therefore$ Area of shaded region = Area (sector OACB) – Area (ΔOAB)

$$= \frac{22}{7} \times 21 \times 21 \times \frac{120}{360} - \frac{441}{4}\sqrt{3}$$ 1

$$= \left(462 - 441\frac{\sqrt{3}}{4}\right)\text{cm}^2 \text{ or } 271.3\,\text{cm}^2 \text{ (approx.)}$$ $\frac{1}{2}$

15. $\Delta ACB \sim \Delta ADC$ (AA similarity)

$\Rightarrow \quad \dfrac{AC}{BC} = \dfrac{AD}{CD}$ ...(1) 1

Also $\Delta ACB \sim \Delta CDB$ (AA similarity)

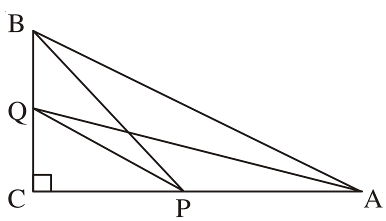
$\Rightarrow \quad \dfrac{AC}{BC} = \dfrac{CD}{BD}$ ...(2) 1

Using equations (1) and (2)

$$\frac{AD}{CD} = \frac{CD}{BD}$$

$\Rightarrow \quad CD^2 = AD \times BD$ 1

OR



Correct Figure $\frac{1}{2}$

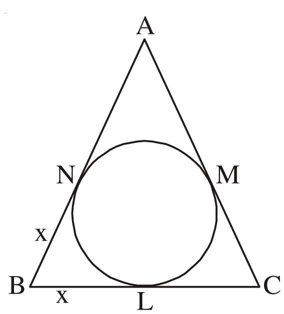
$AQ^2 = CQ^2 + AC^2$ 1

$BP^2 = CP^2 + BC^2$ $\frac{1}{2}$

$\therefore\ AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$

$= PQ^2 + AB^2.$ 1

16.



Let BL = x = BN

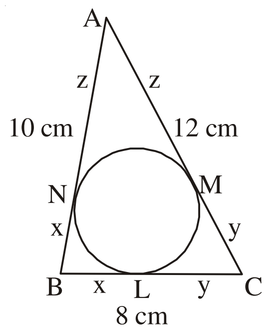
$\left.\begin{array}{l}\therefore\ CL = 8 - x = CM \\ \because\ AC = 12 \Rightarrow AM = 4 + x = AN\end{array}\right\}$ 1

Now AB = AN + NB = 10 $\Rightarrow$ x + 4 + x = 10

$\Rightarrow$ x = 3 1

$\therefore$ BL = 3 cm, CM = 5 cm and AN = 7 cm 1

Alternate method



Let BL = BN = x (tangents from external points are equal) $\frac{1}{2}$

CL = CM = y

AN = AM = z

$\therefore$ AB + BC + AC = 2x + 2y + 2z = 30

$\Rightarrow$ x + y + z = 15 ...(i) 1

Also x + z = 10, x + y = 8 and y + z = 12

Subtracting from equation (i)

y = 5, z = 7 and x = 3 $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

$\therefore$ BL = 3 cm, CM = 5 cm and AN = 7 cm.

17. Length of canal covered in 30 min = 5000 m. $\frac{1}{2}$

$\therefore$ Volume of water flown in 30 min = $6 \times 1.5 \times 5000 \text{ m}^3$ 1

If 8 cm standing water is needed

then area irrigated = $\frac{6\times1.5\times5000}{.08} = 562500 \text{ m}^2$. $1+\frac{1}{2}$

18. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b are coprime positive integers and b ≠ 0.

So $\sqrt{2} = \frac{a}{b}$

$\Rightarrow \quad a^2 = 2b^2$ 1

Thus a^2 is a multiple of 2

$\Rightarrow$ a is a multiple of 2. $\frac{1}{2}$

Let a = 2 m for some integer m

$\therefore \quad b^2 = 2m^2$ $\frac{1}{2}$

Thus b^2 is a multiple of 2

$\Rightarrow$ b is a multiple of 2 $\frac{1}{2}$

Hence 2 is a common factor of a and b.

