Directorate of Education, GNCT of Delhi

Practice Paper -1

(2023-24)

Class – XII

Mathematics (Code: 041)

Time: 3 hours **Maximum Marks: 80**

<u>General Instructions :</u>

1. This Question paper contains - **five sections A,B,C,D,E**. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.(20 Marks)
3. **Section B**has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.(10 Marks)
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.(18 Marks)
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.(20 Marks)
6. **Section E has 3 Source based/Case based/passage based/integrated units of assessment (4 marks each) with sub parts.(12 Marks)**

Section – A

Question Number 1-18 are of MCQ type question one mark each.

1. The domain of the function $\cos^{-1}(2x-1)$ is : **1**

(a) $[0,1]$	(b) $[-1,1]$
(c) $(-1,1)$	(d) $[0,\pi]$

2. If A = $\begin{bmatrix} 0 & a & b \\ 2 & 1 & c \\ 3 & 4 & 5 \end{bmatrix}$ is a symmetric matrix , then the value of (a+b+c) is ; **1**

(a) 9	(b)8
(c) 7	(d) 6

3. If a matrix $A=[0 \quad 1 \quad 2]_{1x3}$ then the matrix AA^T (where A^T is transpose of A) is: **1**

(a) $[0]$	(b) $[3]$
(c) $[5]$	(d) $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

4.	If $A=\begin{bmatrix}2 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 1\end{bmatrix}$ then the value of |adj A| is : (a) 6 (b) 1/6 (c) 31 (d) 216	**1**
5.	If matrices A ,B and C are such that $A_{p\times 4}.B_{q\times 5}=C_{2\times 5}$, then the value of p^2-q^2 is : (a) -12 (b) 12 (c) 16 (d) -16	**1**
6.	The graph of $x\leqslant 3$ and $y\geqslant 3$ lie in : (a) Ist and 2nd quadrant (b) 2nd and 3rd quadrant (c) 3rd and 4th quadrant (d) Ist and 4th quadrant	**1**
7.	Sum of order and degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{3}}.\left(\frac{dy}{dx}\right)^{\frac{1}{3}}=0$ is : (a) 6 (b) 5 (c) 3 (d) 2	**1**
8.	*Derivative of* $sec^{-1}\frac{\sqrt{x}+1}{\sqrt{x}-1}+\sin^{-1}\frac{\sqrt{x}+1}{\sqrt{x}-1}$ w.r.t x is: (a) 0 (b) 1 (c) x (d) x^2	**1**
9.	$\int\frac{x^3}{x+1}dx$ is equal to : (a) $x+\frac{x^2}{2}+\frac{x^3}{3}-\log\|1-x\|+C$ (b) $x+\frac{x^2}{2}-\frac{x^3}{3}-\log\|1-x\|+C$ (c) $x-\frac{x^2}{2}-\frac{x^3}{3}-\log\|1+x\|+C$ (d) $x-\frac{x^2}{2}+\frac{x^3}{3}-\log\|1+x\|+C$	**1**
10.	Integrating factor of $x\frac{dy}{dx}+2y=x^2$ is : (a) x^3 (b) x^2 (c) x^4 (d) x	**1**

