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Secondary School Examination
March 2019

MARKING SCHEME – MATHEMATICS (SUBJECT CODE -041)

PAPER CODE: 30/3/1, 30/3/2, 30/3/3

General Instructions: -

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 1-80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.

12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 30/3/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $(x + 5)^2 = 2(5x - 3) \Rightarrow x^2 + 31 = 10$ $\frac{1}{2}$

$D = -124$ $\frac{1}{2}$

2. $\frac{27}{2^3 \cdot 5^4 \cdot 3^2} = \frac{3}{2^3 \cdot 5^4}$ $\frac{1}{2}$

It will terminate after 4 decimal places $\frac{1}{2}$

OR

$429 = 3 \times 11 \times 13$ 1

3. $S_{10} = \frac{10}{2}[2\times 6 + 9 \times 6]$ $\frac{1}{2}$

$= 330$ $\frac{1}{2}$

4. $AB = 5$

$\Rightarrow \sqrt{(x-0)^2 + (-4-0)^2} = 5$ $\frac{1}{2}$

$x^2 + 16 = 25$

$x = \pm 3$ $\frac{1}{2}$

5. Length of chord $= 2\sqrt{a^2 - b^2}$ 1

6. $PQ = 5$ cm $\frac{1}{2}$

$\tan\theta = \frac{PQ}{PR} = \frac{5}{9}$ $\frac{1}{2}$

OR

$\sec\alpha = \sqrt{1 + \tan^2\alpha}$ $\frac{1}{2}$

$= \sqrt{1 + \frac{25}{144}} = \frac{13}{12}$ $\frac{1}{2}$

SECTION B

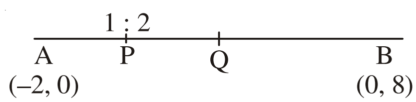
7. Diagonals of parallelogram bisect each other

$\therefore \quad \left(\frac{3+a}{2}, \frac{1+b}{2}\right) = \left(\frac{5+4}{2}, \frac{1+3}{2}\right)$ 1

$3 + a = 9, \ 1 + b = 4$

So $\quad a = 6, \ b = 3$ $\frac{1}{2}+\frac{1}{2}$

OR



P divides AB in the ratio 1 : 2

$\therefore$ Coordinates of P are $\left(\frac{0-4}{3}, \frac{8+0}{2}\right) = \left(\frac{-4}{3}, \frac{8}{3}\right)$ 1

Q divides AB in the ratio 2 : 1

$\therefore$ Coordinates of Q are $\left(\frac{0-2}{3}, \frac{16+0}{3}\right) = \left(\frac{-2}{3}, \frac{16}{3}\right)$ 1

8. $3x - 5y = 4$ ...(1)

$9x - 2y = 7$

$9x - 15y = 12$

$9x - 2y = 7$

$-\quad +\quad -$

$-13y = 5 \Rightarrow y = -5/13$ 1

From (1), $x = 9/13$ $\therefore$ solution is $\left(\frac{9}{13}, \frac{-5}{13}\right)$ 1

9. HCF (65, 117) = 13 1

$13 = 65n - 117$ $\frac{1}{2}$

Solving, we get, $n = 2$ $\frac{1}{2}$

OR

Required minimum distance $= \text{LCM}(30, 36, 40)$ 1

$30 = 2 \times 3 \times 5$ $\quad = 2^3 \times 3^2 \times 5$

$36 = 2^2 \times 3^2$ $\quad = 360$ cm 1

$40 = 2^3 \times 5$

10. Composite numbers on a die are 4 and 6

$\therefore$ P (composite number) $= \frac{2}{6}$ or $\frac{1}{3}$ 1

Prime numbers are 2, 3 and 5

$\therefore$ P(prime number) $= \frac{3}{6}$ or $\frac{1}{2}$ 1

11. $x^2 - 8x + 18 = 0$

$x^2 - 8x + 16 + 2 = 0$ 1

$(x - 4)^2 = -2$ $\frac{1}{2}$

Square of a number can't be negative

$\therefore$ The equation has no solution. $\frac{1}{2}$

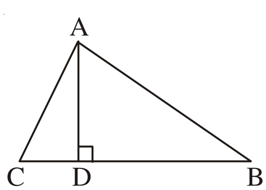
12. Total number of possible outcomes = 34 $\frac{1}{2}$

Favourable number of outcomes is (7, 14, 21, 28 and 35) = 5 1

P(multiple of 7) $= \frac{5}{34}$ $\frac{1}{2}$

SECTION C

13.



$AB^2 = AD^2 + BD^2$ Correct Figure $\frac{1}{2}$

$AC^2 = AD^2 + CD^2$ 1

$AB^2 - AC^2 = BD^2 - CD^2$

$= (3CD)^2 - CD^2$

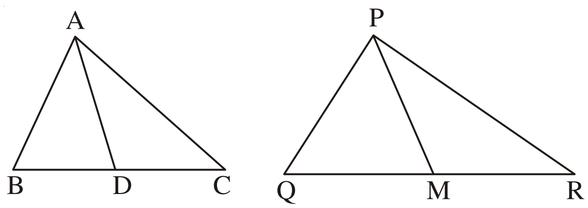
$= 8\ CD^2$ 1

$$= 8 \times \left(\frac{1}{4}BC\right)^2$$

$$\Rightarrow 2AB^2 - 2AC^2 = BC^2$$

$$\text{or } 2AB^2 = 2AC^2 + BC^2 \qquad \frac{1}{2}$$

OR



Correct Figure $\frac{1}{2}$

$\Delta ABC \sim \Delta PQR$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \qquad \frac{1}{2}$$

$$\frac{AB}{PQ} = \frac{2BD}{2QM} \text{ or } \frac{BD}{QM} \qquad 1$$

Also $\angle B = \angle Q$

$$\therefore \Delta ABD \sim \Delta PQM \qquad \frac{1}{2}$$

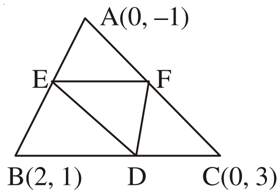
$$\text{So } \frac{AB}{PQ} = \frac{AD}{PM} \qquad \frac{1}{2}$$

14.

$$\begin{array}{r|l|l}
x^3 - 3x + 1 & \cancel{x^5} - 4x^3 + \cancel{x^2} + 3x + 1 & x^2 - 1 \\
 & \cancel{x^5} - 3x^3 + \cancel{x^2} & \\
 & - \quad + \quad - & \\
\hline
 & \cancel{-x^3} + \cancel{3x} + 1 & \\
 & \cancel{-x^3} + \cancel{3x} - 1 & \\
 & + \quad - \quad + & \\
\hline
 & 2 &
\end{array} \qquad 2\frac{1}{2}$$

Since remainder $\neq 0$ $\therefore$ g(x) is not a factor of p(x) $\frac{1}{2}$

15.