This contradicts the fact that a and b are coprimes

Hence $\sqrt{2}$ is an irrational number. $\frac{1}{2}$

19. Sum of zeroes = k + 6 1

Product of zeroes = 2(2k – 1) 1

Hence $k + 6 = \frac{1}{2} \times 2(2k - 1)$

$\Rightarrow \quad k = 7$ 1

20. Let the required point on y-axis be (0, b) $\frac{1}{2}$

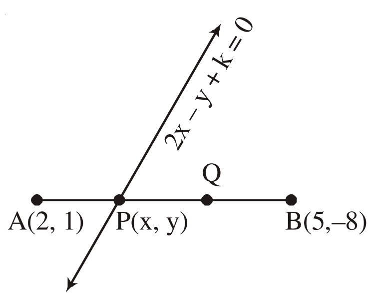
$\therefore \quad (5 - 0)^2 + (-2 - b)^2 = (-3 - 0)^2 + (2 - b)^2$ 1

$\Rightarrow \quad 29 + 4b + b^2 = 13 + b^2 - 4b$

$\Rightarrow \quad b = -2$ 1

$\therefore$ Required point is (0, –2) $\frac{1}{2}$

OR



AP : PB = 1 : 2 $\frac{1}{2}$

$x = \frac{4+5}{3} = 3$ and $y = \frac{2-8}{3} = -2$ $\frac{1}{2} + \frac{1}{2}$

Thus point P is (3, –2). $\frac{1}{2}$

Point (3, –2) lies on 2x – y + k = 0

$\Rightarrow \quad 6 + 2 + k = 0$

$\Rightarrow \quad k = -8.$ 1

21. Let sum of the ages of two children be x yrs and father's age be y yrs.

$\therefore \quad y = 3x$ ...(1) 1

and $\quad y + 5 = 2(x + 10)$ ...(2) 1

Solving equations (1) and (2)

$x = 15$

and $y = 45$

Father's present age is 45 years. 1

OR

Let the fraction be $\frac{x}{y}$

$\therefore \quad \frac{x-2}{y} = \frac{1}{3}$ ...(1) 1

and $\frac{x}{y-1} = \frac{1}{2}$ ...(2) 1

Solving (1) and (2) to get x = 7, y = 15.

$\therefore$ Required fraction is $\frac{7}{15}$ 1

22. LHS $= \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$ 1

$= (\sin^2\theta + \cos^2\theta) + \operatorname{cosec}^2\theta + \sec^2\theta + \frac{2\sin\theta}{\sin\theta} + 2\frac{\cos\theta}{\cos\theta}.$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$ $1\frac{1}{2}$

$= 7 + \cot^2\theta + \tan^2\theta = \text{RHS}$ $\frac{1}{2}$

OR

$\text{LHS} = \left(1 + \frac{1}{\tan A} - \operatorname{cosec} A\right)(1 + \tan A + \sec A)$

$= \frac{1}{\tan A}(\tan A + 1 - \sec A)(1 + \tan A + \sec A)$ 1

$= \frac{1}{\tan A}[(1 + \tan A)^2 - \sec^2 A]$ 1

$= \frac{1}{\tan A}[1 + \tan^2 A + 2\tan A - 1 - \tan^2 A]$

$= 2 = \text{RHS}$ 1

Alternate method

$$\text{LHS} = \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$ 1

$$= (\sin A + \cos A - 1)(\cos A + \sin A + 1)\cdot\frac{1}{\cos A \sin A}$$

$$= \left[(\sin A + \cos A)^2 - 1\right]\times\frac{1}{\sin A \cos A}$$ 1

$$= (1 + 2\sin A \cos - 1)\times\frac{1}{\sin A \cos A}$$ $\frac{1}{2}$

$$= 2 = \text{RHS}$$ $\frac{1}{2}$

SECTION D

23. $$\text{LHS} = \frac{\sin^2 A/\cos^2 A}{\dfrac{\sin^2 A}{\cos^2 A}-1}+\frac{1/\sin^2 A}{\dfrac{1}{\cos^2 A}-\dfrac{1}{\sin^2 A}}$$ 1

