Class X Mathematics

CBSE Board, Set – 1

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 31 questions divided into four sections – A, B, C and D.

(iii) Section A contains 4 questions of 1 mark each, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.

(iv) Use of calculators is not permitted.

Section - A

Q1. If x = - –, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.

Sol. 1 Since it is given that x = – is a solution of the given quadratic equation so it must satisfy the given equation.

So putting x = – is equation.

3 (– –) (– –)

– (– –)

–

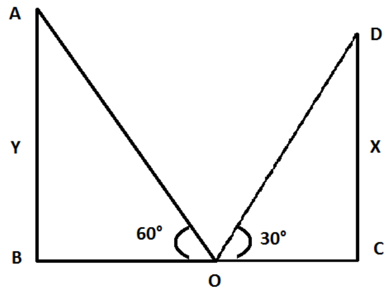
– ——

–

Hence the value of k is –.

Q2. The top of the two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find x : y.

Sol. 2



Let AB and CD be the two towers of height y and x respectively subtending angles of 60° & 30°.

We are required to find the ratio x:y.

Let O be the mid-point of the line joining their feet.

$OB = OC = - BC$ ... (i)

Now, in Δ AOB

$\tan 60° = - \quad —$

$\tan 60° = \frac{}{(-)BC}$ {from (i)}

$\Rightarrow (\sqrt{3}/2)\ BC = y$ ... (ii)

And in Δ DOC

$\tan 30° = - = —$

$\tan 30° = \frac{}{(\quad 2)BC}$ {from (i)}

$\frac{}{2\sqrt{3}} \quad x$ ... (iii)

$x : y = \frac{}{2\sqrt{3}} : (\frac{\sqrt{3}}{})BC$

$x : y = \frac{2BC\sqrt{3}}{2\sqrt{3}BC} = \frac{}{\sqrt{}} \quad \frac{}{\sqrt{3}}$

$x : y = 1 : 3$

Q3. A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant.

Sol.3 There are 26 English alphabets in total, and so total out comes = 26, out of these 26, 5 are vowels .

⇒ Number of Consonants = favorable outcomes = 26 – 5 = 21.

Here the required probability i.e. prob (chosen is a consonant) =—.

Q4. In Fig. 1, PA and PB are tangents to the circle with centre O such that ∠ APB = 50°. Write the measure of ∠OAB.

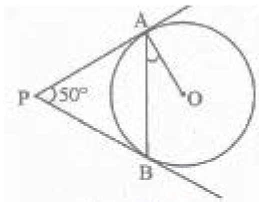
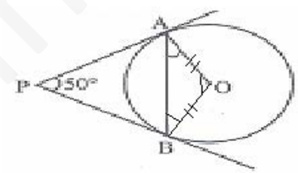


Figure 1

Sol.4 In Δ AOB, OA = OB (radius)



∴ ∠ OAB = ∠ OBA

= x (let)

In fig. , OAPB is quadrilateral, ∠ OAP and ∠ OBP is right angle (radius is perpendicular to the tangent at point of contact)

Using angle sum properly of quadrilateral,

∴ ∠ AOB + ∠OBP + ∠BPA + ∠OAP = 360°

∠ AOB 90° + 50° + 90° = 360°

⇒ ∠ AOB + 230° = 360°

⇒ ∠ AO 360° - 230°

= 130°

In Δ OAB,

∠OAB + ∠AOB + ∠OBA = 180°

⇒ x + 130° + x = 180°

⇒ x + 130° = 180°

⇒ x = 50°

⇒ x 25°

Hence ∠ OAB = 25°

Section - B

Q5. In Fig.2, AB is the diameter of a circle with centre O and at is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$.