Coordinates of mid points are

D(1, 2)

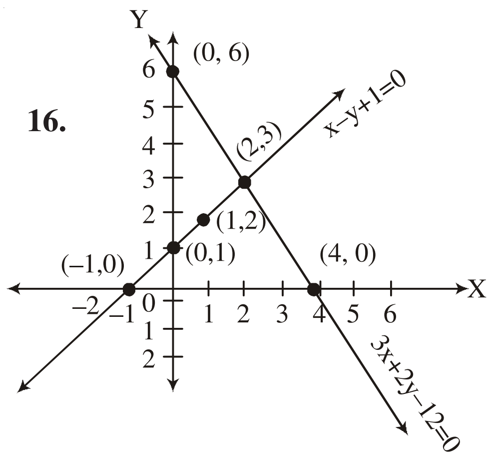
E (1, 0)

F(0 ,1) $1\frac{1}{2}$

Area of $\Delta DEF = \frac{1}{2}[1(0-1)+1(1-2)+0]$ 1

$= \frac{1}{2}(-2) = 1\,\text{sq. unit}$ $\frac{1}{2}$

16.



Correct graph 2

Solution is

x = 2, y = 3 $\frac{1}{2}+\frac{1}{2}$

17. Let us assume that $\sqrt{3}$ be a rational number

$\sqrt{3} = \frac{p}{q}$ where p and q are co-primes and $q \neq 0$ $\frac{1}{2}$

$\Rightarrow$ $p^2 = 3q^2$ ...(1)

$\therefore$ 3 divides p^2

i.e., 3 divides p also ...(2)

Let p = 3m, for some integer m 1

From (1), $9m^2 = 3q^2$

$\Rightarrow$ $q^2 = 3m^2$

$\therefore$ 3 divides q^2 i.e., 3 divides q also ...(3) 1

From (2) and (3), we get that 3 divides p and q both which is a contradiction to the fact that p and q are co-primes. $\frac{1}{2}$

Hence our assumption is wrong

$\therefore \sqrt{3}$ is irrational

OR

$1251 - 1 = 1250,\ 9377 - 2 = 9375,\ 15628 - 3 = 15625$ 1

Required largest number = HCF (1250, 9375, 15625)

$$\left.\begin{array}{l} 1250 = 2 \times 5^4 \\ 9375 = 3 \times 5^4 \\ 6250 = 2 \times 5^5 \end{array}\right\}$$ $1\frac{1}{2}$

$\therefore$ HCF (1250, 9375, 15625) $= 5^4 = 625$ $\frac{1}{2}$

18. A, B, C are interior angles of ΔABC

$\therefore \quad A + B + C = 180°$ $\frac{1}{2}$

(i) $\sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{180° - A}{2}\right)$

$= \sin\left(90° - \frac{A}{2}\right)$

$= \cos\frac{A}{2}$ $1\frac{1}{2}$

(ii) $\tan\left(\frac{B+C}{2}\right) = \tan\left(\frac{90°}{2}\right)$ $(\because \angle A = 90°)$

$= \tan 45°$ 1

$= 1$

OR

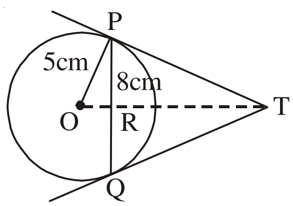
$\tan(A + B) = 1 \ \therefore\ A + B = 45°$ 1

$\tan(A - B) = \frac{1}{\sqrt{3}} \ \therefore\ A - B = 30°$ 1

Solving, we get $\angle A = 37\frac{1°}{2}$ or $37.5°$ $\frac{1}{2}$

$\angle B = 7\frac{1°}{2}$ or $7.5°$ $\frac{1}{2}$

19.



Let TR be x cm and TP be y cm

OT is $\perp$ bisector of PQ

So PR = 4 cm

In ΔOPR, $OP^2 = PR^2 + OR^2$

$\therefore$ OR = 3 cm 1

In ΔPRT, $y^2 = x^2 + 4^2$ ...(1) $\frac{1}{2}$

In ΔOPT, $(x + 3)^2 = 5^2 + y^2$ $\frac{1}{2}$

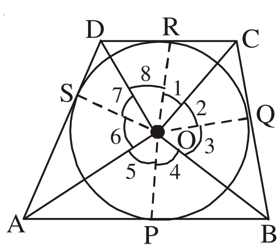
$\therefore (x + 3)^2 = 5^2 + x^2 + 16$ [using (1)]

Solving we get $x = \frac{16}{3}$ cm $\frac{1}{2}$

$$\left.\begin{array}{l} \text{From (1), } y^2 = \dfrac{256}{9} + 16 = \dfrac{400}{9} \\ \text{So } y = \dfrac{20}{3}\text{ cm} \end{array}\right\}$$ $\frac{1}{2}$

OR

$\Delta ROC \cong \Delta QOC$ $\frac{1}{2}$



$$\left.\begin{array}{l} \therefore \angle 1 = \angle 2 \\ \text{Similarly } \angle 4 = \angle 3 \\ \angle 5 = \angle 6 \\ \angle 8 = \angle 7 \end{array}\right\}$$ 1

$\angle ROQ + \angle QOP + \angle POS + \angle SOR = 360°$ $\frac{1}{2}$

$\therefore 2\angle 1 + 2\angle 4 + 2\angle 5 + 2\angle 8 = 360$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180°$

So, $\angle DOC + \angle AOB = 180°$

and $\angle AOD + \angle BOC = 180°$. 1

20. Volume of water flowing through canal in 30 minutes

$= 5000 \times 6 \times 1.5 = 45000 \text{ m}^3$ $1\frac{1}{2}$

Area $= 45000 \div \frac{8}{100}$

$= 562500 \text{ m}^2$ $1\frac{1}{2}$

21.

Number of days	**Number of students (fi)**	x_i	f_ix_i	
0-6	10	3	30	
6-12	11	9	99	
12-18	7	15	105	
18-24	4	21	84	
24-30	4	27	108	Correct Table 2
30-36	3	33	99	
36-42	1	39	39	
Total	40		564	

$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{564}{40}$

$= 14.1$ 1

22. Total area cleaned = 2 × Area of sector

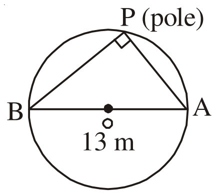
$= 2 \times \frac{\pi r^2 \theta}{260°}$ 1

