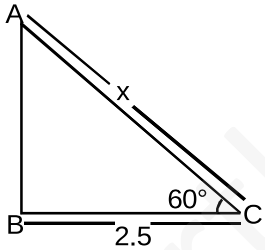
eVidyarthi
FREE Education

1. Given : AB is diameter
$\angle CAB = 30°$
To find $\angle PCA$
construction : Join OC
sol : $\therefore$ In ΔAOC
as AO = OC
$\therefore \angle OAC = \angle OCA = 30°$
$\angle OCP = 90°$ [Radius make an angle of 90° with tangent at point of contact]
$\therefore \angle PCA + \angle OCA = 90°$
$\therefore \angle PCA + 30° = 90°$
$\therefore \angle PCA = 60°$

2. k + 9, 2k – 1 and 2k + 7 are in A.P.
$\therefore a_2 - a_1 = a_3 - a_2$ [where a_1, a_2 and a_3 are the 1st, 2nd and 3rd term of the A.P.]
$2k - 1 - k - 9 = 2k + 7 - 2k + 1$
$k - 10 = 8$
$k = 18$

3. In ΔABC



$\cos 60° = \frac{BC}{AC}$

$\frac{1}{2} = \frac{2.5}{AC}$
$AC = 2.5 \times 2$
$AC = 5$ m
$\therefore$ length of the ladder is 5 m

4. We have to draw a card from 52 playing cards so the total event of drawing a card is = 52 and the event of getting red card and queen is = 26 + 2 = 28
Acc to question
The probability of getting
neither red card nor a queen $= P(\overline{A}) = 1 - P(A)$

$= P(\overline{A}) = 1 - \frac{28}{52} = \frac{6}{13}$

5. Let –5, α be the roots of $2x^2 + px - 15 = 0$

so sum of roots $= -5 + \alpha = -\frac{P}{2}$

and product of roots $= -5 \times \alpha = \frac{-15}{2}$

$\therefore \alpha = \frac{3}{2}$
If $\alpha = 3/2$ then
P = 7
and $P(x^2 + x) + k = 0$ have equal roots
so D = 0
$\Rightarrow P^2 - 4Pk = 0$
$\Rightarrow P(P - 4k) = 0$
$P = 0$ & $P - 4k = 0$
so $4k = p$

$k = \frac{P}{4} = \frac{7}{4}$

6. $x_1 = \frac{(2x-7)+(1\times 2)}{2+1}$

$x_1 = \frac{-14+2}{3} = \frac{-12}{3} = -4$

$y_1 = \frac{(2\times 4)+(1\times -2)}{2+1} = \frac{8-2}{3} = \frac{6}{3} = 2$

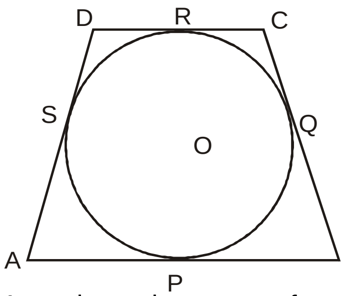
(x,y) = (–4, 2) coordinates of Q.

coordinates of point $P(x_2 y_2)$
$\Rightarrow$ mid of AQ is P

So $x_2 = \frac{2+(-4)}{2} = \frac{-2}{2} = -1$

$y_2 = \frac{-2+2}{2} = 0$, y = 0

7.



