BLUE PRINT : SA-I (IX) : MATHEMATICS

Unit/Topic	1	2	3	4	Total
Number System	1(1)	2(1)	6(2)*	8(2)*	17(6)
Algebra Polynomials	3(3)	4(2)	6(2)	12(3)	25(10)
Geometry Euclids Geom, Lines and Angles, Triangles	2(2)	4(2)*	15(5)*	16(4)	37(13)
Coordinate Geometry	–	2(1)	–	4(1)	6(2)
Mensuration	2(2)	–	3(1)	–	5(3)
Total	8(8)	12(6)	30(10)	40(10)	90(34)

SAMPLE QUESTION PAPER, SA-I
CLASS : IX

Time : 3hrs.   MM : 90

SECTION - A

Question numbers 1 to 8 carry 1mark each. For each question, four alternative choices have been provided of which only one is correct. You have to select the correct choice.

1. Which of the following is a rational number?

   (A) $\dfrac{-2}{3}$ (B) $\dfrac{-1}{\sqrt{5}}$ (C) $\dfrac{13}{\sqrt{5}}$ (D) $\dfrac{\sqrt{2}}{3}$

2. The value of k, for which the polynomial x^3-3x^2+3x+k has 3 as its zero, is

   (A) -3 (B) 9   (C) -9 (D) 12

3. Which of the following is a zero of the polynomial x^3+3x^2-3x-1?

   (A) -1 (B) -2 (C) 1   (D) 2

4. The factorisation of $-x^2+5x-6$ yields:

   (A) $(x-2)(x-3)$ (B) $(2+x)(3-x)$ (C) $-(x-2)(3-x)$ (D) $-(2-x)(3-x)$

5. In fig.1, $\angle DBC$ equals

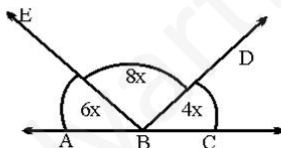   (A) $40°$ (B) $60°$ (C) $80°$ (D) $100°$

Fig.1

6. In fig.2, ABC is an equilateral triangle and BDC is an isosceles right triangle, right angled at D. $\angle ABD$ equals

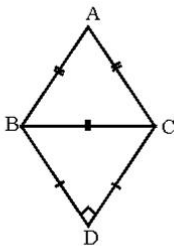   (A) $45°$ (B) $60°$ (C) $105°$ (D) $120°$

Fig.2

7. The sides of a triangle are 12cm, 16cm and 20cm. Its area is

   (A) $48cm^2$ (B) $96cm^2$ (C) $120cm^2$ (D) $160cm^2$

8. The side of an isosceles right triangle of hypotenuse $4\sqrt{2}cm$ is

   (A) 8cm   (B) 6cm   (C) 4cm   (D) $4\sqrt{3}$cm

Question numbers 9 to 14 carry 2 marks each :

9. If $x = 7 + \sqrt{40}$, find the value of $\sqrt{x} + \dfrac{1}{\sqrt{x}}$

10. Factorise the polynomial: $8x^3 - (2x-y)^3$

11. Find the value of 'a' for which $(x-1)$ is a factor of the polynominal $a^2x^3 - 4ax + 4a - 1$
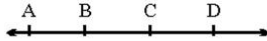

Fig. 3

12. In Fig.3, if AC=BD, show that AB=CD. State the Euclid's postulate/axiom used for the same.
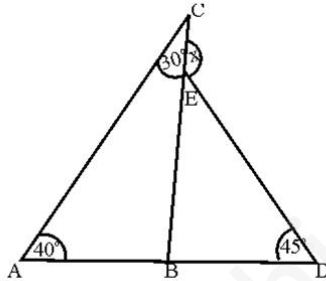

13. In Fig.4 find the value of x.   **Fig. 4**

OR

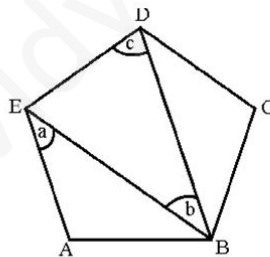In Fig.5, ABCDE is a regular pentagon. Find the relation between 'a' ,'b' and 'c'

Fig. 5

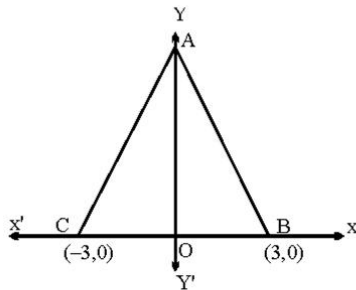14. In Fig.6, ABC is an equilateral triangle. The coordinates of vertices B and C are (3,0) and (−3,0)

Fig. 6

respectively. Find the coordinates of its vertex A.