$= 2 \times \frac{22}{7} \times 21 \times 21 \times \frac{120°}{360°}$ 1

$= 924 \text{ cm}^2$ 1

SECTION D

23. Correct Figure $\frac{1}{2}$



$PB - PA = 7$ m

Let AP be x m $\therefore$ $PB = (x + 7)$ m $\frac{1}{2}$

$AB^2 = PB^2 + AB^2$

$\therefore\ 13^2 = (x + 7)^2 + x^2$

$x^2 + 7x - 60 = 0$ 1

$= (x + 12)\ (x - 5) = 0$ 1

$\therefore\ x = 5, -12$ Rejected

$\therefore$ Situation is possible $\frac{1}{2}$

$\therefore$ Distance of pole from gate A = 5 m

and distance of pole from gate B = 12 m. $\frac{1}{2}$

24. $ma_m = na_n$

$\Rightarrow\quad ma + m(m - 1)d = na + n(n - 1)d$ 1

$\Rightarrow\quad (m - n)a + (m^2 - m - n^2 + n)d = 0$ 1

$(m - n)a + [(m - n)\ (m + n) - (m - n)d] = 0$ 1

Dividing by $(m - n)$

So, $a + (m + n - 1)d = 0$

or $a_{m+n} = 0$ 1

OR

Let first three terms be $a - d$, a and $a + d$ $\frac{1}{2}$

$a - d + a + a + d = 18$

So $a = 6$ $\frac{1}{2}$

$(a - d)\ (a + d) = 5d$

$\Rightarrow \quad 6^2 - d^2 = 5d$ 1

or $\quad d^2 + 5d - 36 = 0$

$(d + 9)(d - 4) = 0$

so $\quad d = -9$ or 4 1

For $d = -9$ three numbers are 15, 6 and -3 $\frac{1}{2}$

For $d = 4$ three numbers are 2, 6 and 10 $\frac{1}{2}$

25. Correct construction of ΔABC 2

Correct construction of triangle similar to ΔABC 2

26. (a) Total surface area of block

= TSA of cube + CSA of hemisphere – Base area of hemisphere 1

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2$

$= \left(6 \times 6^2 + \frac{22}{7} \times 2.1 \times 2.1\right) \text{cm}^2$ $\frac{1}{2}$

$= (216 + 13.86)\ \text{cm}^2$

$= 229.86\ \text{cm}^2$ $\frac{1}{2}$

(b) Volume of block

$= 6^3 + \frac{2}{3} \times \frac{22}{7} \times (2.1)^3$ 1

$= (216 + 19.40)\ \text{cm}^3$

$= 235.40\ \text{cm}^3$ 1

OR

Volume of frustum = $12308.8\ \text{cm}^3$

$\therefore \quad \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2) = 12308.8$

$\Rightarrow \quad \frac{1}{3} \times 3.14 \times h(20^2 + 12^2 + 20 \times 12) = 12308.8$ 1

$h = \frac{12308.8 \times 3}{784 \times 3.14}$

$h = 15$ cm 1

$l = \sqrt{15^2 + (20-12)^2} = 17$ cm. 1

Area of metal sheet used = $\pi l\ (r_1 + r_2) + \pi r_2^2$

$= 3.14[17 \times 32 + 12^2]$

$= 3.14 \times 688\ cm^2$

$= 2160.32\ cm^2$ 1

27. Correct figure, given, to prove and construction $\frac{1}{2} \times 4 = 2$

Correct proof. 2

OR

Correct figure, given, to prove and construction $\frac{1}{2} \times 4 = 2$

Correct proof. 2

28. $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing by $\cos^2\theta$

$\sec^2\theta + \tan^2\theta = 3\tan\theta$ 1

$\Rightarrow\quad 1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow\quad 2\tan^2\theta - 3\tan\theta + 1 = 0$ 1

$(\tan\theta - 1)(2\tan\theta - 1) = 0$ 1

So $\tan\theta = 1$ or $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$

Alternate method

$1 + \sin^2\theta = 3\sin\theta\cos\theta$

$\sin^2\theta + \cos^2\theta + \sin^2\theta - 3\sin\theta\cos\theta = 0$ 1

Dividing by $\cos^2\theta$

$\Rightarrow\quad 2\tan^2\theta - 3\tan\theta + 1 = 0$ 1

$\Rightarrow\quad (\tan\theta - 1)(2\tan\theta - 1) = 0$ 1

So $\tan\theta = 1$ or $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$

29. **Class interval** | **Cumulative Frequency**

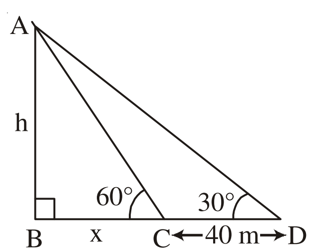
Class interval	Cumulative Frequency
More than or equal to 20	100
More than or equal to 30	90
More than or equal to 40	82
More than or equal to 50	70
More than or equal to 60	46
More than or equal to 70	40
More than or equal to 80	15

Correct Table $1\frac{1}{2}$

Plotting of points (20, 100), (30, 90), (40, 82), (50, 70), (60, 46), (70, 40) and (80, 15) $1\frac{1}{2}$

Joining the points to get a curve 1

30. Correct Figure 1



Let AB = h be the height of tower

In ΔABC, $\frac{h}{x} = \tan 60°$

$h = x\sqrt{3}$ 1

In ΔABD, $\frac{h}{x+40} = \tan 30°$

$\Rightarrow h\sqrt{3} = x + 40$ $\frac{1}{2}$

$3x = x + 40$

$\therefore x = 20$ $\frac{1}{2}$

So, height of tower $= h = 20\sqrt{3}$ m $\frac{1}{2}$

$= 20 \times 1.732$ m

$= 34.64$ m $\frac{1}{2}$

QUESTION PAPER CODE 30/3/2
EXPECTED ANSWER/VALUE POINTS
SECTION A

1. Length of chord = $2\sqrt{a^2 - b^2}$ 1

2. PQ = 5 cm $\frac{1}{2}$

$\tan\theta = \frac{PQ}{PR} = \frac{5}{9}$ $\frac{1}{2}$

OR

$\sec\alpha = \sqrt{1+\tan^2\alpha}$ $\frac{1}{2}$

$= \sqrt{1+\frac{25}{144}} = \frac{13}{12}$ $\frac{1}{2}$

3. $(x + 5)^2 = 2(5x - 3) \Rightarrow x^2 + 31 = 10$ $\frac{1}{2}$

$D = -124$ $\frac{1}{2}$

4. $\frac{27}{2^3 \cdot 5^4 \cdot 3^2} = \frac{3}{2^3 \cdot 5^4}$ $\frac{1}{2}$

It will terminate after 4 decimal places $\frac{1}{2}$

OR

$429 = 3 \times 11 \times 13$ 1

5. $S_{10} = \frac{10}{2}[2\times 6 + 9\times 6]$ $\frac{1}{2}$

$= 330$ $\frac{1}{2}$

6. AB = 10

$(13 - 5)^2 + (m + 3)^2 = 10$

$(m + 3)^2 = 100 - 64 = 6^2$ $\frac{1}{2}$

$m + 3 = 6$

$m = 3$ $\frac{1}{2}$

SECTION B

7. Composite numbers on a die are 4 and 6

$\therefore$ P (composite number) = $\frac{2}{6}$ or $\frac{1}{3}$ 1

Prime numbers are 2, 3 and 5

$\therefore$ P(prime number) = $\frac{3}{6}$ or $\frac{1}{2}$ 1

8. Total number of possible outcomes = 34 $\frac{1}{2}$

Favourable number of outcomes is (7, 14, 21, 28 and 35) = 5 1

P(multiple of 7) = $\frac{5}{34}$ $\frac{1}{2}$

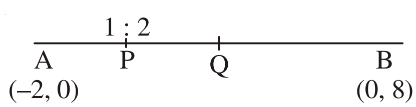
9. Diagonals of parallelogram bisect each other