As we know that tangent from same external points are equal
$\therefore$ SD = DR ...(1)
CQ = CR ...(2)
QB = BP ...(3)
AS = AP ...(4)
Adding equation (1), (2), (3) & (4)
SD + CQ + QB + AS = DR + CR + BP + AP
AD + BC = AB + DC Hence proved

8. To proove : ΔABC is a triangle isosceles triangle

Proof : $AB = \sqrt{(3-6)^2 + (0+4)^2}$ (By using distance formula)

$AB = \sqrt{9+16} = \sqrt{25} = 5$

$\therefore$ AB = 5

$AC = \sqrt{(3+1)^2 + (0-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\therefore$ AC = 5

$BC = \sqrt{(6+1)^2 + (4-3)^2} = \sqrt{49+1} = \sqrt{50}$

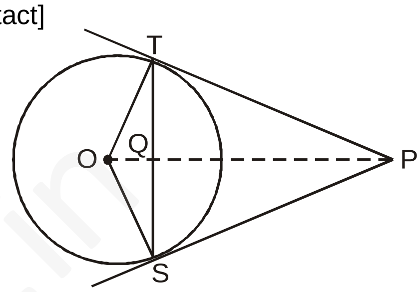
$\therefore BC = 5\sqrt{2}$

Now as AB = AC
$\therefore$ ΔABC is isosceles and $(AB)^2 + (AC)^2 = (BC)^2$
$\therefore$ By converse of pythagoras theorem ΔABC is a right angle isosceles triangle.

9. Let the first term and common difference of the A.P. be a and d respectively.
Then, $a_n = a + (n-1)d$
$a_4 = a + (4-1)d = 0$
$a_4 = a + 3d = 0$
a + 3d = 0
$\therefore$ a =–3d ...... (1)

$a_{25} = a + (25-1)\,d$
$a_{25} = a + 24\,d$
By equation ....(1)
$a_{25} = -3d + 24d$
$a_{25} = 21\,d$
$a_{11} = a + (11-1)\,d$
$a_{11} = a + 10d$
By equation .....(1)
$a_{11} = -3d + 10d$
$\therefore a_{11} = 7d$
multiply both sides by 3
$3a_{11} = 21d$
$\therefore 3a_{11} = a_{25}$ Hence proved

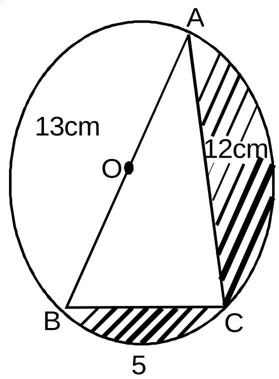
10. In ΔOTP
OT = r, OP = 2r [Given]
$\angle OTP = 90°$ [radius is perpendicular to tangent at the pair of contact]
Let $\angle TPO = \theta$



$$\therefore \sin\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$$

$\therefore \theta = 30°$
$\therefore$ In ΔTOP $\angle TOP = 60°$ [By angle sum property]
$\angle TOP = \angle SOP$ [As Δ's are congruent]
$\therefore \angle SOP$ is also 60°
$\therefore \angle TOS = 120°$ In ΔOTS as OT = OS $\therefore$ [$\angle OST = \angle OTS$]
$\angle OTS + \angle OST + \angle SOT = 180 \Rightarrow 2\angle OST + 120 = 180°$
$\therefore \angle OTS + \angle OST = 30°$

11. $AB^2 = BC^2 + AC^2$
$\Rightarrow 169 = BC^2 + 144$
$25 = BC^2$
BC = 5
Area of shaded region = Area of semicircle
– Area of ΔABC



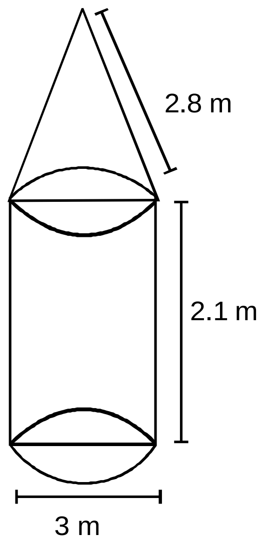
$$= \frac{\pi r^2}{2} - \frac{1}{2} \times BC \times AC$$

$$= \frac{1}{2}[3.14 \times \frac{13}{2} \times \frac{13}{2}] - (5 \times 12)$$

$$= \frac{1}{2}(132.665 - 60)$$

$= 36.3325\ cm^2$

12. Total CSA of tent
$= 2\pi rh + \pi rl$



$$= \frac{22}{7}[(2 \times \frac{3}{2} \times 2.1) + (\frac{3}{2} \times 1.4)]$$

$$\Rightarrow \frac{22}{7} \times 10.5 = 33\ m^2$$

Total CSA oftent = $33\ m^2$
$1\ m^2$ cost $\rightarrow$ Rs. 500
$33\ m^2$ cost $\rightarrow$ Rs. 500 × 33 = 16500 Rs
So total cost of canvas needed to make the text is Rs 16500