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A}+\frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$ 1

$$= \frac{1}{\sin^2 A - \cos^2 A}$$ 1

$$= \frac{1}{1-2\cos^2 A}$$ 1

24. Here $a = 3$, $a_n = 83$ and $S_n = 903$

Therefore $83 = 3 + (n - 1)d$

$\Rightarrow \quad (n - 1)d = 80$ ...(i) 1

Also $903 = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(6+80) = 43n$ (using (i)) $1+\frac{1}{2}$

$\Rightarrow \quad n = 21$

and $\quad d = 4$ $1\frac{1}{2}$

25. Correct construction of ΔABC — 2

Correct construction of triangle similar to ΔABC. — 2

26.

Class	**Frequency**	**Cumulative freq.**		
0-10	f_1	f_1		
10-20	5	$5 + f_1$		
20-30	9	$14 + f_1$		
30-40	12	$26 + f_1$		
40-50	f_2	$26 + f_1 + f_2$		
50-60	3	$29 + f_1 + f_2$		
60-70	2	$31 + f_1 + f_2$	Correct Table	1
	40			

Median = 32.5 $\Rightarrow$ median class is 30-40. — $\frac{1}{2}$

Now $32.5 = 30 + \frac{10}{12}(20 - 14 - f_1)$ — 1

$\Rightarrow f_1 = 3$ — 1

Also $31 + f_1 + f_2 = 40$

$\Rightarrow f_2 = 6$ — $\frac{1}{2}$

OR

Less than type distribution is as follows

Marks	No. of students		
Less than 5	2		
Less than 10	7		
Less than 15	13		
Less than 20	21		
Less than 25	31		
Less than 30	56		
Less than 35	76		
Less than 40	94		
Less than 45	98		
Less than 50	100	Correct Table	$1\frac{1}{2}$

Plotting of points (5, 2), (10, 7) (15, 13), (20, 21), (25, 31), (30, 56),

(35, 76), (40, 94), (45, 98), (50, 100) $1\frac{1}{2}$

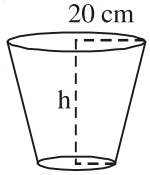
Joining to get the curve $\frac{1}{2}$

Getting median from graph (approx. 29) $\frac{1}{2}$

27. Correct given, to prove, figure and construction $\frac{1}{2}\times 4=2$

Correct proof. 2

28.



Volume of the bucket = 12308.8 cm^3

Let r_1 = 20 cm, r_2 = 12 cm

$\therefore \quad V = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$\therefore \quad 12308.8 = \frac{3.14 \times h}{3}(400 + 144 + 240)$ 1

$\Rightarrow h = \frac{12308.8 \times 3}{3.14 \times 784} = 15$ cm 1

Now $l^2 = h^2 + (r_1 - r_2)^2 = 225 + 64 = 289$

$\Rightarrow \quad l = 17$ cm. 1

Surface area of metal sheet used = $\pi r_2^2 + \pi l\,(r_1 + r_2)$

= 3.14 (144 + 17 × 32)

= 2160.32 cm^2. 1

29. Let the smaller tap fills the tank in x hrs

$\therefore$ the larger tap fills the tank in (x – 2) hrs.

Time taken by both the taps together = $\frac{15}{8}$ hrs.

Therefore $\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$ 2

$\Rightarrow \quad 4x^2 - 23x + 15 = 0$ $\frac{1}{2}$

$\Rightarrow \quad (4x - 3)(x - 5) = 0$

$x \neq \frac{3}{4} \quad \therefore\ x = 5$ 1

Smaller and larger taps can fill the tank seperately in 5 hrs and 3 hrs resp. $\frac{1}{2}$