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SECTION – C

</div>

Question numbers 15 to 24 carry 3 marks each:

15. **Evaluate :** $\left\{\sqrt{5+2\sqrt{6}}\right\}+\left\{\sqrt{8-2\sqrt{15}}\right\}$

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OR

</div>

If $a=9-4\sqrt{5}$, Find the value of $a^2+\dfrac{1}{a^2}$

16. Simplify the following:

$$\dfrac{7+3\sqrt{5}}{3+\sqrt{5}}-\dfrac{7-3\sqrt{5}}{3-\sqrt{5}}$$

<div align="center">

OR

</div>

If $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=a+\sqrt{15}\,b$, find the values of a and b

17. Factorise the following:

$12(x^2+7x)^2-8(x^2+7x)(2x-1)-15(2x-1)^2$

18. Show that 2 and $-\dfrac{1}{3}$ are the zeroes of the polynomial $3x^3-2x^2-7x-2$.

Also, find the third zero of the polynomial

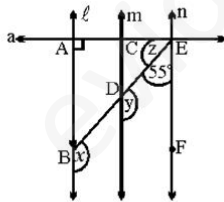19. In Fig.7, $\ell\parallel m\parallel n$ and $a\perp\ell$. If $\angle BEF=55°$, Find the values of x, y and z

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Fig.7   **OR**

</div>

In Fig.8, $\ell\parallel m\parallel n$. From the figure find the value of (y+x): (y-x)
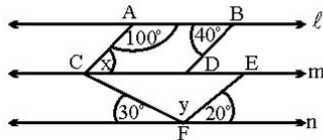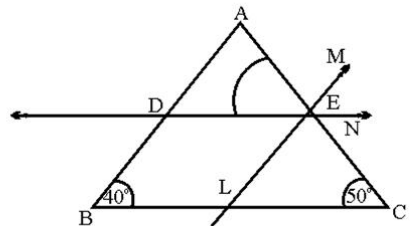

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Fig.8

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20. In Fig.9, DE∥BC and MF∥AB.
    Find (i) $\angle ADE+\angle MEN$ (ii) $\angle BDE$ (iii) $\angle BLE$

<div align="center">

Fig.9

</div>

21. In Fig.10, PS is the bisector of $\angle QPR$ and $PT \perp RQ$. Show that $\angle TPS = \frac{1}{2}(\angle R - \angle Q)$
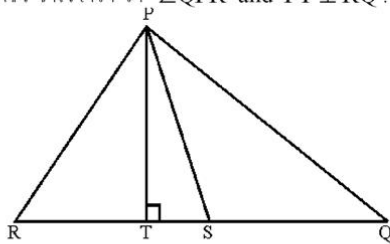

Fig.10

22. In Fig.11, $\triangle ABC$ and $\triangle ABD$ are such that AD=BC, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Prove that BD = AC
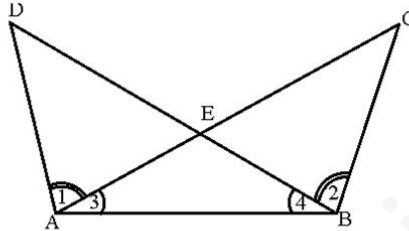

Fig.11

23. In Fig.12, AB||CD. If $\angle BAE = 50°$ and $\angle AEC = 20°$, find $\angle DCE$
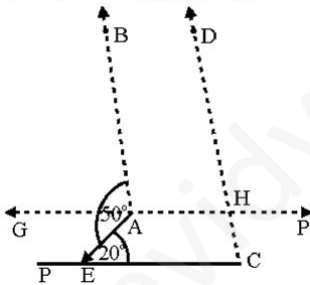

Fig.12

24. Find the area of a triangle whose perimeter is 180cm and two of its sides are 80cm and 18cm. Also calculate the altitude of the triangle corresponding to the shortest side.