13. Given : Coordinates of

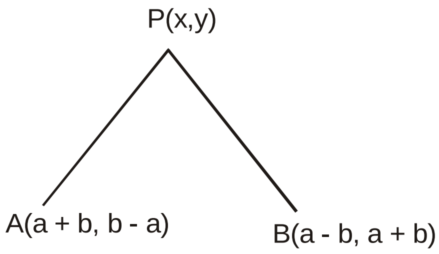
P(x,y)

A(a + b, b – a)

B (a – b, a + b)

To prove = bx = ay

According to question



PA = PB

$(PA)^2 = (PB)^2$

so accoding to distance formula

$[x-(a+b)]^2 + [y-(b-a)]^2 = [(x-(a-b)]^2 + [y-(a+b)]^2$

$(a+b)^2 - 2(a+b)x + (b-a)^2 - 2(b-a)y = (a-b)^2 - 2(a-b)x + (a+b)^2 - 2(a+b)y$

$2[(a+b)x + (b-a)y] = 2[(a-b)x + (a+b)y]$

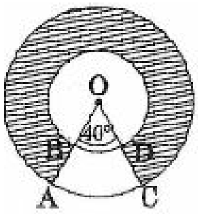
$(a+b)x + (b-a)y = (a-b)x + (a+b)y$

$(a+b)x - (a-b)x = (a+b)y - (b-a)y$

$(a+b-a+b)x = (a+b-b+a)y$

$2bx = 2ay$

bx = ay hence prove

14. 

Shaded area = Area of larger major sector – area of smaller major sector

$$= \pi(14)^2 \times \frac{40}{360} - \pi(7)^2\left(\frac{40}{360}\right)$$

$$= \pi \times \frac{40}{360}(14^2 - 7^2)$$

$$= \frac{22}{7} \times \frac{1}{9}(147) = 51.3 \text{ cm}^2$$

15. $$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27}$$

$$\frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27}$$

$$\frac{a_1 + \frac{(n-1)}{2}d_1}{a_2 + \frac{(n-1)}{2}d_2} = \frac{7n+1}{4n+27} \quad ..(1)$$

Put $\frac{n-1}{2} = m - 1$

$n - 1 = 2m - 2$

$n = 2m - 2 + 1$

$= 2m - 1$

$$\frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$= \frac{14m-7+1}{8m-4+27} = \frac{14m-6}{8m+23}$$

16. Let x – 2 = t

$$\frac{1}{t(t+1)} + \frac{1}{t(t-1)} = \frac{2}{3}$$

$$= \frac{t-1+t+1}{t(t+1)(t-1)} = \frac{2}{3}$$

$3t = t(t + 1)(t - 1)$
$3t = t(t^2 - 1)$
$3t = t^3 - t$
$t^3 - 4t = 0$
$t(t^2 - 4) = 0$
$t = 0 \quad t^2 - 4 = 0$
$t = \pm\sqrt{4}$
$t = \pm 2$
$x - 2 = 0 \quad \& \; x - 2 = \pm 2$
$x = 2 \quad x = 0, 4$

17. Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 24$

Volume of cone = volume of cylinder

$\frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 24 = \frac{22}{7} \times 10 \times 10 \times h$

h = 2 cm

18. The rise in the level of water will be due to the volume of sphere

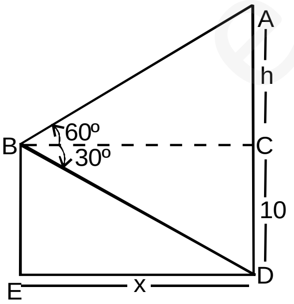
$\therefore \frac{4}{3}\pi(6)^3 = \pi x^2 \times 3\frac{5}{9}$