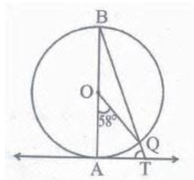


Figure 2

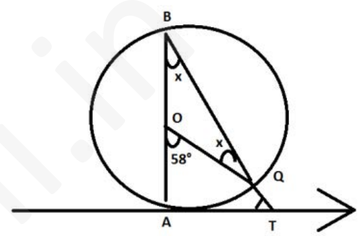
Sol. 5 Since A B is the diameter of the circle

So, $\angle AOQ + \angle QOB = 180°$ (linear pair)

$58° + \angle QOB = 180°$

$\angle QOB = 180° - 58°$

$= 122°$



In Δ OBQ,

$OB = OQ$ (radius)

$\angle OBQ = \angle OQB = x$

$\angle BOQ + \angle OBQ + \angle OQB = 180°$

$122° + x + x = 180°$

$2x = 58°$

$\Longrightarrow x = 29°$

$\angle OQT = 180° - \angle OQB$

$= 180° - 29°$

$\angle OQT = 151°$

AOQT is quadrilateral

$\angle D = 58°$, $\angle BAT = 90°$ $\angle OQT = 151°$

So, $\angle ATQ = 260° - (58° + 90° + 151°)$

$= 360° - 299°$

$\angle ATQ = 61°$

$\Rightarrow \angle ATQ = 61°$

Q6. Solve the following quadratic equation for x:

$4x^2 - 4a^2x + (a^4 - b^4) = 0.$

Sol.6 The given quadratic equation is $4x^2 - 4a^2x + (a^4 - b^4)$

Hence, the constant term is $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$

Coefficient of middle term is $= -4a^2$

$= -\{2(a^2 + b^2) + 2(a^2 - b^2)\}$

Now equation is

$4x^2 - 4a^2x + (a^4 - b^4) = 0.$

$4x^2 - \{2(a^2+b^2) + 2(a^2 - b^2)\ x + (a^2 - b^2)(a^2 + b^2) = 0$

$4x^2 - 2(a^2 + b^2)x - 2(a^2 - b^2)x - (a^2 - b^2)(a^2 + b^2) = 0$

$\Rightarrow \{4x^2 - 2(a^2 + b^2)x\} - \{2(a^2 - b^2)x - (a^2 - b^2)(a^2 + b^2)\} = 0$

$\Rightarrow \quad x\{2x - (a^2 + b^2)\} - (a^2 - b^2)\{2x - (a^2 + b^2)\} = 0$

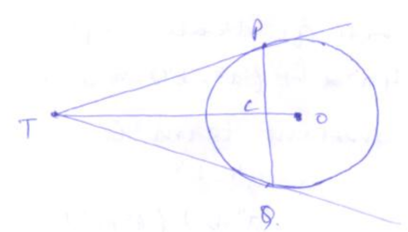
$\Rightarrow \{2x - (a^2 + b^2)\}\{2x - (a^2 - b^2)\} = 0$

$\Rightarrow 2x - (a^2 + b^2) = 0$ or $2x - (a^2 - b^2) = 0.$

$\Rightarrow x \quad \text{———}$ or $x = \text{———}$

Q7. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

Sol.7 Suppose OT intersects PQ at C.



Then, in Δ QTC, Δ PTC

TP = TQ (Tangents from an external point are equal)

$\angle PTC = \angle QTC$ (TP & TQ are equally inclined to OT)

& TC = TC (common)

By SAS criterion of similarity, are have

Δ PTC $\cong \Delta$QTC.

$\Rightarrow$ PC = CQ and $\angle$ PCT = $\angle$ QCT

But $\angle$ PCT + $\angle$ QCT = 180°

$\Rightarrow$ PCT = $\angle$ QCT = 90°

$\Rightarrow$ OT $\perp$ PQ

Hence OT is the right bisector of line segment PQ.

Q8. Find the middle term of the A.P. 6, 13, 20, ---, 216.

Sol. 8 The given A·P is,

6, 13, 20, ... 216.

Here a = 6, d = 13 – 6 = 7.

$a_n = 216$. (nth term)

$\Longrightarrow$ a + (n – 1) d = 216.

6 + (n – 1) 7 = 216

$\Longrightarrow$ 6 + 7n – 7 = 216

$\Longrightarrow$ 7n – 1 = 216

$\Longrightarrow$ 7n = 217

$\Longrightarrow$ n = 31.

which is odd.