SECTION-D

Question numbers 25 to 34 carry 4 marks each:

25. If $x = \dfrac{1}{2 - \sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 5$

OR

Simplify : $\dfrac{1}{1 + \sqrt{2}} + \dfrac{1}{\sqrt{2} + \sqrt{3}} + \dfrac{1}{\sqrt{3} + \sqrt{4}} + \ldots\ldots + \dfrac{1}{\sqrt{8} + \sqrt{9}}$

26. If $x = \dfrac{\sqrt{p+2q}+\sqrt{p-2q}}{\sqrt{p+2q}-\sqrt{p-2q}}$, then show that $qx^2-px+q=0$

<div align="center">OR</div>

If $x = \dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ and $y = \dfrac{\sqrt{2}-1}{\sqrt{2}+1}$, find the value of x^2+y^2+xy

27. If x^3+mx^2-x+6 has $(x-2)$ as a factor, and leaves a remainder n when divided by $(x-3)$, find the values of m and n.

28. Prove that $(x+y)^3 + (y+z)^3 + (z+x)^3 - 3(x+y)(y+z)(z+x) = 2(x^3+y^3+z^3-3xyz)$

29. If A and B be the remainders when the polynomials $x^3+2x^2-5ax-7$ and $x^3+ax^2-12x+6$ are divided by $(x+1)$ and $(x-2)$ respectively and $2A+B=6$, find the value of 'a'

30. From Fig.13, find the coordinates of the points A,B,C,D,E and F. Which of the points are mirror images in   (i) x-axis   (ii) y-axis
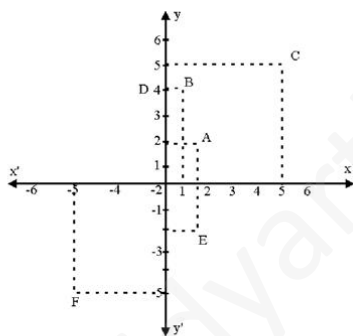

Fig.13

31. In Fig.14, $QT \perp PR$, $\angle TQR = 40°$ and $\angle SPR = 30°$. Find the values of x,y and z

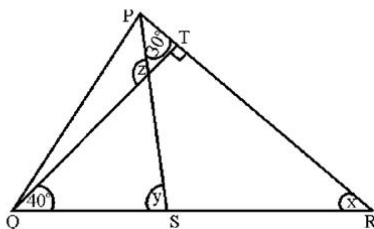**Fig.14**

32. In Fig.15, ABCD is a square and EF is parallel to diagonal BD and EM=FM Prove that
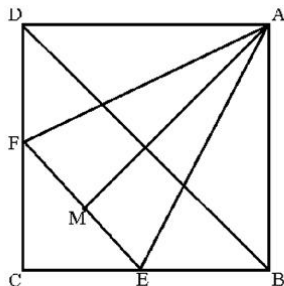
(i) DF=BE   (ii) AM bisects $\angle BAD$

Fig.15

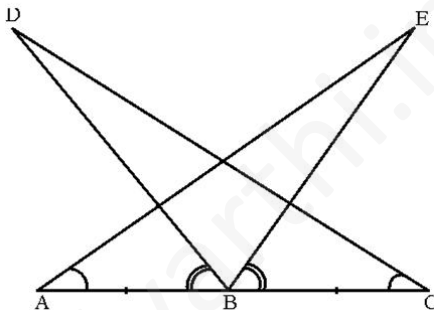33. In Fig.16, AB=BC, $\angle A = \angle C$ and $\angle ABD = \angle CBE$. Prove that CD=AE

Fig.16

34. In Fig.17, AB=AC, D is a point in the interior of $\triangle ABC$ such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$ of $\triangle ABC$
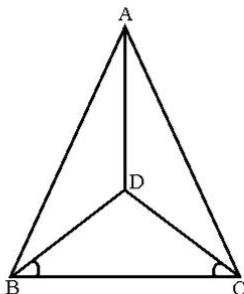
Fig.17

SAMPLE QUESTION PAPER, SA-I
MARKING SCHEME
CLASS : IX

Time : 3hrs. MM : 90

SECTION - A

1. (A)　　2. (C)　　3. (C)　　4. (D)

5. (A)　　6. (C)　　7. (B)　　8. (C)　　　　　　　　1x8=8

SECTION - B

9. $x = 7 + \sqrt{40} = 7 + 2\sqrt{10} = (\sqrt{5})^2 + (\sqrt{2})^2 + 2(\sqrt{5})(\sqrt{2}) = (\sqrt{5} + \sqrt{2})^2$　　½

