Solutions, Maths 2016

Ans 1. $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ $0 < \alpha < \frac{\pi}{2}$

$A + A^T = \sqrt{2}\ I_2$

$$\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

$$2\cos\alpha = \sqrt{2}$$

$$\cos\alpha = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\alpha = \frac{\pi}{4}$$

Ans 2. $|3A| = k|A|$

$|3A| = 27|A|$

$k = 27$

Ans 3. for unique solution $|A| \neq 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

$C_2 \to C_2 - C_1$; $C_3 \to C_3 - C_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & -1 & -3 \\ 3 & -1 & k-3 \end{vmatrix} \neq 0$$

expansion along R_1

$-(k-3) - 3 \neq 0$

$-k + 3 - 3 \neq 0$

$k \neq 0$

Ans 4. $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$

in Cartesian form

$2x + y - z - 5 = 0$

$2x + y - z = 5$

$$\frac{2x}{5} + \frac{y}{5} - \frac{z}{5} = 1$$

$$\frac{x}{5/2}+\frac{y}{5}+\frac{z}{-5}=1$$

Intercept cutt of on the axes $\left(\frac{5}{2}, 5, -5\right)$

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

a = $\frac{5}{2}$ b = 5 c = –5

a + b + c = 5/2

Ans 5. $\left(\hat{i}+3\hat{j}+9\hat{k}\right)\times\left(3\hat{i}-\lambda\,\hat{j}+\mu\,\hat{k}\right)=\vec{0}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 9 \\ 3 & -\lambda & \mu \end{vmatrix}=\vec{0}$$

$\hat{i}\,(3\mu+9\lambda)-\hat{j}\,(\mu-27)+\hat{k}\,(-\lambda-9)=\vec{0}$

$3\mu + 9\lambda = 0$ ——(1) $27 - \mu = 0$ ——(2) $-\lambda - 9 = 0$ ——(3)

by eq^n (2) & (3) $\mu = 27$ $\lambda = -9$

λ, μ value satisfy the eq^n (1)

So $\mu = 27,\ \lambda = -9$

Ans 6. $\vec{a}=4\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}-2\hat{j}+\hat{k}$

$\vec{a}+\vec{b}=\left(4\hat{i}-\hat{j}+\hat{k}\right)+\left(2\hat{i}-2\hat{j}+\hat{k}\right)$

$= 6\hat{i}-3\hat{j}+2\hat{k}$

unit vector parallel to $\left(\vec{a}+\vec{b}\right)=\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$

$$=\frac{6\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{36+9+4}}$$

$$=\frac{6\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{49}}$$

$$=\frac{6}{7}\hat{i}-\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}$$

Ans 7. $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$

$\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$

$$\tan^{-1}\left(\frac{x-1+x+1}{1-(x-1)(x+1)}\right)=\tan^{-1}\left(\frac{3x-x}{1+3x^2}\right)$$

$$\frac{2x}{1-\left(x^2-1\right)}=\frac{2x}{1+3x^2}$$

$x\,(1+3x^2)=x\,(2-x^2)$

$x\,(1+3x^2-2+x^2)=0$

$x(4x^2 - 1) = 0$

$x = 0$; $4x^2 - 1 = 0$

$x^2 = \frac{1}{4}$

$x = \pm \frac{1}{2}$

$x = 0, \pm \frac{1}{2}$

OR

L.H.S.

$$\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) - \tan^{-1}\left(\frac{4x}{1-4x^2}\right)$$

$$\tan^{-1}\left(\frac{\frac{6x-8x^3}{1-12x^2} - \frac{4x}{1-4x^2}}{1+\frac{6x-8x^3}{1-12x^2}\times\frac{4x}{1-4x^2}}\right)$$

$$\tan^{-1}\left(\frac{(6x-8x^3)(1-4x^2)-4x(1-12x^2)}{(1-12x^2)(1-4x^2)+(6x-8x^3)\,4x}\right)$$

$$\tan^{-1}\left(\frac{6x-24x^3-8x^3+32x^5-4x+48x^3}{1-4x^2-12x^2+48x^4+24x^2-32x^4}\right)$$

$$\tan^{-1}\left(\frac{32x^5+16x^3+2x}{16x^4+8x^2+1}\right)$$

$$\tan^{-1}\left\{2x\frac{(16x^4+8x^2+1)}{16x^4+8x^2+1}\right\}$$

$\tan^{-1} 2x$

L.H.S. = R.H.S.