Hence the middle term is $\left(\frac{}{}\right)$ term

$= \left(\frac{}{}\right)$ term

= 16th term

a_{16} = a + 15 d

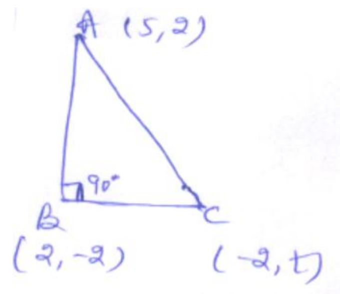
= 6 + 15 · 7

= 6 + 105

= 111 Hence the middle term is 111.

Q9. If A (5, 2), B (2,-2) and C (-2, t) are the vertices of a right angled triangle with ∠B = 90°. Then find the value of t.

Sol.9 A (5,2), B (2,-2), C(-2, t) are verticals of right angles triangle



$\angle B = 90°$

Using distance formula

$AB = \sqrt{(5 \quad 2) \qquad (2 \quad 2)}$

$BC = \sqrt{(2 \quad 2) \qquad (- \qquad t)}$

$AC = \sqrt{(5+2)^2 + (2+t)}$

Now, using Pythagoras theorem in Δ ABC, we have

$AB^2 + BC^2 = AC^2$.

$(3^2 + 4^2) + (4^2 + (2+t)^2) = T^2 + (2 - t)^2$

$\Rightarrow 9 + 16 + 16 + 4 + t^2 + 4t = 49 + 4 + t^2 - 4t$

$\Rightarrow 41 + 4 + 4t = 53 - 4t$

$\Rightarrow 8t = 53 - 4t$

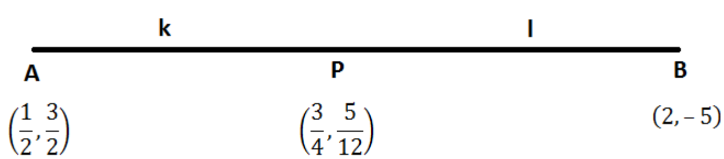
$\Rightarrow 8t = 8$

$\Rightarrow t = 1$

Hence the value & t is 1.

Q10. Find the ratio in which the point $P\left(-,-\right)$ and B (2,-5).

Sol. 10



Let us assume that the point P divides AB in the ratio k : 1.

Then using section formula, we have

$P = \left(\overline{}, \overline{} \right)$

$(-,-) = \left(\overline{}, \overline{} \right)$

$- \quad \overline{}$

and — $\overline{}$

$- \quad \overline{}$

$3\,(k + 1) = (2 \quad -)\,4.$

$3\,k + 3 = 8\,k + 2.$

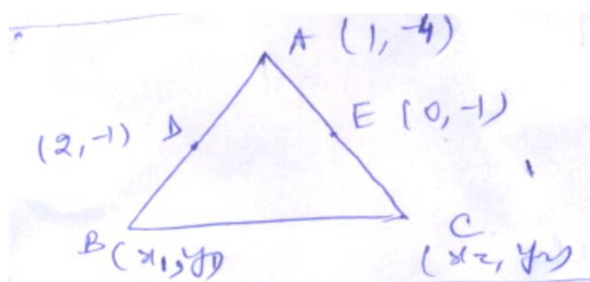
$5\,k = 1. \quad k = -$

Hence the required ratio is k = 1 i.e. – = 1 or 1: 5.

<u>Section - C</u>

Q11. Find the area of the triangle ABC with A (1, –4) and mid-points of sides through A being (2, – 1) and (0, –1).

Sol.11 Given:



ΔABC in which

A (1, -4)& mid – Pts of sides through

A being (2, -1) & (0, -1).

Let the co-ordinates of B (x_1, y_1) & C (x_2, y_2).

Co-ordinates of points B = (——, ——)

(2, -1) = (——, ——)

⇒ —— = 2 & —— = -1

⇒ $x_1 = 3$ & $y_1 = 2$

So, Point B = (3,2).