$\frac{4}{3} \times 6 \times 6 \times 6 = x^2 \times \frac{32}{9}$

x = 9

∴ diameter = 2x = 18 cm

19. Let x be distance of cliff from man and h + 10 be height of hill which is required.
In right triangle ACB,



$\Rightarrow \tan 60^\circ = \frac{AC}{BC} = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow x = \frac{h}{\sqrt{3}}$ ....(i)

In right triangle BCD,

$\tan 30^\circ = \frac{CD}{BC} = \frac{10}{x} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{x}$

$\Rightarrow x = 10\sqrt{3}$ ...(ii)

From (i) & (ii)

$$\frac{h}{\sqrt{3}} = 10\sqrt{3}$$

$\Rightarrow$ h = 30 m

$\therefore$ Height of cliff = h + 10 = 30 + 10 = 40 m.

Distance of ship from cliff = x = $10\sqrt{3}$ m

= 10 (1.732) = 17.32 m

20. Sample space while tossing 3 coins

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

(i) Favourable cases = {HHT, HTH, THH}

$$P(\text{exactly 2 heads}) = \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}} = \frac{3}{8}$$

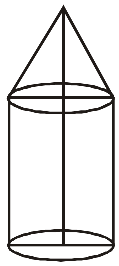
(ii) Favourable cases = {HHH, HHT, HTH, THH}

$$P(\text{at least 2 heads}) = \frac{4}{8} = \frac{1}{2}$$

(iii) favourable cases = {HTT, THT, TTH, TTT}

$$P\ (\text{at least 2 tails}) = \frac{4}{8} = \frac{1}{2}$$

21.



Given r = 2.8 ; h = 3.5 m

(ht. of cone) h_1 = 2.1 m

$\therefore$ $l = \sqrt{r^2 + (h_1)^2} = 3.5$ m

Area of convas required per tent

= [CSA of cone + CSA of cylinder]

= $\pi rl + 2\pi rh$

= πr [3.5 + 7]

$$= \frac{22}{7} \times \frac{28}{10} \times \frac{105}{10} = \frac{462}{5}\ m^2$$

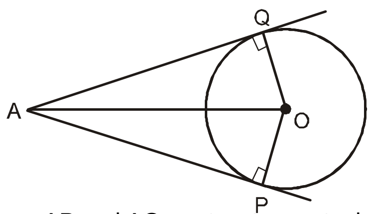
cost of canvas per tent = Rs. $\frac{462}{5} \times 120$ = Rs. 11088

Total cost of 1500 tents = Rs. 11088 $\times$ 1500

Amount shared by each schoo;

$$= \text{Rs. } \frac{11088 \times 1500}{50} = \text{Rs. } 332640$$

22.



Given : AP and AQ are two tangents drawn from a point A to a circle C (O, r).

To prove : AP = AQ.

Construction : Join OP, OQ and OA.

Proof : In ΔAOQ and ΔAPO

$\angle OQA = \angle OPA$

[Tangent at any point of a circle is perp. to radius through the point of contact]

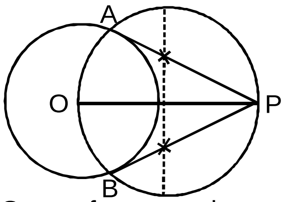
AO = AO [Common]

OQ = OP [Radius]

So, by R.H.S. criterion of congruency $\Delta AOQ \cong \Delta AOP$

$\therefore$ AQ = AP [By CPCT] **Hence Proved.**

23.