Ans 8. $[10 \quad 3]\begin{bmatrix} x \\ y \end{bmatrix} = [145]$

$[3 \quad 10]\begin{bmatrix} x \\ y \end{bmatrix} = [180]$

$10x + 3y = 145$

$3x + 10y = 180$

by solving the equations we get

$x = 10, y = 15$

but Typist charge 2 Rs. Per Page from a Poor student shyam

amount taken by shyam = $2 \times 5 = 10$ Rs.

but from another person, he take for

5 Pages = 15 × 5

= 75 Rs.

amount differ by = 75 – 10

= 65 Rs. Less.

sympathy are reflect this problem

Ans 9. at x = 0 function is continuous,

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$

R.H.L.

$$= \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(0+h) = \lim_{h\to 0} \frac{\sqrt{1+b\,h}-1}{h} \times \frac{\sqrt{1+b\,h}+1}{\sqrt{1+b\,h}+1}$$

$$= \lim_{x\to 0} \frac{1+bh-1}{h\left(\sqrt{1+bh}+1\right)}$$

$$= \lim_{x\to 0} \frac{bh}{h\left(\sqrt{1+bh}+1\right)}$$

$$= \lim_{x\to 0} \frac{b}{\sqrt{1+bh}+1}$$

$$= \frac{b}{2}$$

f(0) = 2

L.H.L.

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$$

$$= \lim_{h\to 0} \frac{\sin(a+1)\,(0-h)+\sin(0-h)}{0-h}$$

$$= \lim_{h\to 0} \frac{\sin\left(-(a+1)\,h\right)+\sin(-h)}{-h}$$

$$= \lim_{h\to 0} \frac{\sin\,(a+1)\,h}{h} + \frac{2\sin h}{h}$$

= (a+1) + 2

a + 3 = 2 $\frac{b}{2} = 2$

a = –1 b = 4

Ans 10. x cos (a+y) = cos y

$$x = \frac{\cos y}{\cos\,(a+y)}$$

$$\frac{dx}{dy} = \frac{\cos(a+y)\times(-\sin y) - \cos y\left(-\sin(a+y)\right)}{\cos^2(a+y)}$$

$$\frac{dx}{dy} = \frac{-\sin y\cos(a+y)+\cos y\sin(a+y)}{\cos^2(a+y)}$$

$\because \quad \sin(A-B) = \sin A\cos B - \cos A\sin B$

$$\frac{dx}{dy} = \frac{\sin a}{\cos^2(a+y)} \quad \text{So } \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

$$\sin a\frac{d^2y}{dx^2} = 2\cos(a+y) \times (-\sin(a+y))\frac{dy}{dx}$$

$$\sin a\frac{d^2y}{dx^2} + 2\cos(a+y) \times (\sin(a+y))\frac{dy}{dx} = 0$$

$$\sin a\frac{d^2y}{dx^2} + \sin 2(a+y))\frac{dy}{dx} = 0$$

OR

$$y = \sin^{-1}\left(\frac{6x-4\sqrt{1-4x^2}}{5}\right)$$

$$y = \sin^{-1}\left(\frac{6x}{5}-\frac{4}{5}\sqrt{1-4x^2}\right)$$

$$y = \sin^{-1}\left(2x\times\frac{3}{5}-\frac{4}{5}\sqrt{1-(2x)^2}\right)$$

$$\sin^{-1}p - \sin^{-1}q = \sin^{-1}\left(p\sqrt{1-q^2} - q\sqrt{1-p^2}\right)$$

$$p = 2x \quad q = \frac{4}{5}$$

$$y = \sin^{-1}2x - \sin^{-1}\frac{4}{5}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x)^2}}\times 2.1 - 0$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$