OR

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Given $\frac{30}{x-y} + \frac{44}{x+y} = 10$ ...(i) 1

and $\frac{40}{x-y} + \frac{55}{x+y} = 13$ ...(ii) 1

Solving (i) and (ii) to get

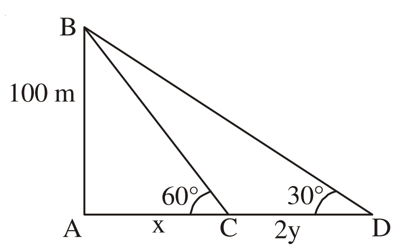
$x + y = 11$ ..(iii)

and $x - y = 5$ ...(iv)

Solving (iii) and (iv) to get x = 8, y = 3. 1+1

Speed of boat = 8 km/hr & speed of stream = 3 km/hr.

30. Correct Figure 1



Let the speed of the boat be y m/min

$\therefore\ CD = 2y$

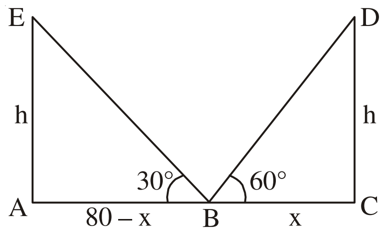
$\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$ 1

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x+2y} \Rightarrow x + 2y = 100\sqrt{3}$ 1

$\therefore\ y = \frac{100\sqrt{3}}{3} = 57.73$ 1

or speed of boat = 57.73 m/min.

OR



Correct Figure 1

Let BC = x so AB = 80 – x

where AC is the road.

$\tan 60° = \sqrt{3} = \dfrac{h}{x} \Rightarrow h = x\sqrt{3}$ 1

and $\tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{h}{80-x} \Rightarrow h\sqrt{3} = 80 - x$ 1

Solving equation to get

$x = 20,\ h = 20\sqrt{3}$

$\therefore$ AB = 60 m, BC = 20 m and h = $20\sqrt{3}$ m. 1

QUESTION PAPER CODE 30/1/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. LCM $(x^3y^2, xy^3) = x^3y^3$. 1

2. Numbers are 12, 15, 18, ..., 99 $\frac{1}{2}$

$\therefore \quad 99 = 12 + (n - 1) \times 3$

$\Rightarrow \quad n = 30$ $\frac{1}{2}$

3. AB = 1 + 2 = 3 cm $\frac{1}{2}$

$\Delta ABC \sim \Delta ADE$

$\therefore \quad \frac{ar\,(ABC)}{ar\,(ADE)} = \frac{AB^2}{AD^2} = \frac{9}{1}$ $\frac{1}{2}$

$\therefore \quad ar(\Delta ABC) : ar(\Delta ADE) = 9 : 1$

4. Let the point A be (x, y)

$\therefore \quad \frac{1+x}{2} = 2$ and $\frac{4+y}{2} = -3$ $\frac{1}{2}$

$\Rightarrow \quad x = 3$ and $y = -10$

$\therefore$ Point A is (3, –10) $\frac{1}{2}$

5. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow \quad 16 - 4k \geq 0$ $\frac{1}{2}$

$\Rightarrow \quad k \leq 4$ $\frac{1}{2}$

OR

Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow$ Product of roots = 1 $\frac{1}{2}$

$\Rightarrow \quad \frac{k}{3} = 1 \Rightarrow k = 3$ $\frac{1}{2}$

6. $\tan 2A = \cot (90° - 2A)$

$\therefore \quad 90° - 2A = A - 24°$ $\frac{1}{2}$

$\Rightarrow \quad A = 38°$ $\frac{1}{2}$

OR

$\sin 33° = \cos 57°$ $\frac{1}{2}$

$\therefore \quad \sin^2 33° + \sin^2 57° = \cos^2 57° + \sin^2 57° = 1$ $\frac{1}{2}$