$\therefore \quad \left(\frac{3+a}{2}, \frac{1+b}{2}\right) = \left(\frac{5+4}{2}, \frac{1+3}{2}\right)$ 1

$3 + a = 9,\ 1 + b = 4$

So $a = 6,\ b = 3$ $\frac{1}{2}+\frac{1}{2}$

OR



P divides AB in the ratio 1 : 2

$\therefore$ Coordinates of P are $\left(\frac{0-4}{3}, \frac{8+0}{2}\right) = \left(\frac{-4}{3}, \frac{8}{3}\right)$ 1

Q divides AB in the ratio 2 : 1

$\therefore$ Coordinates of Q are $\left(\frac{0-2}{3}, \frac{16+0}{3}\right) = \left(\frac{-2}{3}, \frac{16}{3}\right)$ 1

10. $3x - 5y = 4$ ...(1)

$9x - 2y = 7$

$9x - 15y = 12$

$9x - 2y = 7$

$-\quad +\quad -$

$-13y = 5 \Rightarrow y = -5/13$ 1

From (1), x = 9/13 $\therefore$ solution is $\left(\frac{9}{13}, \frac{-5}{13}\right)$ 1

11. HCF (65, 117) = 13 1

13 = 65n – 117 $\frac{1}{2}$

Solving, we get, n = 2 $\frac{1}{2}$

OR

Required minimum distance = LCM (30, 36, 40) 1

$30 = 2 \times 3 \times 5$ $\quad = 2^3 \times 3^2 \times 5$

$36 = 2^2 \times 3^2$ $\quad = 360$ cm 1

$40 = 2^3 \times 5$

12. $k^2 - 6x - 1 = 0$

Since the roots are not real $\therefore$ D < 0 1

$(-6)^2 - 4 \times k \times (-1) < 0$

$k < -9$ 1

SECTION C

13. A, B, C are interior angles of ΔABC

$\therefore \quad A + B + C = 180°$ $\frac{1}{2}$

(i) $\sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{180° - A}{2}\right)$

$= \sin\left(90° - \frac{A}{2}\right)$

$= \cos\frac{A}{2}$ $1\frac{1}{2}$

(ii) $\tan\left(\frac{B+C}{2}\right) = \tan\left(\frac{90°}{2}\right)$ $\quad (\because \angle A = 90°)$

$= \tan 45°$ 1

$= 1$

OR

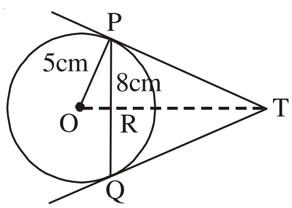
$\tan(A + B) = 1 \therefore A + B = 45°$ 1

$\tan(A - B) = \dfrac{1}{\sqrt{3}} \therefore A - B = 30°$ 1

Solving, we get $\angle A = 37\dfrac{1°}{2}$ or $37.5°$ $\frac{1}{2}$

$\angle B = 7\dfrac{1°}{2}$ or $7.5°$ $\frac{1}{2}$

14.



Let TR be x cm and TP be y cm

OT is $\perp$ bisector of PQ

So PR = 4 cm

In ΔOPR, $OP^2 = PR^2 + OR^2$

$\therefore$ OR = 3 cm 1

In ΔPRT, $y^2 = x^2 + 4^2$ ...(1) $\frac{1}{2}$

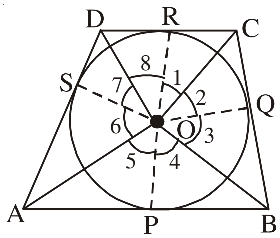
In ΔOPT, $(x + 3)^2 = 5^2 + y^2$ $\frac{1}{2}$

$\therefore (x + 3)^2 = 5^2 + x^2 + 16$ [using (1)]

Solving we get $x = \dfrac{16}{3}$ cm $\frac{1}{2}$

$\left.\begin{array}{l} \text{From (1), } y^2 = \dfrac{256}{9} + 16 = \dfrac{400}{9} \\ \text{So } y = \dfrac{20}{3} \text{ cm} \end{array}\right\}$ $\frac{1}{2}$

OR



$\Delta ROC \cong \Delta QOC$ $\frac{1}{2}$

$$\left.\begin{array}{l}\therefore \angle 1 = \angle 2 \\ \text{Similarly } \angle 4 = \angle 3 \\ \angle 5 = \angle 6 \\ \angle 8 = \angle 7\end{array}\right\}$$ 1

$\angle ROQ + \angle QOP + \angle POS + \angle SOR = 360°$ $\frac{1}{2}$

$\therefore\ 2\angle 1 + 2\angle 4 + 2\angle 5 + 2\angle 8 = 360$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180°$

So, $\angle DOC + \angle AOB = 180°$

and $\angle AOD + \angle BOC = 180°$. 1

15.

Number of days	**Number of students (fi)**	x_i	f_ix_i	
0-6	10	3	30	
6-12	11	9	99	
12-18	7	15	105	
18-24	4	21	84	
24-30	4	27	108	Correct Table 2
30-36	3	33	99	
36-42	1	39	39	
Total	40		564	

$$\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{564}{40}$$

$= 14.1$ 1

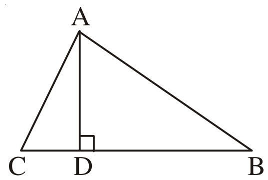
16. Total area cleaned = 2 × Area of sector

$= 2 \times \frac{\pi r^2 \theta}{260°}$ 1

$= 2 \times \frac{22}{7} \times 21 \times 21 \times \frac{120°}{360°}$ 1

$= 924\ cm^2$ 1

17. $AB^2 = AD^2 + BD^2$ Correct Figure $\frac{1}{2}$



$AC^2 = AD^2 + CD^2$ 1

$AB^2 - AC^2 = BD^2 - CD^2$

$= (3CD)^2 - CD^2$

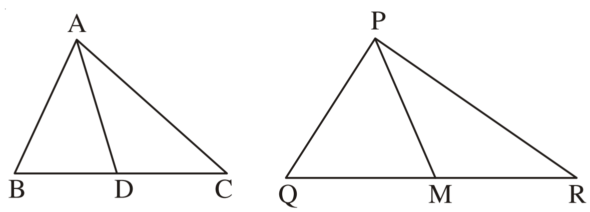
$= 8\ CD^2$ 1

$= 8 \times \left(\frac{1}{4}BC\right)^2$

$\Rightarrow 2AB^2 - 2AC^2 = BC^2$

or $2AB^2 = 2AC^2 + BC^2$ $\frac{1}{2}$

OR



Correct Figure $\frac{1}{2}$

$\Delta ABC \sim \Delta PQR$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ $\frac{1}{2}$