Steps of constructions are as follows

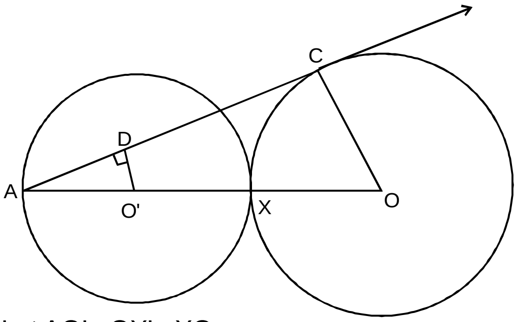
(1) Draw a circle of radius 4 cm

(2) Let O be its centre and P be any external point such that OP = 8 cm

(3) Join OP and then taking OP as diameter draw a circle intersecting the given circle at two points A and B. Join AP and BP.

(4) Hence, AP and BP are the required tangents

24.



Let AO'= OX'= XO = r

$\therefore$ Radius is always perpendicular to tangent, $\therefore \angle ACO = 90°$

In ΔADO and ΔACO

$\angle DAO' = \angle CAO$ [Common]

$\angle ADO' = \angle ACO$ [each 90°]

$\therefore$ By AA similarity criteria

$\Delta ADO' \sim \Delta ACO$

$$\Rightarrow \frac{DO'}{CO} = \frac{AO'}{AO} = \frac{r}{3r} = \frac{1}{3}$$

25. We have

$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$$

$$\Rightarrow \frac{x+2+2(x+1)}{x^2+3x+2} = \frac{4}{x+4}$$

$\Rightarrow (3x + 4)(x + 4) = 4(x^2 + 3x + 2)$

$\Rightarrow 3x^2 + 16x + 16 = 4x^2 + 12x + 8$

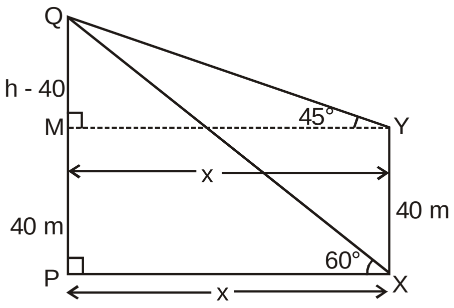
$\Rightarrow x^2 - 4x - 8 = 0$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$

$x = 2 + 2\sqrt{3}$ or $2 - 2\sqrt{3}$

26.



Let PQ be the tower
Let PQ = h
Clearly
XY = PM = 40 m
QM = (h – 40)
Let PX = MY = x

In ΔMQY, $\tan 45° = \frac{QM}{MY} \Rightarrow 1 = \frac{h-40}{x}$

$\Rightarrow x = h - 40$ ...(i)

In ΔQPX, $\tan 60° = \frac{QP}{PX} \Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = x\sqrt{3}$ ...(ii)

From (i) and (ii) we get

$x = x\sqrt{3} - 40$

$(\sqrt{3} - 1)x = 40$

$PX = x = \frac{40}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = 20(\sqrt{3}+1)$

$PQ = h = 20(\sqrt{3}+1)\sqrt{3} = 20\sqrt{3}(\sqrt{3}+1)$

27. $\underbrace{1,2,3,\ldots\ldots x-1}_{S}, x, \underbrace{x+1, \ldots\ldots\ldots, 49}_{S'}$

$S = 1 + 2 + 3 + \ldots\ldots. + (x - 1)$

$= \left(\frac{x-1}{2}\right)[1 + x - 1]$

$= \left(\frac{x-1}{2}\right)(x)$

$S = (x + 1) + (x + 2) + \ldots.. + 49$

$= \left(\frac{49-x}{2}\right)(x + 1 + 49)$

$= \frac{49-x}{2}(x + 50)$

S = S'