SECTION B

7. Required numbers are

14, 21, 28, 35, ..., 98. 1

$98 = 14 + (n - 1) \times 7$ $\frac{1}{2}$

$\Rightarrow \quad n = 13$ $\frac{1}{2}$

OR

Given $S_n = n^2$

$S_1 = a_1 = 1$ $\frac{1}{2}$

$S_2 = a_1 + a_2 = 4$

$\Rightarrow a_2 = 3$ $\frac{1}{2}$

$\therefore d = a_2 - a_1 = 2$ $\frac{1}{2}$

$a_{10} = 1 + 18 = 19$ $\frac{1}{2}$

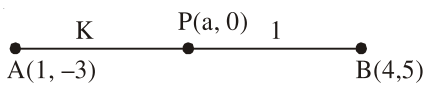
8. Total number of outcomes = 8 $\frac{1}{2}$

Favourable number of outcomes (HHH, TTT) = 2 $\frac{1}{2}$

Prob. (getting success) = $\frac{2}{8}$ or $\frac{1}{4}$ $\frac{1}{2}$

$\therefore$ Prob. (losing the game) $= 1 - \frac{1}{4} = \frac{3}{4}$. $\frac{1}{2}$

9. Let the required point be (a, 0) and required ratio AP : PB = k : 1 $\frac{1}{2}$



$$\therefore \quad a = \frac{4k+1}{k+1}$$

$$0 = \frac{5k-3}{k+1}$$

$\Rightarrow$ $k = \frac{3}{5}$ or required ratio is 3 : 5 1

Point P is $\left(\frac{17}{8}, 0\right)$ $\frac{1}{2}$

10. Total number of outcomes = 6.

(i) Prob. (getting a prime number (2, 3, 5)) $= \frac{3}{6}$ or $\frac{1}{2}$ 1

(ii) Prob. (getting a number between 2 and 6 (3, 4, 5)) $= \frac{3}{6}$ or $\frac{1}{2}$. 1

11. System of equations has infinitely many solutions

$\therefore \quad \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$ $\frac{1}{2}$

$\Rightarrow \quad c^2 = 36 \Rightarrow c = 6$ or $c = -6$ ...(1) $\frac{1}{2}$

Also $-3c = 3c - c^2 \Rightarrow c = 6$ or $c = 0$ ...(2) $\frac{1}{2}$

From equations (1) and (2)

$c = 6$. $\frac{1}{2}$

12. Using Euclid's Algorithm

$$\left.\begin{aligned} 7344 &= 1260 \times 5 + 1044 \\ 1260 &= 1044 \times 1 + 216 \\ 1044 &= 216 \times 4 + 180 \\ 216 &= 180 \times 1 + 36 \\ 180 &= 36 \times 5 + 0 \end{aligned}\right\}$$ $1\frac{1}{2}$

HCF of 1260 and 7344 is 36. $\frac{1}{2}$

OR

Using Euclid's Algorithm

$a = 4q + r,\ 0 \le r < 4$

$\Rightarrow \quad a = 4q,\ a = 4q + 1,\ a = 4q + 2$ and $a = 4q + 3$. 1

Now $a = 4q$ and $a = 4q + 2$ are even numbers. $\frac{1}{2}$

Therefore when a is odd, it is of the form

$a = 4q + 1$ or $a = 4q + 3$ for some integer q. $\frac{1}{2}$

SECTION C

13. Let $p(x) = 3x^3 + 10x^2 - 9x - 4$.

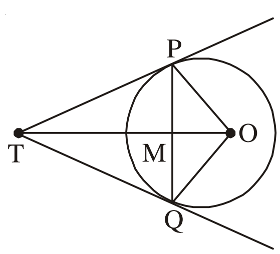
One of the zeroes is 1, therefore dividing p(x) by (x – 1)

$p(x) = (x - 1)(3x^2 + 13x + 4)$ $1\frac{1}{2}$

$= (x - 1)(x + 4)(3x + 1)$ 1

All zeroes are $x = 1$, $x = -4$ and $x = -\frac{1}{3}$. $\frac{1}{2}$

14.



Join OQ, TP = TQ $\therefore$ TM $\perp$ PQ and bisects PQ

Hence PM = 4 cm.