$\frac{AB}{PQ} = \frac{2BD}{2QM}$ or $\frac{BD}{QM}$ 1

Also $\angle B = \angle Q$

$\therefore \Delta ABD \sim \Delta PQM$ $\frac{1}{2}$

So $\frac{AB}{PQ} = \frac{AD}{PM}$ $\frac{1}{2}$

18.

$$
\begin{array}{r|l|l}
x^3 - 3x + 1 & \cancel{x^5} - 4x^3 + \cancel{x^2} + 3x + 1 & x^2 - 1 \\
 & \cancel{x^5} - 3x^3 + \cancel{x^2} & \\
 & \underline{-\quad\ +\qquad -\qquad\qquad\qquad} & \\
 & \quad -\cancel{x^3} + 3\cancel{x} + 1 & \\
 & \quad \cancel{-x^3} + \cancel{3x} - 1 & \\
 & \underline{\quad +\qquad -\qquad +\qquad\qquad} & \\
 & \qquad\qquad\quad 2 &
\end{array}
$$

$2\frac{1}{2}$

Since remainder $\neq 0$ $\therefore$ g(x) is not a factor of p(x) $\frac{1}{2}$

19. Let us assume that $\sqrt{3}$ be a rational number

$\sqrt{3} = \frac{p}{q}$ where p and q are co-primes and $q \neq 0$ $\frac{1}{2}$

$\Rightarrow \quad p^2 = 3q^2 \qquad ...(1)$

$\therefore \quad$ 3 divides p^2

i.e., 3 divides p also ...(2)

Let p = 3m, for some integer m 1

From (1), $9m^2 = 3q^2$

$\Rightarrow \quad q^2 = 3m^2$

$\therefore \quad$ 3 divides q^2 i.e., 3 divides q also ...(3) 1

From (2) and (3), we get that 3 divides p and q both which is a contradiction to the fact that p and q are co-primes. $\frac{1}{2}$

Hence our assumption is wrong $\therefore$ $\sqrt{3}$ is irrational

OR

1251 − 1 = 1250, 9377 − 2 = 9375, 15628 − 3 = 15625 1

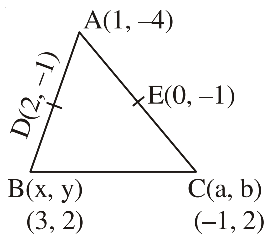
Required largest number = HCF (1250, 9375, 15625)

$$
\left.\begin{array}{l}
1250 = 2 \times 5^4 \\
9375 = 3 \times 5^4 \\
6250 = 2 \times 5^5
\end{array}\right\}
$$

$1\frac{1}{2}$

$\therefore \quad$ HCF (1250, 9375, 15625) $= 5^4 = 625$ $\frac{1}{2}$

20.



$$\left(\frac{x+1}{2}, \frac{y-4}{2}\right) = (2, -1)$$

$\therefore$ $x = 3, y = 2$ 1

$$\left(\frac{1+a}{2}, \frac{-4+b}{2}\right) = (0, -1)$$

$a = -1, b = 2$ 1

Area of $\Delta ABC = \frac{1}{2}[1(2-2)+3(2+4)-1(-4-2)]$

$= \frac{1}{2} \times 24 = 12 \text{ sq. units}$ 1

21. Let the numbers be 5x and 6x $\frac{1}{2}$

$$\frac{5x-7}{6x-7} = \frac{4}{5}$$ 1

Solving, we get x = 7 $\frac{1}{2}$

$\therefore$ Numbers are 35 and 42 $\frac{1}{2}+\frac{1}{2}$

22. Volume of water flowing through pipe in half an hour

$= \pi r^2 \times 1260 \text{ m}^3$ ...(1) $\frac{1}{2}$

Volume of water raised in cylinder

$= \pi \times \frac{40}{100} \times \frac{40}{100} \times \frac{315}{100} \text{ m}^3$ ...(2) 1

$(1) = (2) \Rightarrow r^2 = \frac{4}{10} \times \frac{4}{10} \times \frac{315}{100 \times 2160}$

$= \frac{4}{100 \times 100} \text{ m}^2 = 4 \text{ cm}^2$ 1

$\Rightarrow$ r = 2 cm, $\therefore$ diameter = 4 cm $\frac{1}{2}$

SECTION D

23. (a) Total surface area of block

= TSA of cube + CSA of hemisphere – Base area of hemisphere 1

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2$

$= \left(6 \times 6^2 + \frac{22}{7} \times 2.1 \times 2.1\right) \text{cm}^2$ $\frac{1}{2}$

$= (216 + 13.86)\ \text{cm}^2$

$= 229.86\ \text{cm}^2$ $\frac{1}{2}$

(b) Volume of block

$= 6^3 + \frac{2}{3} \times \frac{22}{7} \times (2.1)^3$ 1

$= (216 + 19.40)\ \text{cm}^3$

$= 235.40\ \text{cm}^3$ 1

OR

Volume of frustum = 12308.8 cm^3

$\therefore \quad \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2) = 12308.8$

$\Rightarrow \quad \frac{1}{3} \times 3.14 \times h(20^2 + 12^2 + 20 \times 12) = 12308.8$ 1

$h = \frac{12308.8 \times 3}{784 \times 3.14}$

$h = 15$ cm 1

$l = \sqrt{15^2 + (20-12)^2} = 17$ cm. 1

Area of metal sheet used $= \pi l\,(r_1 + r_2) + \pi r_2^2$

$= 3.14[17 \times 32 + 12^2]$

$= 3.14 \times 688\ \text{cm}^2$

$= 2160.32\ \text{cm}^2$ 1

24. Correct figure, given, to prove and construction $\frac{1}{2}\times 4=2$

Correct proof. 2

OR

Correct figure, given, to prove and construction $\frac{1}{2}\times 4=2$

Correct proof. 2

25.