$\Rightarrow \sqrt{x} = \sqrt{5} + \sqrt{2}$, $\dfrac{1}{\sqrt{x}} = \dfrac{1}{\sqrt{5} + \sqrt{2}} = \dfrac{\sqrt{5} - \sqrt{2}}{3}$　　½

$\therefore \sqrt{x} + \dfrac{1}{\sqrt{x}} = \dfrac{3(\sqrt{5} + \sqrt{2}) + (\sqrt{5} - \sqrt{2})}{3} = \dfrac{1}{3}\left[4\sqrt{5} + 2\sqrt{2}\right]$　　½

$= \dfrac{2}{3}\left[2\sqrt{5} + \sqrt{2}\right]$　　½

10. $8x^3 - (2x{-}y)^3 = (2x)^3 - (2x{-}y)^3$　　½

$= [2x{-}(2x{-}y)][(2x)^2 + (2x{-}y)^2 + 2x(2x{-}y)]$　　½

$= y\,[4x^2 + 4x^2 + y^2 - 4xy + 4x^2 - 2xy]$　　½

$= y\,[12x^2 + y^2 - 6xy]$　　½

11. $P(x) = a^2 x^3 - 4ax + 4a - 1$

$P(1) = 0 \Rightarrow a^2 - 4\cancel{a} + 4\cancel{a} - 1 = 0 \Rightarrow a = \pm 1$　　**1+1**

12. $AC = BD \Rightarrow AC - BC = BD - BC$

$\Rightarrow AB = CD$　　**1+½**
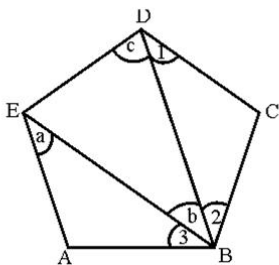
Euclid's Axiom : If equals are subtracted from equals, the remainders are equal　　**½**

13. $\angle ABC = 180° - (40° + 30°) = 110° \Rightarrow \angle CBD = 70°$　　**1**

$x = \angle CBD + \angle BDE = 70° + 45° = 115°$　　**1**

OR

ABCD is a regular pentagon

$\Rightarrow \angle BCD = 108°$ ½

$\Rightarrow \angle 1 = \angle 2 = 36°$ $[BC=CD]$

$\angle C + \angle 1 = 108°$ $\Rightarrow \angle C = 72°$ ½

$\angle EAB = 108°$ $\Rightarrow \angle a = 36°$

$\angle b = 108° - (\angle 2 + \angle 3) = 108° - 72° = 36°$ ½

$\Rightarrow \angle a + \angle b = 72° = \angle C$ ½

14. $AB = 6$ unit $\Rightarrow AC = BC = 6$ units

$OA = 3$ units and $\angle AOC = 90°$ ½

$\Rightarrow OC^2 = AC^2 - OA^2 = 36 - 9 = 27$

$\Rightarrow OC = 3\sqrt{3}$ units 1

$\therefore$ Coordinates of C are $(O, 3\sqrt{3})$ ½

SECTION - C

15. $\sqrt{5 + 2\sqrt{6}} = \sqrt{3 + 2 + 2\sqrt{6}}$ ½

$= \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3}\sqrt{2}} = \sqrt{(\sqrt{3} + \sqrt{2})^2} = \sqrt{3} + \sqrt{2}$ ½+½

$= \sqrt{3} + \sqrt{2}$

Also, $\sqrt{8 - 2\sqrt{15}} = \sqrt{(\sqrt{5})^2 + (\sqrt{3})^2 - 2\sqrt{5}\sqrt{3}} = \sqrt{(\sqrt{5} - \sqrt{3})^2} = \sqrt{5} - \sqrt{3}$ ½+½

$\therefore$ Required sum $= (\sqrt{3} + \sqrt{2}) + (\sqrt{5} - \sqrt{3}) = \sqrt{2} + \sqrt{5}$ ½

OR

$a = 9 - 4\sqrt{5}$, $\dfrac{1}{a} = \dfrac{1}{9 - 4\sqrt{5}} = \dfrac{9 + 4\sqrt{5}}{9^2 - (4\sqrt{5})^2} = 9 + 4\sqrt{5}$ 1