$\therefore\ OM = \sqrt{25-16} = \sqrt{9} = 3\,\text{cm}$ 1

Let TM = x $\therefore$ $PT^2 = x^2 + 16$ (ΔPMT)

$PT^2 = (x + 3)^2 - 25$ (ΔPOT)

Hence $x^2 + 16 = (x + 3)^2 - 25 = x^2 + 9 + 6x - 25$ 1

$\Rightarrow 6x = 32 \Rightarrow x = \frac{16}{3}$

Hence $PT^2 = \frac{256}{9} + 16 = \frac{400}{9}$

$\Rightarrow PT = \frac{20}{3}\,\text{cm}$ 1

15. Let us assume $\frac{2+\sqrt{3}}{5}$ be a rational number.

Let $\frac{2+\sqrt{3}}{5} = \frac{a}{b}$ (b ≠ 0, a and b are integers)

$\Rightarrow \quad \sqrt{3} = \frac{5a-2b}{b}$ 1

$\because$ a, b are integers

$\therefore \quad \frac{5a-2b}{b}$ is a rational number 1

i.e. $\sqrt{3}$ is a rational number

which contradicts the fact that $\sqrt{3}$ is irrational

Therefore is $\frac{2+\sqrt{3}}{5}$ is an irrational number. 1

16. LHS = $\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$ 1

$= (\sin^2\theta + \cos^2\theta) + \text{cosec}^2\theta + \sec^2\theta + \frac{2\sin\theta}{\sin\theta} + 2\frac{\cos\theta}{\cos\theta}.$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$ $1\frac{1}{2}$

$= 7 + \cot^2\theta + \tan^2\theta = \text{RHS}$ $\frac{1}{2}$

OR

LHS = $\left(1+\frac{1}{\tan A} - \text{cosec}\,A\right)(1+\tan A+\sec A)$

$= \frac{1}{\tan A}(\tan A + 1 - \sec A)\,(1+\tan A+\sec A)$ 1

$= \frac{1}{\tan A}[(1+\tan A)^2 - \sec^2 A]$ 1

$= \frac{1}{\tan A}[1+\tan^2 A + 2\tan A - 1 - \tan^2 A]$

$= 2 = \text{RHS}$ 1

Alternate method

$$\text{LHS} = \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right) \qquad 1$$

$$= (\sin A+\cos A-1)(\cos A+\sin A+1)\cdot\frac{1}{\cos A\sin A}$$

$$= \left[(\sin A+\cos A)^2-1\right]\times\frac{1}{\sin A\cos A} \qquad 1$$

$$= (1+2\sin A\cos -1)\times\frac{1}{\sin A\cos A} \qquad \frac{1}{2}$$

$$= 2 = \text{RHS} \qquad \frac{1}{2}$$

17. Let sum of the ages of two children be x yrs and father's age be y yrs.

$\therefore$ $y = 3x$ ...(1) 1

and $y + 5 = 2(x + 10)$ ...(2) 1

Solving equations (1) and (2)

$x = 15$

and $y = 45$

Father's present age is 45 years. 1

OR

Let the fraction be $\frac{x}{y}$

$\therefore$ $\frac{x-2}{y} = \frac{1}{3}$ ...(1) 1

and $\frac{x}{y-1} = \frac{1}{2}$ ...(2) 1

Solving (1) and (2) to get $x = 7$, $y = 15$.

$\therefore$ Required fraction is $\frac{7}{15}$ 1

18. Let the required point on y-axis be (0, b) $\frac{1}{2}$

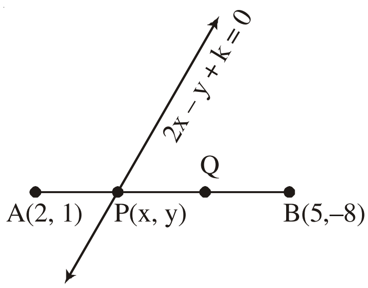
$\therefore \quad (5-0)^2 + (-2-b)^2 = (-3-0)^2 + (2-b)^2$ 1