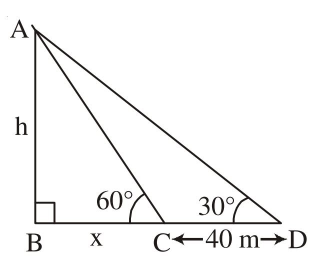
Class interval	**Cumulative Frequency**
More than or equal to 20	100
More than or equal to 30	90
More than or equal to 40	82
More than or equal to 50	70
More than or equal to 60	46
More than or equal to 70	40
More than or equal to 80	15

Correct Table $1\frac{1}{2}$

Plotting of points (20, 100), (30, 90), (40, 82), (50, 70), (60, 46), (70, 40) and (80, 15) $1\frac{1}{2}$

Joining the points to get a curve 1

26. Correct Figure 1



Let AB = h be the height of tower

In ΔABC, $\frac{h}{x} = \tan 60°$

$h = x\sqrt{3}$ 1

In ΔABD, $\frac{h}{x+40} = \tan 30°$

$\Rightarrow h\sqrt{3} = x + 40$ $\frac{1}{2}$

$3x = x + 40$

$\therefore x = 20$ $\frac{1}{2}$

So, height of tower $= h = 20\sqrt{3}$ m $\frac{1}{2}$

$= 20 \times 1.732$ m

$= 34.64$ m $\frac{1}{2}$

27. $ma_m = na_n$

$\Rightarrow$ $ma + m(m-1)d = na + n(n-1)d$ 1

$\Rightarrow$ $(m-n)a + (m^2 - m - n^2 + n)d = 0$ 1

$(m-n)a + [(m-n)(m+n) - (m-n)d] = 0$ 1

Dividing by $(m - n)$

So, $a + (m + n - 1)d = 0$

or $a_{m+n} = 0$ 1

OR

Let first three terms be $a - d$, a and $a + d$ $\frac{1}{2}$

$a - d + a + a + d = 18$

So $a = 6$ $\frac{1}{2}$

$(a - d)(a + d) = 5d$

$\Rightarrow$ $6^2 - d^2 = 5d$ 1

or $d^2 + 5d - 36 = 0$

$(d + 9)(d - 4) = 0$

so $d = -9$ or 4 1

For $d = -9$ three numbers are 15, 6 and -3 $\frac{1}{2}$

For $d = 4$ three numbers are 2, 6 and 10 $\frac{1}{2}$

28. Let the number of books be x

$\frac{80}{x} - \frac{80}{x+4} = 1$ 2

$x^2 + 4x - 320 = 0$ 1

$(x + 20)(x - 16) = 0$

$x = -20,\ 16$
(rejected)

$\therefore$ Number of books = 16 1

29. Correct construction of circle. 1

Correct construction of tangents. 3

30. LHS $= \dfrac{1}{1+\sin^2\theta} + \dfrac{1}{1+\cos^2\theta} + \dfrac{1}{1+\sec^2\theta} + \dfrac{1}{1+\text{cosec}^2\theta}$

$= \dfrac{1}{1+\sin^2\theta} + \dfrac{1}{1+\cos^2\theta} + \dfrac{1}{1+\dfrac{1}{\cos^2\theta}} + \dfrac{1}{1+\dfrac{1}{\sin^2\theta}}$ 1

$= \dfrac{1}{1+\sin^2\theta} + \dfrac{1}{1+\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta+1} + \dfrac{\sin^2\theta}{\sin^2\theta+1}$ 1

$= \dfrac{1+\sin^2\theta}{1+\sin^2\theta} + \dfrac{1+\cos^2\theta}{1+\cos^2\theta}$ 1

$= 1 + 1 = 2$

= R.H.S. 1

QUESTION PAPER CODE 30/3/3
EXPECTED ANSWER/VALUE POINTS
SECTION A

1. PQ = 5 cm $\frac{1}{2}$

$\tan\theta = \frac{PQ}{PR} = \frac{5}{9}$ $\frac{1}{2}$

OR

$\sec\alpha = \sqrt{1+\tan^2\alpha}$ $\frac{1}{2}$

$= \sqrt{1+\frac{25}{144}} = \frac{13}{12}$ $\frac{1}{2}$

2. Length of chord = $2\sqrt{a^2 - b^2}$ 1

3. AB = 5

$\Rightarrow \sqrt{(x-0)^2 + (-4-0)^2} = 5$ $\frac{1}{2}$

$x^2 + 16 = 25$

$x = \pm 3$ $\frac{1}{2}$

4. $\frac{27}{2^3 \cdot 5^4 \cdot 3^2} = \frac{3}{2^3 \cdot 5^4}$ $\frac{1}{2}$

It will terminate after 4 decimal places $\frac{1}{2}$

OR

$429 = 3 \times 11 \times 13$ 1

5. $(x + 5)^2 = 2(5x - 3) \Rightarrow x^2 + 31 = 10$ $\frac{1}{2}$

$D = -124$ $\frac{1}{2}$

6. $S_{10} = \frac{10}{2}[2\times 3 + 9\times 3]$ $\frac{1}{2}$

$= 5 \times 33 = 165$ $\frac{1}{2}$

SECTION B

7. HCF (65, 117) = 13   1

13 = 65n − 117 $\frac{1}{2}$

Solving, we get, n = 2 $\frac{1}{2}$

OR

Required minimum distance = LCM (30, 36, 40)   1

$30 = 2 \times 3 \times 5$ $= 2^3 \times 3^2 \times 5$

$36 = 2^2 \times 3^2$ = 360 cm   1

$40 = 2^3 \times 5$

8. Composite numbers on a die are 4 and 6

∴ P (composite number) = $\frac{2}{6}$ or $\frac{1}{3}$ 1

Prime numbers are 2, 3 and 5

∴ P(prime number) = $\frac{3}{6}$ or $\frac{1}{2}$ 1

9. $x^2 - 8x + 18 = 0$

$x^2 - 8x + 16 + 2 = 0$ 1

$(x - 4)^2 = -2$ $\frac{1}{2}$

Square of a number can't be negative

∴ The equation has no solution. $\frac{1}{2}$

10. Total number of possible outcomes = 34 $\frac{1}{2}$

Favourable number of outcomes is (7, 14, 21, 28 and 35) = 5   1

P(multiple of 7) = $\frac{5}{34}$ $\frac{1}{2}$

11. $3x + 4y = 10 \quad \Rightarrow 3x + 4y = 10$

$2x - 2y = 2 \quad \Rightarrow 4x - 4y = 10$

On solving, $7x = 14 \quad \therefore \; x = 2$ 1

So, $y = 1$ 1

Solution is (2, 1)

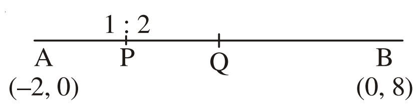
12. Diagonals of parallelogram bisect each other

$$\therefore \quad \left(\frac{3+a}{2}, \frac{1+b}{2}\right) = \left(\frac{5+4}{2}, \frac{1+3}{2}\right)$$ 1

$3 + a = 9, \; 1 + b = 4$

So $a = 6, \; b = 3$ $\frac{1}{2}+\frac{1}{2}$

OR



P divides AB in the ratio 1 : 2

$\therefore$ Coordinates of P are $\left(\frac{0-4}{3}, \frac{8+0}{2}\right) = \left(\frac{-4}{3}, \frac{8}{3}\right)$ 1

Q divides AB in the ratio 2 : 1

$\therefore$ Coordinates of Q are $\left(\frac{0-2}{3}, \frac{16+0}{3}\right) = \left(\frac{-2}{3}, \frac{16}{3}\right)$ 1

SECTION C

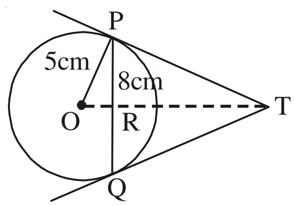
13.