$\therefore a + \dfrac{1}{a} = (9 - 4\sqrt{5}) + (9 + 4\sqrt{5}) = 18$ ½

$a^2 + \dfrac{1}{a^2} = (a + \dfrac{1}{a})^2 - 2 = (18)^2 - 2$ 1

$= 324 - 2 = 322$ ½

16. $\dfrac{7+3\sqrt{5}}{3+\sqrt{5}} - \dfrac{7-3\sqrt{5}}{3-\sqrt{5}} = \dfrac{(7+3\sqrt{5})\,(3-\sqrt{5}) - (7-3\sqrt{5})(3+\sqrt{5})}{9-5}$ **1**

$= \dfrac{1}{4}\left[21+2\sqrt{5}-15-(21-2\sqrt{5}-15)\right]=\dfrac{1}{4}\left[6+2\sqrt{5}-6+2\sqrt{5}\right]=\sqrt{5}$ **1+1**

OR

LHS $= \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \dfrac{(\sqrt{5}+\sqrt{3})^2}{5-3} = \dfrac{1}{2}\left[5+3+2\sqrt{15}\right]$ **1**

$= 4+\sqrt{15} = a+\sqrt{15}\,b$ **1**

$\Rightarrow a = 4,\ b = 1$ **1**

17. Let $x^2+7x = p,\ 2x-1 = q$

   $\therefore$ Given expression $= 12p^2 - 8pq - 15q^2$ **½**

   $= 12p^2 - 18pq + 10pq - 15q^2$

   $= 6p(2p-3q) + 5q(2p-3q)$

   $= (6p+5q)(2p-3q)$ **1+½**

   $\therefore$ Factors are : $[\,6(x^2+7x)+5(2x-1)\,][\,2(x^2+7x)-3(2x-1)\,]$ **1**

       $= (6x^2+52x-5)(2x^2+8x+3)$

18. $p(x) = 3x^3 - 2x^2 - 7x - 2$

   $p(2) = 3(2)^3 - 2(2)^2 - 14 - 2 = 24 - 8 - 16 = 0 \Rightarrow 2$ is a zero of $p(x)$ **1**

$p\left(\dfrac{-1}{3}\right) = 3\left(\dfrac{-1}{3}\right)^3 - 2\left(\dfrac{-1}{3}\right)^2 - 7\left(\dfrac{-1}{3}\right) - 2 = \dfrac{-1}{9} - \dfrac{2}{9} + \dfrac{7}{3} - 2 = 0 \Rightarrow \dfrac{-1}{3}$ is a zero of $p(x)$

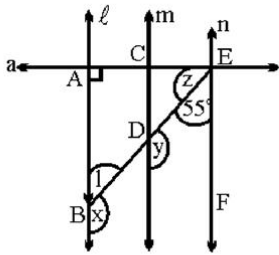$(x-2)\,(x+\dfrac{1}{3})$ or $(x-2)\,(3x+1)$ is a factor of $p(x)$ **1**

or $3x^2 - 5x - 2$ is a factor of $p(x)$

$(3x^3 - 2x^2 - 7x - 2) \div (3x^2 - 5x - 2) = x+1$ **½**

   $\therefore x = -1$ is the third zero of $p(x)$ **½**

19.

$\ell \parallel n \Rightarrow \angle CEF = 90°$

$\Rightarrow Z = (90° - 55°) = 35°$ **1**

$\Rightarrow \angle x = 90° + z = 90° + 35° = 125°$ **1**

$\angle y = \angle x = 125°$ **1**

OR

$y = 180° - (30° + 20°) = 130°$ ½

$\ell \parallel m \Rightarrow x + 100° = 180°$ **1**

$\Rightarrow x = 80°$

$\therefore x + y = 130° + 80° = 210°$ **1**

$y - x = 130° - 80° = 50°$

$\Rightarrow (y + x) : (y - x) = 21 : 5$ ½

20. DE $\parallel$ BC and AB is a transversal

$\Rightarrow \angle ADE = 40°$ ½

DE $\parallel$ BC and LE $\parallel$ AB $\Rightarrow$ DBLE is a $\parallel$ gm

$\therefore \angle DEL = \angle MEN = 40°$ ½

$\therefore$ (i) $\angle ADE + \angle MEN = 2 \times 40° = 80°$ **1**

(ii) $\angle BDE = 180° - 40° = 140°$ ½+½

(iii) $\angle BLE = \angle BDE = 140°$ For fig. . . . .   ½

21.
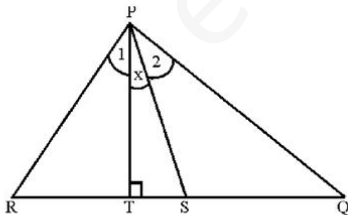
$\angle 1 + \angle x = \angle 2$ (Given) . . . .   ½