11.	The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}=\frac{d^2y}{dx^2}$ is: (a) 4 (b) $\frac{3}{2}$ (c) Not defined (d) 2	**1**
12.	The projection of $2\hat{i}+3\hat{j}-6\hat{k}$ on the vector $\hat{i}-2\hat{j}+3\hat{k}$ is : (a) $\frac{2}{\sqrt{14}}$ (b) $\frac{1}{\sqrt{14}}$ (c) $\frac{3}{\sqrt{14}}$ (d) $\frac{-2}{\sqrt{14}}$	**1**
13.	Area of the parallelogram whose diagonals are $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}-3\hat{j}+4\hat{j}$ is given by : (a) $10\sqrt{3}$ (b) $5\sqrt{3}$ (c) 8 (d) 4	**1**
14.	If $\lvert\vec{a}+\vec{b}\rvert=\lvert\vec{a}-\vec{b}\rvert$ then the angle between $\vec{a}$ and $\vec{b}$ is : (a) $\frac{\pi}{2}$ (b) 0 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$	**1**
15.	The direction ratios of line are 1 ,3 ,5 then its direction cosines are : (a) $\frac{1}{\sqrt{35}},\frac{3}{\sqrt{35}},\frac{5}{\sqrt{35}}$ (b) $\frac{1}{9},\frac{1}{3},\frac{5}{9}$ (c) $\frac{5}{\sqrt{35}},\frac{3}{\sqrt{35}},\frac{1}{\sqrt{35}}$ (d) None of these	**1**
16.	: For two independent events A & B $P(A\cup B)=\frac{2}{3}$, $P(A)=\frac{2}{5}$, then P(B) is equal to: (a) $\frac{5}{9}$ (b) $\frac{4}{9}$ (c) $\frac{2}{9}$ (d) $\frac{3}{9}$	**1**

17. The minimum value of Z=x+y subjet to the constraints $x \le 20$, $y \ge 10$ and $x, y \ge 0$, is : **1**

(a) 0	(b) 10
(c) 20	(d) 30

18. If the objective function for the LPP is Z=11x+7y and the corner points of the bounded feasible regions are (3 ,2), (0, 5), (0 ,3)then the minimum value of Z occurs at : **1**

(a) (3, 2)	(b) (0, 5)
(c) (0, 3)	(d) does not exist

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion(A) :** $c\cos^{-1}(\cos(\frac{7\pi}{6})) = \frac{5\pi}{6}$ **1**

Reason (R) : $\cos^{-1}(\cos x) = x\ for\ all\ x \in (10, \pi)$

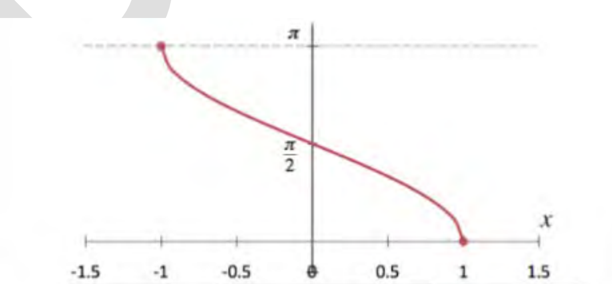
20. **Assertion(A) :**If a line makes angles α, β, γ with the positive direction of coordinate axes then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ **1**

Reason (R) :) : Sum of squares of direction cosines of a line is 1

(Section B)

This section contains 5 Very Short Answer (VSA)-type questions of 2 marks each.

21. The graph of an inverse trigonometric function f(x) is given below, observe the graph and answer the following questions **2**



(i)What is the value of $f(\frac{-1}{2})$?

(ii)If $f(x) = \frac{\pi}{4}$, then find the value of x.

22. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one- one **2**

23.	If $y=x^y$,then find $\frac{dy}{dx}$ **OR** If $y=\sin^{-1}(\frac{1}{\sqrt{1+x^2}})+\tan^{-1}(\frac{\sqrt{1+x^2}-1}{x})$ find $\frac{dy}{dx}$	**2**
24.	A particle moves along the curve $x^2=2y$. At what point , ordinate increases at the same rate as abscissa increases ?	**2**
25.	Find $\int \frac{logx}{(1+logx)^2}dx$ **OR** Find the value of $\int_0^1 \tan^{-1}(\frac{1-2x}{1+x-x^2})dx$	**2**
	Section C **This section contains 6 Short Answer (SA)-type questions of 3marks each.**	
26 .	If $x=a\sin^2\theta$, $y=a\cos^2\theta$, then find $\frac{d^2y}{dx^2}$	**3**
27.	A bag A contains 4 black balls and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown . If 1 or 2 appear on it , then bag A is chosen , otherwise bag B . If two balls are drawn at random (Without replacement) from the selected bag , find the probability of one of them being red and another black. **OR** From a lot of 15 bulbs which include 5 defectives , a sample of two bulbs is drawn at random (without replacement).Find the probability distribution of the number of defective bulbs.	**3**
28.	Evaluate $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$ OR Find $\int e^x.sinx\,dx$	**3**
29.	Find the general solution of $(1+x^2)dy+2xy\,dx=cotx\,dx$ **OR** Solve following differential equation $(x^2-y^2)dx+2xy\,dy=0$	**3**
30.	Solve the following Linear programming problem graphically : Maximize : Z=4x+y subject to the constraints $x+y\le 50, 3x+y\le 90, x\ge 0, y\ge 0$	**3**
31.	Find the interval in which function $f(x)=2x^3-9x^2+12x+15$ is strictly increasing and strictly decreasing	**3**
	खंड डी /(SECTION D) This section contains **four Long Answer (LA)-type** questions of 5marks each.	
32.	Find the area of the region included between the curves $4y=3x^2$ and the line 2y=3x+12	**5**
33.	Let N denote the set of all natural numbers and R be the relation on $N\times N$ defined by $(a,b)\in R(c,d)$ If ad(b+c)=bc(a+d). Prove that R is an equivalence relation.	**5**
34.	Find the shortest distance between the lines given by $\vec{r}=(8+3\lambda)\hat{i}-(9+16\lambda)\hat{j}+(10+7\lambda)\hat{k}$ and $\vec{r}=15\hat{i}+29\hat{j}+5\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$ **OR** Find the vector and cartesian equations of the line which is perpendicular to the lines with equations and and passes through the point (1, 1, 1). Also find the angle between the given	**5**

	lines.	
35.	Evaluate the product AB , where : $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$ Hence solve the system of linear equations x-y=3 2x+3y+4z=17 y+2z=7 **OR** If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, find $A^2-5A+4I$ and hence find a matrix X such that $A^2-5A+4I+X=0$	5
	<u>(Section E)</u> **Source based/Case based/passage based/integrated units of assessment Questions**	
36.	A bike is running on the road along the line $\frac{x-6}{1}=\frac{2-y}{2}=\frac{z-2}{2}$ while an aeroplane is flying in the space along the line $\frac{x+4}{3}=\frac{y}{-2}=\frac{z+1}{-2}$  Based on the information given above answer the following questions . 1(i) Write the equations of both the lines in vector form. (II) Find a vector perpendicular to both the given lines . (iii) Find shortest distance between both skew lines. OR (III) For which value of λ the lines $\frac{x-6}{1}=\frac{2-y}{2}=\frac{z-2}{2}$ and $\frac{x+4}{3}=\frac{y}{-2}=\frac{z+1}{-2}$ Intersect each other	**1+1+2**
37.	In a smart city Indore a residential society comprising of 100 houses, there were 60 childrens between the ages 10-15 yearsThey were inspired by their teacher to start composting to ensure that biodegradable waste is recyled. For this purpose instead of each child doing it for only his/her house childrens convinced the Residents welfare association to do it as a society initiative. For this they identified a square area ina local park . Local authorities charged amount of ₹50 per sq metre for space so that there is no misuse of the space and Resident welfare association takes it seriously . Association hired a labourer for digging out 250 m^3 and he charged ₹400 x or $X(depth)^2$. Association will like to have minimum cost .	**1+1+2**

Based on the information given above answer the following:

(i)If the side of square plot is x metre and its depth is h metre then find the cost C for the pit

.(II) Find the value of h(in metre)for which $\frac{dc}{dh}=0$.

(iii) What is the value of $\frac{d^2c}{dh^2}$?

OR

(iii)Find the value of x)(in metre)for minimum cost.

38. In a war between Russia and Ukraine , UK provided Ukraine two types of new anti-aircraft guns named A and B which were used by Ukrainian forces to stop Russia's 'suicide drones' . The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time. **2+2**

Based on the information given above answer the following questions.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?