Number of days	Number of students (fi)	x_i	$f_i x_i$	
0-6	10	3	30	
6-12	11	9	99	
12-18	7	15	105	
18-24	4	21	84	
24-30	4	27	108	Correct Table 2
30-36	3	33	99	
36-42	1	39	39	
Total	40		564	

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{564}{40}$$

$= 14.1$ 1

14. Let TR be x cm and TP be y cm



OT is $\perp$ bisector of PQ

So PR = 4 cm

In ΔOPR, $OP^2 = PR^2 + OR^2$

$\therefore$ OR = 3 cm 1

In ΔPRT, $y^2 = x^2 + 4^2$ ...(1) $\frac{1}{2}$

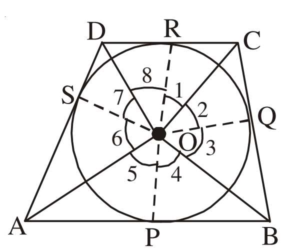
In ΔOPT, $(x + 3)^2 = 5^2 + y^2$ $\frac{1}{2}$

$\therefore (x + 3)^2 = 5^2 + x^2 + 16$ [using (1)]

Solving we get $x = \frac{16}{3}$ cm $\frac{1}{2}$

$$\left.\begin{array}{l}\text{From (1), } y^2 = \frac{256}{9} + 16 = \frac{400}{9} \\ \text{So } y = \frac{20}{3} \text{ cm}\end{array}\right\} \quad \frac{1}{2}$$

OR



$\Delta ROC \cong \Delta QOC$ $\frac{1}{2}$

$$\left.\begin{array}{l}\therefore \angle 1 = \angle 2 \\ \text{Similarly } \angle 4 = \angle 3 \\ \angle 5 = \angle 6 \\ \angle 8 = \angle 7\end{array}\right\} \quad 1$$

$\angle ROQ + \angle QOP + \angle POS + \angle SOR = 360°$ $\frac{1}{2}$

$\therefore 2\angle 1 + 2\angle 4 + 2\angle 5 + 2\angle 8 = 360$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180°$

So, $\angle DOC + \angle AOB = 180°$

and $\angle AOD + \angle BOC = 180°$. 1

15. A, B, C are interior angles of ΔABC

$\therefore \quad A + B + C = 180°$ $\frac{1}{2}$

(i) $\sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{180° - A}{2}\right)$

$= \sin\left(90° - \frac{A}{2}\right)$

$= \cos\frac{A}{2}$ $1\frac{1}{2}$

(ii) $\tan\left(\frac{B+C}{2}\right) = \tan\left(\frac{90°}{2}\right)$ $\quad (\because \angle A = 90°)$

$= \tan 45°$ 1

$= 1$

OR

$\tan(A + B) = 1 \ \therefore\ A + B = 45°$ 1

$\tan(A - B) = \frac{1}{\sqrt{3}} \ \therefore\ A - B = 30°$ 1

Solving, we get $\angle A = 37\frac{1°}{2}$ or $37.5°$ $\frac{1}{2}$

$\angle B = 7\frac{1°}{2}$ or $7.5°$ $\frac{1}{2}$

16. Let us assume that $\sqrt{3}$ be a rational number

$\sqrt{3} = \frac{p}{q}$ where p and q are co-primes and $q \neq 0$ $\frac{1}{2}$

$\Rightarrow \quad p^2 = 3q^2$ ...(1)

$\therefore \quad$ 3 divides p^2

i.e., 3 divides p also ...(2)

Let $p = 3m$, for some integer m 1

From (1), $9m^2 = 3q^2$

$\Rightarrow \quad q^2 = 3m^2$

$\therefore \quad$ 3 divides q^2 i.e., 3 divides q also ...(3) 1

From (2) and (3), we get that 3 divides p and q both which is a contradiction to the fact that p and q are co-primes. $\frac{1}{2}$

Hence our assumption is wrong $\therefore\ \sqrt{3}$ is irrational

OR

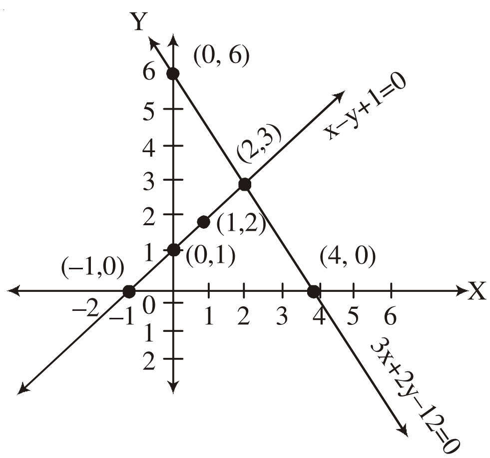
$1251 - 1 = 1250,\ 9377 - 2 = 9375,\ 15628 - 3 = 15625$ 1

Required largest number = HCF (1250, 9375, 15625)

$$\left.\begin{aligned} 1250 &= 2 \times 5^4 \\ 9375 &= 3 \times 5^4 \\ 6250 &= 2 \times 5^5 \end{aligned}\right\}$$ $1\frac{1}{2}$

$\therefore$ HCF (1250, 9375, 15625) $= 5^4 = 625$ $\frac{1}{2}$

17. Correct graph 2



Solution is

$x = 2,\ y = 3$ $\frac{1}{2}+\frac{1}{2}$

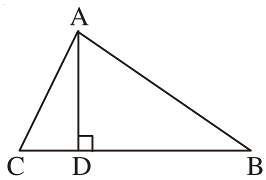
18. Volume of water flowing through canal in 30 minutes

$= 5000 \times 6 \times 1.5 = 45000\ \text{m}^3$ $1\frac{1}{2}$

Area $= 45000 \div \dfrac{8}{100}$

$= 562500\ \text{m}^2$ $1\frac{1}{2}$

19. $AB^2 = AD^2 + BD^2$ Correct Figure $\frac{1}{2}$



$AC^2 = AD^2 + CD^2$ 1

$AB^2 - AC^2 = BD^2 - CD^2$

$= (3CD)^2 - CD^2$

$= 8\ CD^2$ 1

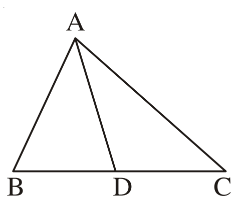
$= 8 \times \left(\frac{1}{4}BC\right)^2$