Ans 11. $y = x^3 + 2x - 4$

$\left(\frac{dy}{dx}\right)_{C_1} = 3x^2 + 2$ ....(1)

eq^n of tangent : $y - y_1 = m(x - x_1)$ ......(2)

is $\perp$ to $x + 14y + 3 = 0$

$$m = \left(\frac{dy}{dx}\right)_{C_1} = 3x^2 + 2$$

$14y = -x - 3$

$y = \frac{-x-3}{14}$

$m \times \frac{-1}{14} = -1$

$m = 14$

$3x^2 + 12 = 14$

$3x^2 = 12$

$x^2 = 14 \qquad x = \pm 2$

if $\quad x = 2 \qquad\qquad x = -2$

$y = 2^3 + 2.2 - 4 \qquad y = -8 - 4 - 4$

$y = 8 + 4 - 4 \qquad y = -16$

$y = 8$

$P_1 (2, 8) \qquad P_2 (-2, -16)$

eq^n of tangent at $P_1 (2, 8)$

$y - 8 = 14 (x - 2)$

$y - 8 = 14x - 28$

$14x - y = 20$

eq^n of tangent at $P_2 (-2, -16)$

$y + 16 = 14 (x + 2)$

$14x - y = 16 - 28$

$14x - y = -12$

Ans 12.

$$I = \int \frac{(2x-5)e^{2x}}{(2x-3)^3} dx$$

$$= \int e^{2x-3} \cdot e^3 \left[\frac{(2x-3)-2}{(2x-3)^3}\right] dx$$

$$= \int e^{2x-3} \cdot e^3 \left[\frac{1}{(2x-3)^2} - \frac{2}{(2x-3)^3}\right] dx$$

Let $f(x) = \frac{1}{(2x-3)^2}$ whose, $2x - 3 = t$

$\Rightarrow 2dx = dt$

$$I = e^3 \int e^t \left[\frac{t-2}{t^3}\right] \frac{dt}{2}$$

$$= \int e^3 e^t \left[f(t) + f^1(t)\right] \frac{dt}{2}$$

$$= \frac{e^3}{2} \cdot e^t \cdot f(t) \qquad + C$$

$$= \frac{e^3}{2} e^{2x-3} \cdot \frac{1}{(2x-3)^2} + C$$

$$= \frac{e^{2x}}{2(2x-3)^2} + C$$

OR

$$I = \int \frac{x^2+x+1}{(x^2+1)(x+2)} dx$$

Let $\frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$

$$= \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$$

$$= \frac{(A+B)x^2+(2B+C)x+(2C+A)}{(x+2)(x^2+1)}$$

$\therefore$ A + B = 1 ....(1)

2B + C = 1 ....(2)

2C + A = 1 ....(3)

$\Rightarrow$ A = 1 – 2C ....(4)

From (2)

$B = \frac{1-C}{2}$ ....(5)

Put (4) and (5) in (1) we get.

$$1 - 2C + \frac{1-C}{2} = 1$$

$\Rightarrow$ 2 – 4C + 1 – C = 2

$\Rightarrow$ 5C = 1

$\Rightarrow$ $C = \frac{1}{5}$

$\therefore$ $A = 1 - 2C = 1 - \frac{2}{5} = \frac{3}{5}$

$B = \frac{1-C}{2} = \frac{1-\frac{1}{5}}{2} = \frac{\frac{4}{5}}{2} = \frac{2}{5}$

$\therefore$ $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3/5}{(x+2)} + \frac{\frac{2}{5}x+\frac{1}{5}}{(x^2+1)}$

$\therefore$ $I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$

$\Rightarrow$ $I = \int \frac{3/5}{(x+2)} + \frac{\frac{2}{5}x+\frac{1}{5}}{(x^2+1)} dx$

$\Rightarrow \quad I = \frac{3}{5}\log(x+2) + \frac{1}{5}\left[\int \frac{2x}{x^2+1}dx + \int \frac{1}{x^2+1}dx\right]$

$\Rightarrow \quad I = \frac{3}{5}\log(x+2) + \frac{1}{5}\log\left(x^2+1\right) + \tan^{-1}x + C$

Ans 13. $I = \int_{-2}^{2} \frac{x^2}{1+5^x}dx$ ....(1)

Using Property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$I = \int_{-2}^{2} \frac{(-x)^2}{1+5^{-x}}dx$

$\Rightarrow \quad I = \int_{-2}^{2} \frac{5^x x^2}{1+5^x}\,dx$ ....(2)