$\Rightarrow \quad 29 + 4b + b^2 = 13 + b^2 - 4b$

$\Rightarrow \quad b = -2$ 1

$\therefore$ Required point is (0, –2) $\frac{1}{2}$

OR



AP : PB = 1 : 2 $\frac{1}{2}$

$x = \frac{4+5}{3} = 3$ and $y = \frac{2-8}{3} = -2$ $\frac{1}{2}+\frac{1}{2}$

Thus point P is (3, –2). $\frac{1}{2}$

Point (3, –2) lies on $2x - y + k = 0$

$\Rightarrow \quad 6 + 2 + k = 0$

$\Rightarrow \quad k = -8.$ 1

19. Modal class is 30-40 $\frac{1}{2}$

$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 30 + \left(\frac{16-10}{32-10-12}\right) \times 10$ 2

$= 36.$ $\frac{1}{2}$

20. Length of canal covered in 30 min = 5000 m. $\frac{1}{2}$

$\therefore$ Volume of water flown in 30 min = $6 \times 1.5 \times 5000 \text{ m}^3$ 1

If 8 cm standing water is needed

then area irrigated = $\frac{6 \times 1.5 \times 5000}{.08} = 562500 \text{ m}^2$. $1+\frac{1}{2}$

21. $\Delta ACB \sim \Delta ADC$ (AA similarity)

$\Rightarrow \quad \frac{AC}{BC} = \frac{AD}{CD}$ ...(1) 1

Also $\Delta ACB \sim \Delta CDB$ (AA similarity)

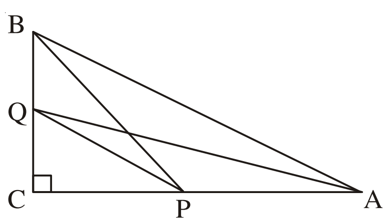
$\Rightarrow \quad \frac{AC}{BC} = \frac{CD}{BD}$ ...(2) 1

Using equations (1) and (2)

$\frac{AD}{CD} = \frac{CD}{BD}$

$\Rightarrow \quad CD^2 = AD \times BD$ 1

OR



Correct Figure $\frac{1}{2}$

$AQ^2 = CQ^2 + AC^2$ 1

$BP^2 = CP^2 + BC^2$ $\frac{1}{2}$

$\therefore\ AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$

$= PQ^2 + AB^2.$ 1

22. $AC = \sqrt{64+36} = 10$ cm.

$\therefore$ Radius of the circle (r) = 5 cm. 1

Area of shaded region = Area of circle – Ar(ABCD) $\frac{1}{2}$

$= 3.14 \times 25 - 6 \times 8$ 1

$= 78.5 - 48$

$= 30.5\ cm^2.$ $\frac{1}{2}$

SECTION D

23. $\sec^2\theta = \left(x + \frac{1}{4x}\right)^2 = x^2 + \frac{1}{16x^2} + \frac{1}{2}$ 1

$\therefore \quad \tan^2\theta = \sec^2\theta - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2}$ 1

$\Rightarrow \quad \tan^2\theta = \left(x - \frac{1}{4x}\right)^2$

$\Rightarrow \quad \tan\theta = \left(x - \frac{1}{4x}\right)$ or $\left(\frac{1}{4x} - x\right)$ 1

Hence $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$ 1

24. Correct given, to prove, figure, construction $\frac{1}{2} \times 4 = 2$

Correct proof. 2

25. Less than type distribution is as follows

Daily income	**Number of workers**
Less than 220	12
Less than 240	26
Less than 260	34
Less than 280	40
Less than 300	50

Correct Table $1\frac{1}{2}$

Plotting of points (220, 12), (240, 26), (260, 34) (280, 40) and (300, 50) $1\frac{1}{2}$

Joining to get curve 1

OR

Daily expenditure	x_i	No. of households (f_i)	$u_i = \frac{x - 225}{50}$	f_iu_i
100-150	125	4	–2	–8
150-200	175	5	–1	–5
200–250	225	12	0	0
250–300	275	2	1	2
300-350	325	2	2	4
		$\Sigma f_i = 25$		$\Sigma f_iu_i = -7$

Correct Table 2

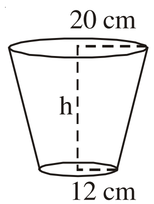
Mean = $225+50\times\left(\frac{-7}{25}\right)=211$ 2

Mean expenditure on food is ₹ 211.