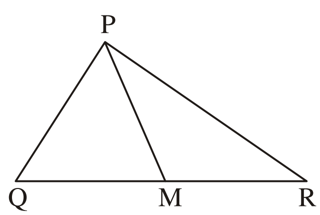
$\Rightarrow 2AB^2 - 2AC^2 = BC^2$

or $2AB^2 = 2AC^2 + BC^2$ $\frac{1}{2}$

OR

Correct Figure $\frac{1}{2}$





$\Delta ABC \sim \Delta PQR$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ $\frac{1}{2}$

$\frac{AB}{PQ} = \frac{2BD}{2QM}$ or $\frac{BD}{QM}$ 1

Also $\angle B = \angle Q$

$\therefore \Delta ABD \sim \Delta PQM$ $\frac{1}{2}$

So $\frac{AB}{PQ} = \frac{AD}{PM}$ $\frac{1}{2}$

20. Area of minor segment = $\frac{\pi r^2 \theta}{360°} - \frac{\sqrt{3}}{4} r^2$ 1

$= 14 \times 14 \left[\frac{22}{7} \times \frac{60°}{360°} - \frac{1.73}{4} \right] cm^2$ 1

$= \frac{14 \times 14}{84} (44 - 36.33)\ cm^2$

$= 17.90\ cm^2$ (approx.) 1

21. $\frac{1}{2}[(k+1)(-3+k) + 4(-k-1) + 7(1+3)] = 6$ 1

$\frac{1}{2}(k^2 - 6k + 21) = 6$ 1

$\Rightarrow \quad k^2 - 6k + 9 = 0$

$(k - 3)^2 = 0$

$\therefore \quad k = 3$ 1

22. $ax^2 + 7x + b$

Sum of zeroes = $\frac{-7}{a} = \frac{-7}{3}$ $1\frac{1}{2}$

$\therefore \quad a = 3$

Product of zeroes = $\frac{b}{a} = -2$

$\therefore \quad b = -6.$ $1\frac{1}{2}$

SECTION D

23.

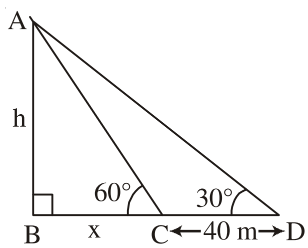
Class interval	**Cumulative Frequency**
More than or equal to 20	100
More than or equal to 30	90
More than or equal to 40	82
More than or equal to 50	70
More than or equal to 60	46
More than or equal to 70	40
More than or equal to 80	15

Correct Table $1\frac{1}{2}$

Plotting of points (20, 100), (30, 90), (40, 82), (50, 70), (60, 46), (70, 40) and (80, 15) $1\frac{1}{2}$

Joining the points to get a curve 1

24. Correct Figure 1



Let AB = h be the height of tower

In ΔABC, $\frac{h}{x} = \tan 60°$

$h = x\sqrt{3}$ 1

In ΔABD, $\frac{h}{x+40} = \tan 30°$

$\Rightarrow h\sqrt{3} = x + 40$ $\frac{1}{2}$

$3x = x + 40$

$\therefore x = 20$ $\frac{1}{2}$

So, height of tower $= h = 20\sqrt{3}$ m $\frac{1}{2}$

$= 20 \times 1.732$ m

$= 34.64$ m $\frac{1}{2}$

25. Correct figure, given, to prove and construction $\frac{1}{2}\times 4 = 2$

Correct proof. 2

OR

Correct figure, given, to prove and construction $\frac{1}{2}\times 4 = 2$

Correct proof. 2

26. $ma_m = na_n$

$\Rightarrow$ $ma + m(m-1)d = na + n(n-1)d$ 1

$\Rightarrow$ $(m-n)a + (m^2 - m - n^2 + n)d = 0$ 1

$(m-n)a + [(m-n)(m+n) - (m-n)d] = 0$ 1

Dividing by $(m - n)$

So, $a + (m + n - 1)d = 0$

or $a_{m+n} = 0$ 1

OR

Let first three terms be $a - d$, a and $a + d$ $\frac{1}{2}$

$a - d + a + a + d = 18$

So $a = 6$ $\frac{1}{2}$

$(a - d)(a + d) = 5d$

$\Rightarrow$ $6^2 - d^2 = 5d$ 1

or $d^2 + 5d - 36 = 0$

$(d + 9)(d - 4) = 0$

so $d = -9$ or 4 1

For $d = -9$ three numbers are 15, 6 and -3 $\frac{1}{2}$

For $d = 4$ three numbers are 2, 6 and 10 $\frac{1}{2}$

27. (a) Total surface area of block

= TSA of cube + CSA of hemisphere – Base area of hemisphere 1

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2$

$= \left(6 \times 6^2 + \frac{22}{7} \times 2.1 \times 2.1\right) cm^2$ $\frac{1}{2}$

$= (216 + 13.86)\ cm^2$

$= 229.86\ cm^2$ $\frac{1}{2}$

(b) Volume of block

$$= 6^3 + \frac{2}{3}\times\frac{22}{7}\times(2.1)^3$$ 1

$= (216 + 19.40)\ cm^3$

$= 235.40\ cm^3$ 1

OR

Volume of frustum = $12308.8\ cm^3$

$$\therefore \quad \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2) = 12308.8$$

$$\Rightarrow \quad \frac{1}{3}\times 3.14\times h(20^2 + 12^2 + 20\times 12) = 12308.8$$ 1

$$h = \frac{12308.8\times 3}{784\times 3.14}$$

$h = 15$ cm 1

$$l = \sqrt{15^2 + (20-12)^2} = 17\text{ cm.}$$ 1

Area of metal sheet used = $\pi l\ (r_1 + r_2) + \pi r_2^2$

$= 3.14[17 \times 32 + 12^2]$

$= 3.14 \times 688\ cm^2$

$= 2160.32\ cm^2$ 1

28. Correct construction of given triangle 2

Correct construction of triangle similar to given triangle 2

29. LHS $= \dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta}$

$$= \frac{\dfrac{\sin^3\theta}{\cos^3\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}} + \frac{\dfrac{\cos^3\theta}{\sin^3\theta}}{1+\dfrac{\cos^2\theta}{\sin^2\theta}}$$ 1

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$$

$$= \frac{\sin^4\theta + \cos^4\theta}{\cos\theta\sin\theta}$$ 1

$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$ 1

$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta} = \sec\theta\,\text{cosec}\,\theta - 2\sin\theta\cos\theta$$ 1

= R.H.S.

30. Let speed of stream be x km/hr.

Speed in downstream = (9 + x) km/hr. $\frac{1}{2}$

Speed in upstream = (9 – x) km/hr. $\frac{1}{2}$

$$\frac{15}{9+x} + \frac{15}{9-x} = 3\frac{45}{60} = 3\frac{3}{4}$$ 1

$$\frac{15(9-x+9+x)}{(9+x)(9-x)} = \frac{15}{4}$$

$\Rightarrow$ $72 = 81 - x^2$ 1

$x^2 = 9$

$x = 3$ or -3 Rejected

$\therefore$ Speed of stream = 3 km/hr 1