Similarly co-ordinates of point C = (——, ——)

(0, -1) = (——, ——)

⇒ —— = 0 & —— = -1.

⇒ $x_2 = -1$ & $y_2 = 2$

Point C = (-1, 2)

Area of Δ ABC = where A (x_1, y_1), B(x_2, y_2) & C (x_3, y_3) is

– $| x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) |$

= – | 1 (2 – 2) + 3 (2 – (-4)) + (-1)(-4 -2)|

= – | 0 + 3.6 + (-1)(-6)|

= – | 18 + 6 |

= – × 24

= 12 sq units.

Q12. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k - 1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.

Sol.12 $kx^2 + 1 - 2(k-1)x + x^2 = 0$

$x^2 (k + 1) - 2 (k - 1)x + 1 = 0$

For equal roots

D = 0

$b^2 - 4ac = 0$

$4(k - 1)^2 - 4(1)(k + 1) = 0$

$4(k^2 + 1 - 2k) - 4k - 4 = 0$

$4k^2 + 4 - 8k - 4k - 4 = 0$

$4k^2 - 8k - 4k = 0$

$k[4k - 12] = 0$

$k = 0, k = 3$

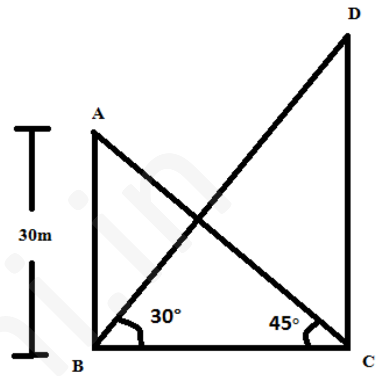
Since nor – zero value is required

$k = 3$(Ans.)

Q13. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

Sol.13 $AB \rightarrow$ Tower

$CD \rightarrow$ Building



In Δ ABC

$\tan 45 = —$

$BC = AB$

$BC = 30$

In Δ BDC

$\tan 30 = —$

$CD \quad \frac{\ }{\sqrt{3}} \quad 10\sqrt{3}$m.

Q14. Two different dice are rolled together. Find the probability of getting:

(i) the sum of numbers on two dice to be 5.

(ii) even numbers on both dice.

Sol.14 (i) Favorable outcomes = (1, 4) (4, 1) (2, 3) (3, 2)

$n(E) = 4$

Sample space $n(S) = 36$

Probability $\frac{n(E)}{n(S)}$ — –

(ii) Favorable outcomes = (2, 2) (2, 4) (2, 6)

(4, 2) (4, 4) (4, 6)

(6, 2) (6, 4) (6, 6)

n (E) = 9

Probability $= \frac{n(E)}{N(S)}$ — –

Q15. If S_n, denotes the sum of first n terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$.

Sol.15 $S_n = \frac{n}{}(20 + (n - 1)d)$

$S_{12} = \frac{12}{} (2a + 11d) = 6(2a + 11d)$ ... (1)

$3(S \quad S_4) \quad -\big(2a + 7d) \quad -(2a \quad 3d)\big]$

$3(S_8 - S_4) = 3\ [4(2a + 7d) - 2(2a + 3d)]$

$[8a + 28d - 4a - 6d]$

$[4a \quad d] \quad 6\ (2a + 11d)$ ... (2)

From (1) and (2)

$S_{12} = 3\ (S_8 - S_4)$

Q16. In Fig. 3, APB and AQO are semicircles, and AO = OB. If the perimeter of the figure is 40 cm, find the area of the shaded region. $\left[\text{User } \pi = —\right]$.