$\left(\frac{x-1}{2}\right)x = \left(\frac{49-x}{2}\right)(x + 50)$

$x^2 - x = 49x + 49 \times 50 - x^2 - 50x$

$2x^2 = 49 \times 50$

$x^2 = 49 \times 25$

$x = 35$

28. Coordinates of D = $\frac{2(4)+(1)}{2+1}$

$= \frac{8+1}{3} = \frac{9}{3} = 3 = \frac{2(6)+(1)(5)}{2+1}$

Coordinates of D = $(3, \frac{17}{3})$

Coordinates of E = $\frac{2(4)+(1)(7)}{2+1} = \frac{8+7}{3} = \frac{15}{3} = 5$

$\frac{2(6)+1(2)}{2+1} = \frac{12+2}{3} = \frac{14}{3}$

area Δ(ADE) $= \frac{1}{2}[4\frac{17}{3} - \frac{14}{3} + 3(\frac{14}{3} - 6) + 5(6 - \frac{17}{3})]$

$= \frac{1}{2}[4 \times 1 + 3\frac{(-4)}{3} + 5 \times \frac{1}{3}]$

$= \frac{1}{2} \times \frac{5}{3} = \frac{5}{6}$

area ΔABC

$= \frac{1}{2}[4(5-2) + 1(2-6) + 7(6-5)]$

$= \frac{1}{2}[4 \times 3 + 1 \times -4 + 7 \times 1]$

$= \frac{1}{2}[12 - 4 + 7] = \frac{15}{2}$

$\Rightarrow \frac{\text{area } \Delta ABC}{\text{area } \Delta ADE} = \frac{\frac{15}{2}}{\frac{5}{6}} = 9$

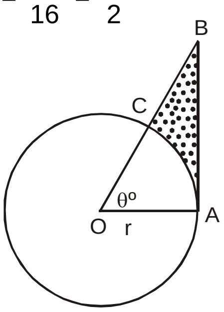
$\therefore$ area ΔABC = 9 area (ΔADE)

29. Total no. of events 16
{1 × 1, 1 × 4, 4 × 9, 1 × 16
2 × 1, 2 × 4, 2 × 9, 2 × 16
3 × 1, 3 × 4, 3 × 9, 3 × 16
4 × 1, 4 × 4, 4 × 9, 4 × 16}
Events when product is less than 16 = 8
{1 × 1, 1 × 4, 1 × 9, 2 × 1, 2 × 4, 3 × 1, 3 × 4, 4 × 1}

$\therefore$ Probability that sproduct of x & y is less than 16 = $\frac{\text{events when product is less than 16}}{\text{Total no. of events}}$

$= \frac{8}{16} = \frac{1}{2}$

30.



(i) length of sector $\widehat{CA} = \pi r \frac{\theta}{180}$

In ΔOAB

$\tan\theta = \frac{AB}{OA}$

$AB = r\tan\theta$

Now, $\sec\theta = \frac{BO}{r}$ so, $BO = r\sec\theta$

length of CO = r

So length of BC = OB – OC

$= r\sec\theta - r$

So perimeter = $\widehat{AC} + AB + BC$

$= \pi r\frac{\theta}{180} + r\tan\theta + r\sec\theta - r$

$= r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$

31. Speed of boat in still water = 24 km/hr

Let the speed of stream be 'x'

Upstream = Speed of boat = 24 – x

$T_{upstream} = \frac{\text{Distance}}{\text{speed}} = \frac{32}{24-x}$

Downstream

Speed of boat = 24 + x

$T_{downstream} = \frac{\text{distance}}{\text{speed}} = \frac{32}{24+x}$

ATP

$T_{upstream} - T_{downstream} = 1$

$\frac{32}{24-x} - \frac{32}{24+x} = 1$

$32\left[\frac{24+x-(24-x)}{(24-x)(24+x)}\right] = 1$

$32[24 + x - 24 + x] = (24 - x)(24 + x)$

$64x = (24)^2 - x^2$

$x^2 + 64x - 576 = 0$

$x^2 + 72x - 8x - 576 = 0$

$x(x + 72) - 8(x + 72) = 0$

$(x - 8)(x + 72) = 0$

$x = 8, -72$

$\therefore$ speed of stream = 8 km/hr