$\angle 1 + \angle R = \angle 2 + x + \angle Q$

$\angle 1 + \angle R = \angle 1 + 2x + \angle Q$ **1**

$\Rightarrow 2x = \angle R - \angle Q \Rightarrow \angle TPS = \dfrac{1}{2} (\angle R - \angle Q)$ **1**

22. It is given that $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

$\angle 1 + \angle 3 = \angle 2 + \angle 4 \Rightarrow \angle DAB = \angle CBA$ ½

In $\Delta's$ DAB and CBA

AD = BC, AB = AB, $\angle DAB = \angle CBA$ **1½**

$\therefore \Delta DAB \cong \Delta CBA \Rightarrow BD = AC$ **1**

23. Draw GAP || PC ½

$\angle GAE = \angle AEC = 20°$ (i) ½

AB || DH and GP is a transversal ½

$\therefore \angle GAB = \angle GHD$ (ii) 1

Agains, GP || CE $\Rightarrow \angle GHD = \angle ECD$ (iii) ½

from (i), (ii) and (iii), we get

$\angle DCE = 30°$ ½

24. Two sides are 80cm, 12cm and perimeter = 180cm ½

$\therefore$ Third side = 180–(98) = 82cm

The sides are 82cm, 80cm, 18cm

Now $(80)^2 = 6400$, $18^2 = 324$ 1

$\Rightarrow (80)^2 + (18)^2 = 6724$

$(82)^2 = 6724$

$\therefore \Delta$ is right angled. ½

$\therefore$ area $= \dfrac{1}{2} \times 80 \times 18 = 720cm^2$ ½

altilude corresponding to shortest side = 80cm ½

SECTION - D

25. $x = \dfrac{1}{2-\sqrt{3}} = \dfrac{2+\sqrt{3}}{(2)^2 - (\sqrt{3})^2} = 2+\sqrt{3}$ 1

$\Rightarrow (x-2)^2 = 3 \Rightarrow x^2-4x+1=0$ ½

$(x^3-2x^2-7x+5) \div (x^2-4x+1) \Rightarrow$ Quotient = x+2, Remainder = 3 1+½

$\therefore x^3 - 2x^2 - 7x + 5 = (x+2)(x^2-4x+1) + 3 = 3$ 1

OR

$\dfrac{1}{\sqrt{2}+1} = \dfrac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)} = \sqrt{2}-1$, $\dfrac{1}{\sqrt{3}+\sqrt{2}} = \sqrt{3}-\sqrt{2}$, $\dfrac{1}{\sqrt{4}+\sqrt{3}} = \sqrt{4}-\sqrt{3}$

$\dfrac{1}{\sqrt{8}+\sqrt{9}} = \sqrt{9}-\sqrt{8}$ 3

$\therefore$ Given expression $= (\sqrt{2} - 1) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + ... + (\sqrt{8} - \sqrt{7}) + (\sqrt{9} - \sqrt{8}$ **1**

$\sqrt{9} - 1 = 3 - 1 = 2$

26. $x = \dfrac{\left[\sqrt{p + 2q} + \sqrt{p - 2q}\right]^2}{p + 2q - p + 2q} = \dfrac{1}{4q}\left[p + 2q + p - 2q + 2\sqrt{p^2 - 4q^2}\right]$ **1+½**

$= \dfrac{1}{2q}\left[p + \sqrt{p^2 - 4q^2}\right] \Rightarrow 2qx - p = \sqrt{p^2 - 4q^2}$ **½+½**

$\Rightarrow 4q^2x^2 + p^2 - 4pqx = p^2 - 4q^2$ **1**

$qx^2 - px + q = 0$ **½**

<center>**OR**</center>

$x = (\sqrt{2} + 1)^2 = 3 + 2\sqrt{2}$, $y = 3 - 2\sqrt{2}$ **1½**

$x + y = 6$, $xy = 9 - 8 = 1$ **1**

$x^2 + y^2 + xy = (x + y)^2 - xy = 36 - 1 = 35$ **1+½**

27. $p(x) = x^3 + mx^2 - x + 6$, $p(2) = 0 \Rightarrow 8 + 4m - 2 + 6 = 0$

$\Rightarrow 4m = -12 \Rightarrow m = -3$ **1+½**

$p(3) = n$, $\therefore$ $n = (3)^3 + (-3)(3)^2 - 3 + 6$ **1+½**

$n = 3$ **1**

28. We know that $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ **½**

Let $a = x + y$, $b = y + z$, $c = z + x$

LHS $= 2(x + y + z)[(x + y)^2 + (y + z)^2 + (z + x)^2 - (x + y)(y + z) - (y + z)(z + x) - (z + x)(x + y)]$ **1**