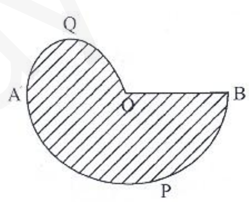
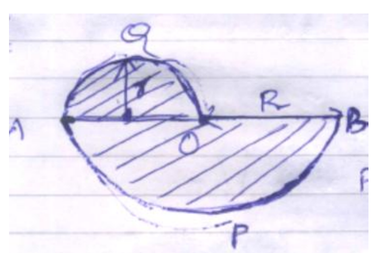


Figure 3

Sol. 16 AO = OB



Perimeter of figure

$= \pi r + \pi R + R$

$r = -$

Perimeter $= — + \pi R + R = \quad \left(— \quad 1\right)$

$40 = R\left(— \quad 1\right)$

R = — — — 7

R = 7 cm. r = – .5 cm

Area of shaded region = — —

= — —

= — —

= — —

= – 77 = 1.25 × 77 = 96.25 cm^2

Q17. In Fig. 4, from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid. (User π = — and $\sqrt{5}$ = 2.236)

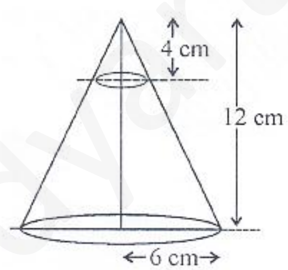
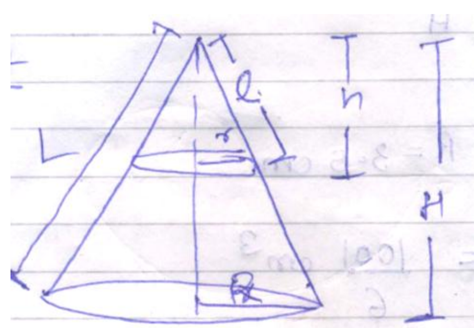


Figure 4

Sol. 17 Using similarity



— –

— –

r = 2 cm

r = 2 cm (H – h) = 8 cm

R = 6 cm

l = $\sqrt{r}$ $\sqrt{4+16}$ $\sqrt{5}$

– –

$L = — \frac{2\sqrt{5}}{} \quad 6\sqrt{5}$

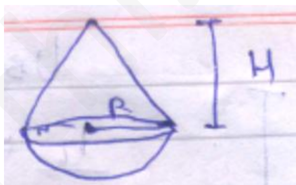
TSA $\pi r^2 + \pi R^2 + \pi(L - l)\,(r + R)$

$= \pi\,[4 \quad 36 + 4\sqrt{5} \times 8]$

$= 8\pi\,[5 + 4\sqrt{5}] = 350.592\ cm^2$

Q18. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is 166 – cm^3. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of Rs. 10 per cm^2. $\left[\text{Use } \pi = —\right]$

Sol. 18 R = 3.5 cm



V = —— cm

Height of cone = H

Volume = – –

—— —— H]

———— .5

6 = H

H = 6 cm

Surface area of hemispherical part to painted $2\pi R^2 =$ $22/7 \times 3.5 \times 3.5$

$= 22 \times 3.5 = 77\ cm^2$

Total Cost = 10×77 = Rs. 770

Q19. In Fig. 5, from a cuboidal solid metallic block, of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. $\left[\text{Use } \pi = —\right]$

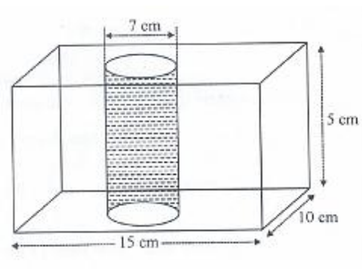


Figure 5

Sol. 19 Surface area of remaining block = Surface area of block - 2 area of circle + C·SA· of cylinder formed

$= 2\,(15 \times 10 + 10 \times 5 + 5 \times 15) - \quad - \times - \quad - \times 5$

$= 2\,[275] - 77 + 110$

$= 583\ cm^2$

Q20. In Fig. 6, find the area of the shaded region [Use = 3.14]