26. Correct construction of ΔABC. 2

Correct construction of triangle similar to triangle ABC. 2

27.



Volume of the bucket = 12308.8 cm^3

Let r_1 = 20 cm, r_2 = 12 cm

$\therefore \quad V = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$\therefore \quad 12308.8 = \frac{3.14\times h}{3}(400+144+240)$ 1

$\Rightarrow h = \frac{12308.8\times 3}{3.14\times 784} = 15$ cm 1

Now $l^2 = h^2 + (r_1 - r_2)^2 = 225 + 64 = 289$

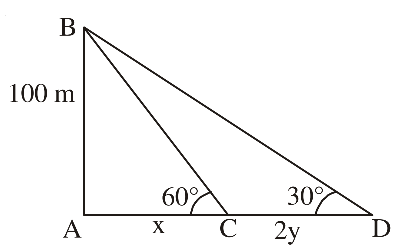
$\Rightarrow \quad l = 17$ cm. 1

Surface area of metal sheet used = $\pi r_2^2 + \pi l\,(r_1 + r_2)$

= 3.14 (144 + 17 × 32)

= 2160.32 cm^2. 1

28.



Correct Figure 1

Let the speed of the boat be y m/min

$\therefore$ CD = 2y

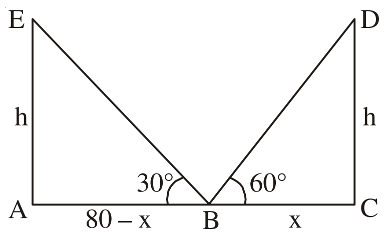
$\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$ 1

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x+2y} \Rightarrow x + 2y = 100\sqrt{3}$ 1

$\therefore y = \frac{100\sqrt{3}}{3} = 57.73$ 1

or speed of boat = 57.73 m/min.

OR



Correct Figure 1

Let BC = x so AB = 80 – x

where AC is the road.

$\tan 60° = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ 1

and $\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{80 - x} \Rightarrow h\sqrt{3} = 80 - x$ 1

Solving equation to get

$x = 20,\ h = 20\sqrt{3}$

$\therefore$ AB = 60 m, BC = 20 m and h = $20\sqrt{3}$ m. 1

29. Let the smaller tap fills the tank in x hrs

$\therefore$ the larger tap fills the tank in (x – 2) hrs.

Time taken by both the taps together = $\frac{15}{8}$ hrs.

Therefore $\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$ 2

$\Rightarrow \quad 4x^2 - 23x + 15 = 0$ $\frac{1}{2}$

$\Rightarrow \quad (4x - 3)(x - 5) = 0$

$x \neq \frac{3}{4} \quad \therefore x = 5$ 1

Smaller and larger taps can fill the tank seperately in 5 hrs and 3 hrs resp. $\frac{1}{2}$

OR

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Given $\frac{30}{x-y} + \frac{44}{x+y} = 10$ ...(i) 1

and $\frac{40}{x-y} + \frac{55}{x+y} = 13$ ...(ii) 1

Solving (i) and (ii) to get

$x + y = 11$ ..(iii)

and $x - y = 5$ ...(iv)

Solving (iii) and (iv) to get $x = 8$, $y = 3$. 1+1

Speed of boat = 8 km/hr & speed of stream = 3 km/hr.

30. $S_4 = 40 \Rightarrow 2(2a + 3d) = 40 \Rightarrow 2a + 3d = 20$ 1

$S_{14} = 280 \Rightarrow 7(2a + 13d) = 280 \Rightarrow 2a + 13d = 40$ 1

Solving to get $d = 2$ $\frac{1}{2}$

and $a = 7$ $\frac{1}{2}$

$\therefore \quad Sn = \frac{n}{2}[14 + (n-1) \times 2]$

$= n(n + 6)$ or $(n^2 + 6n)$ 1