$= 2(x + y + z)[x^2 + y^2 + 2xy + x^2 + y^2 + z^2 + 2yz + z^2 - xy - y^2 - xz - yz - z^2 + 2zx - yz - xy - xz - 2x - x^2 - yz - xy$ **1½**

$= 2(x + y + z)[x^2 + y^2 + z^2 - xy - yz - zx]$ **1**

$= 2(x^3 + y^3 + z^3 + -3xyz)$

29. $p(x) = x^3 + 2x^2 - 5ax - 7$, $q(x) = x^3 + ax^2 - 12x + 6$

It is given that $p(-1) = A$ and $q(2) = B$ **1**

$\therefore$ $A = -1 + 2 + 5a - 7 \Rightarrow A = 5a - 6$ **1**

$B = 8 + 4a - 24 + 6 \Rightarrow B = 4a - 10$ **1**

Also $2A + B = 6 \Rightarrow 10a - 12 + 4a - 10 = 6$ **1**

$\Rightarrow 14a = 28 \Rightarrow a = 2$

30. Coordinates of : A (2,2), B (1,4), C (5,5), **2**

D (-1, 4) , E (2, -2), F (-5, -5)

E is the mirror image of A in x-axis **1**

D is the mirror image of B in y-axis **1**

31. In Δ RPS, $\angle P + \angle S + x = 180°$

$\Rightarrow x = 180° - 100° - 30° = 50°$ **1**

$y = 180° - \angle PSR = 180° - 100° = 80°$ **1½**

$z = y + 40° = 120°$ **1½**
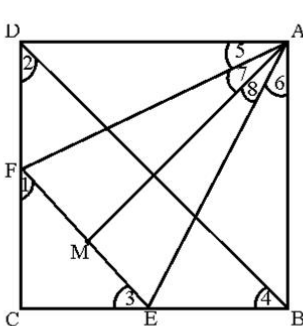
32.

$EF \parallel BD \Rightarrow \angle 1 = \angle 2$ and $\angle 3 = \angle 4$
$\angle 2 = \angle 4 \Rightarrow \angle 1 = \angle 3$
$\therefore DF = BE [\because BC - CE = CD - CF]$ **1**

$\Delta ADF \cong \Delta ABE [AD = AB, FD = BE, \angle D = \angle B = 90°]$ **1**
$\Rightarrow AF = AE$ and $\angle 5 = \angle 6$

$\Delta AMF \cong \Delta AME [AF = AE, FM = EM, AM = AM]$ **½**

$\therefore \angle 7 = \angle 8 \Rightarrow \angle 7 + \angle 5 = \angle 8 + \angle 6 \Rightarrow \angle MAD = \angle MAB$ **1**
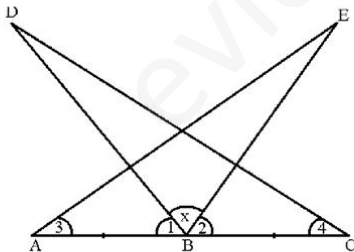
$\Rightarrow AM$ bisects $\angle BAD$ **½**

33.

$\angle 1 = \angle 2$ (Given)
$\therefore \angle 1 + \angle x = \angle 2 + \angle x$ **1+½**
$\Rightarrow \angle ADE = \angle CBD$

In $\Delta's$ ABE and CBD
(i) $\angle 3 = \angle 4$ (Given) (ii) $\angle ADE = \angle CBD$
(iii) $AB = BC$ **2+½**
$\Rightarrow \Delta's$ are $\cong \Rightarrow CD = AE$

$AB = AC \Rightarrow \angle ABC = \angle ACB \quad \ldots (i)$

34.

½

It is given that $\angle DBC = \angle DCB \ldots (ii) \Rightarrow DB = DC$ **1**
from [(i)–(ii))], we get

$\angle ABD = \angle ACD$ **½**

$\Delta's$ ABD and ACD are $\cong$ by (sss) **1**

$\therefore \angle BAD = \angle CAD$

$\Rightarrow AD$ bisects $\angle BAC$ **1**