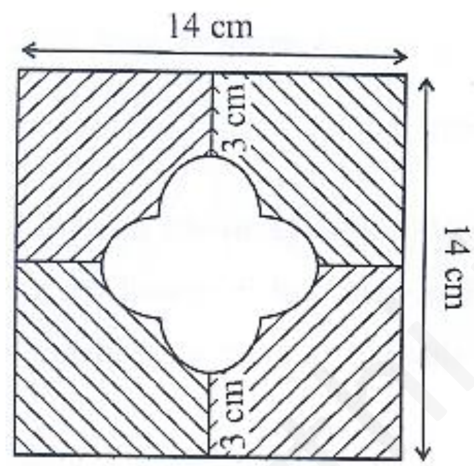
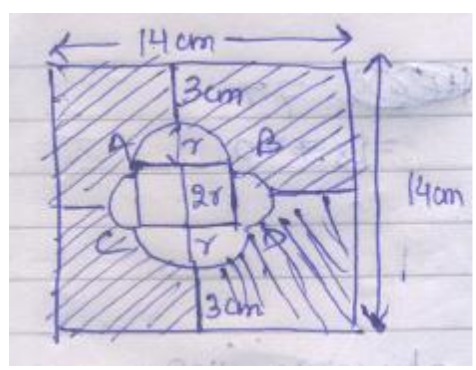


Figure 6

Sol. 20 ABCD will be square with side = diameter of semi-circle



Radius + side + radius = 14 – 3 – 3

$r + 2r + r = 8$

$r = 2$ cm

Area of unshaded figure $= 4^2 + 4$ —

$= 16 + 25{\cdot}12$

$= 41{\cdot}12\ cm^2$

Area of shaded figure $= 14^2 - 41{\cdot}12$

$= 154{\cdot}88\ cm^2$

Section - D

Q21. The numerator if a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is —. Find the original fraction.

Sol.21 Let Denominator be x

Numerator x – 3

Fraction = —

New fraction = —

ATQ

— — —

$\frac{(x\ \ 3)(x\ \ 2)+(x\ \ 1)x}{x\,(x\ \ 2)}$ —

— —

$40x^2 - 40x - 120 = 29x^2 + 58x$

$11x^2 - 98x - 120 = 0$

$11x^2 - 110x + 12x - 120 = 0$

$11x\,(x - 10) + 12\,(x - 10) = 0$

$(11x + 12)\,(x - 10) = 0$

x — Rejected as x is integer/whole

$x = 10$

Original Fraction = —

Q22. Ramkali required Rs. 2500 after 12 weeks to send her daughter to school. She saved Rs. 100 in the first week and increased her weekly saving by Rs. 20 every week. Find whether she will be able to send her daughter to school after 12 weeks.

What value is generated in the above situation?

Sol.22 First weak = 100

Second weak = 120

Third weak = 140

So it will form A.P. with 12 terms

a = 100, d = 20

$— \quad 100 \quad (12 \quad 1) \quad]$

$= 6[200 + 220]$

$= 6 \times 440$

= Rs. 2640 > Rs. 2500

Yes she will be able to send money.

Q23. Solve for x:

$$— \quad \frac{}{2(x \quad 2)} \quad —, x \quad 0, -1, 2$$

Sol.23 $— \quad \frac{}{2(x \quad 2)} \quad —$

$$\frac{4(x \quad 2)+3\,(x \quad 1)}{2\,(x \quad 1)(x \quad 2)} \quad —$$

$$\frac{}{2(x+1)(x-2)} \quad —$$

$5x\,(7x - 5) = 46\,(x + 1)(x \quad 2)$

$35x \quad 25x = 46x \quad 46x - 92$

$11x \quad 21x - 92 \quad 10$

$11x \quad 44x \quad 23x - 92$

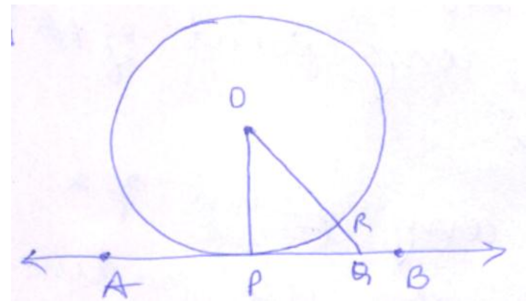
$11x\,(x - 4) \quad (x - 4)$

$x = 4/\ x = \quad —$

Q24. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Sol. 24 Given : A circle C (O, r) and a tangent

AB at a point P



To prove : OP ⊥ AB

Construction: Take any point Q, other than P, on the tangent AB. Join OQ suppose OQ meets the circle at R.

Proof : –

We know that among all line segment joining point O to a point on AB, the shortest one is perpendicular to AB.

Hence to prove OP ⊥ AB, we shall first prove that OP is shorter than any other segment joining O to any point of AB.

Clearly, OP = OR (radius)

Now, $OQ = OR + RQ$

$OQ > OR$

$OQ > OP \quad (\because OP = OR)$

$OP < OQ$

Thus, OP is shorter than any other segment joining O to any point of AB

Hence OP ⊥ AB.

Hence the tangent at any point of a circle is perpendicular to the radius through point of contact.

Q25. In Fig.7, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS.

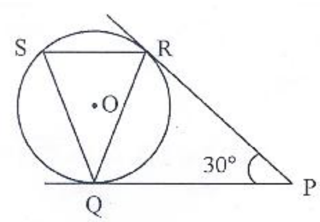
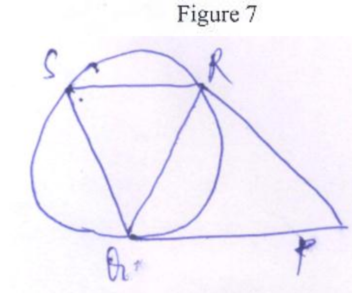


Figure 7

Sol. 25 Since, PQ = PR

$\angle PQR = \angle QPR = \text{———} \quad 75°$

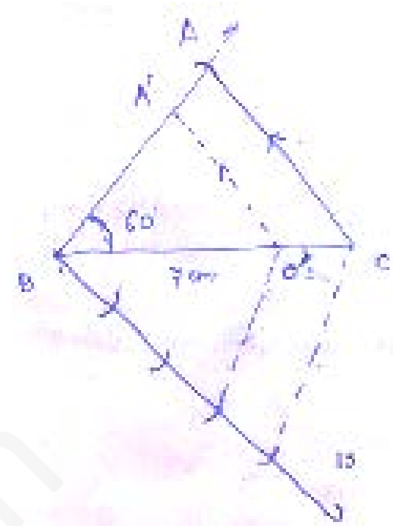


Now, ∠RSQ = ∠RQP = 75° (because angle in equal segment)

Also, ∠SQP = 180 – ∠RSQ (internal angles)

m ∠SQP = 105 $\qquad \therefore \angle RQS = 30°$

Q26. Construct a triangle ABC with BC = 7 cm, ∠B = 60° and AB = 6 cm. Construct another traingle whose sides are – times the corresponding sides of ΔABC.



Sol. 26 A' BC' is required in Δ

1. Draw BC of 7 cm

2. Draw a ray $\overrightarrow{BX}$ such that ∠CBX = 60°

3. Get a point A at distance of 6 cm ∵ AB = 6 cm. Join AC

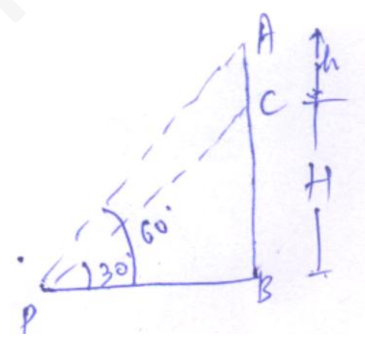
4. Draw arc of equal radius from B to cut ray $\overrightarrow{BX}$ in equal intervals
Draw 4 such arcs

5. Join the 3rd point of intersection with the point C. Mark it as C'

6. Now, Draw line parallel to AC from the point C'

Q27. From a point P on the ground the angle of elevation of the top of a tower is 30° and that of the top of a flag staff fixed on the top of the tower, is 60°. If the length of the flag staff is 5 m, find the height of the tower.

Sol.27 In Δ CBP BC is tower → H



tan 30 = — AC is flag → h

$\frac{}{\sqrt{3}} = \frac{}{}$

BP = H√3 .....(1)

tan60 = — = ——

BP = $\frac{}{\sqrt{3}}$ .....(2)

From (1) and (2)

$\frac{}{\sqrt{3}}$ √ H ⇒ H + h = 3H

h = 2H

Length of flag staff h = 5m = 2H

Length of tower = 2.5m

Q28. A box contains 20 cards numbered from 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is

(i) divisible by 2 or 3

(ii) a prime number

Sol.28 (i) Divisible by 2 will be 10 numbers

Divisible by 3 will be 6 numbers

Divisible by both i.e. divisible by 6 will be counted twice which will be 6, 12, 18

So, numbers divisible by 2 or 3 = 10+6 -3

= 13

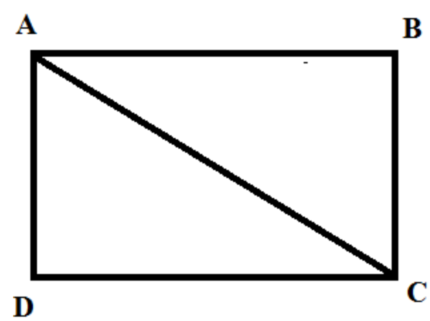
Probability = —

(ii) Prime numbers = 2,3,5,7,,11,17,17,19

Probability = — = -

Q29. If A(−4, 8), B(−3, −4), (0, −5) and D(5, 6) are the vertices of a quadrilateral ABCD, find its area.

Sol.29 A – (-4, 8)



B – (-3, -4)

C – (0,-5)

D – (-5, 6)

Area of quadrilateral = Area ABC + Area ADC

Area ABc = | – [-4 (-4 + 5)+ (-3) (-5-8) + 0 (8+4)]|

= – (-4 + 39)

= — sq unit

Area ADC = | – (-4 [6 + 5) + 5 (-5 -8) + 0 (8 - 6)|]

= | – (-44 + 65)|

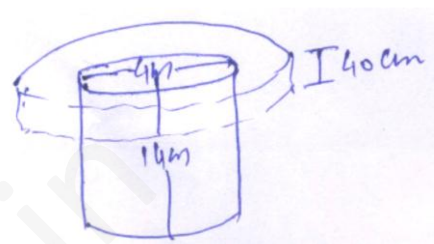
= —— sq units

Area of quad = — + —— = —— = 72 sq units

Q30. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread event all around the well to form a 40 cm high embankment. Find the width of the embankment.

Sol. 30 Diameter = 4

Radius = 2



Height (h) = 14m

Let w be width of embankment

Height (H) = 40m

Volume of earth taken out $= \pi r^2 h$

$= \pi \times 2 \times 2 \times 14$

$= 176 m^2$

Radius of embankment = 2 + w

Volume of earth spread out = Volume of embankment

$176 = \pi (r + w)^2 H - \pi r^2 H$ (No earth being filled here)

$\pi H \left((r \quad w)^2 - r^2\right) = 176$

— $.4 [(r \quad r)(r \quad r)] = 176$

(2r + w) (w) = ——— 140

(4 + w) (w) = 140

= (10 × 14)

w = 10 m

Q31. Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm, If the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.

Sol. 31 Rate of flowing = 2.52 km/hr

= 2.52 × ——

= 0.7 m/s

Let r be radius of pipe

Volume flowing through pipe = πr^2 0.7 —

= 0.7 × — r^2

= $2.2r^2$ —

Time = 30 min = 1800s

Volume flown = 2.2 × r^2 × 1800 m^3

In tank

Height = 3.15 m

Radius = 0.4 m

Volume flown from pipe = volume rised in tank

2.2 × r^2 × 1800 .4 × 0.4 × 3.15

r^2 = ————————

r^2 = ——————

r = ——— —— = 2 cm.

Diameter = 2r = 4 cm.