Add (1) and (2), we get

$2I = \int_{-2}^{2} \frac{1+5^x}{1+5^x}\cdot x^2\,dx$

$= \int_{-2}^{2} x^2\,dx$

$= \left[\frac{x^3}{3}\right]_{-2}^{2}$

$= \frac{1}{3}\,[\,2^3 - (-2)^3\,]$

$= \frac{1}{3}[\,8 - (-8)\,]$

$2I = \frac{16}{3} \qquad \therefore\ I = \frac{8}{3}$

Ans 14. $I = \int (x+3)\sqrt{3-4x-x^2}\,dx$

Let $x+3 = \lambda\,\frac{d}{dx}(3-4x-x^2) + \mu$

$\Rightarrow \quad x+3 = \lambda\,(-2x-4) + \mu$

$\Rightarrow \quad x+3 = -2\lambda x - 4\lambda + \mu$

$\therefore \quad -2\lambda = 1$

$\Rightarrow \quad \lambda = -\frac{1}{2}$

$-4\lambda + \mu = 3$

$\Rightarrow \quad -4\left(-\frac{1}{2}\right) + \mu = 3$

$\Rightarrow \quad 2 + \mu = 3$

$\Rightarrow \quad \mu = 1$

$$\therefore \quad I = \int\left[-\frac{1}{2}\frac{d}{dx}(3-4x-x^2)+1\right]\sqrt{3-4x-x^2}\,dx$$

$$= -\frac{1}{2}\int\frac{d}{dx}(3-4x-x^2)\sqrt{3-4x-x^2}\,dx + \int\sqrt{3-4x-x^2}\,dx$$

$$= -\frac{1}{2}\frac{(3-4x-x^2)^{3/2}}{\frac{3}{2}} + \int\sqrt{3-x^2-4x-4+4}\,dx$$

$$= -\frac{(3-4x-x^2)^{3/2}}{3} + \int\sqrt{7-(x+2)^2}\,dx$$

$$= -\frac{(3-4x-x^2)^{3/2}}{3} + \frac{x+2}{2}\sqrt{7-(x+2)^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + C$$

Ans 15. $\frac{dy}{dx} = -\frac{x+y\cos x}{1+\sin x}$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{\cos x}{1+\sin x}y = -\frac{x}{1+\sin x} \quad ...(1)$$

This is a linear differential equation with

$P = \frac{\cos x}{1+\sin x}$, $Q = \frac{-x}{1+\sin x}$

$$\therefore \quad \text{I.F.} = e^{\int \frac{\cos x}{1+\sin x}dx}$$

$$= e^{\log(1+\sin x)}$$

$$= (1+\sin x)$$

Multiplying both sides of (i) by I.F. = 1+sinx , we get

$$(1+\sin x)\frac{dy}{dx} + y\cos x = -x$$

Integrating with respect to x, we get

$$y(1+\sin x) = \int -x\,dx + C$$

$$\Rightarrow \quad y = \frac{2C-x^2}{2(1+\sin x)} \quad ......(2)$$

Given that y = 1 when x = 0

$\therefore \quad 1 = \dfrac{2C}{2\,(1+0)}$

$\Rightarrow \quad C = 1$ .... (3)

$\therefore$ Put (3) in (2) , we get

$$y = \frac{2 - x^2}{2\,(1+\sin x)}$$

Ans 16. $2ye^{x/y}\,dx + (y - 2x\,e^{x/y})\,dy = 0$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{2x\,e^{x/y} - y}{2y\,e^{x/y}}$$

The given D.E. is a homogeneous differential equation.

$\therefore$ Put $x = vy$

$$\frac{dx}{dy} = v + y\,\frac{dv}{dy}$$

$$v + y\,\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$$

$$\Rightarrow \quad y\,\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v$$

$$\Rightarrow \quad y\,\frac{dv}{dy} = -\frac{1}{2e^v}$$

$$\Rightarrow \quad 2ye^v\,dv = -\,dy$$

$$\Rightarrow \quad 2e^v\,dv = -\frac{1}{y}dy$$

$$\Rightarrow \quad 2\int e^v dv = -\int \frac{1}{y}dy$$

$$\Rightarrow \quad 2e^v = -\log|y| + \log C$$

$$\Rightarrow \quad 2e^v = \log\left|\frac{c}{y}\right|$$

$$\Rightarrow \quad 2e^{x/y} = \log\left|\frac{c}{y}\right|$$

Given that at $x = 0$, $y = 1$.

$$\therefore \quad 2\cdot e^0 = \log\left|\frac{c}{1}\right|$$

$$\Rightarrow \quad C = e^2$$

$$\therefore \quad 2e^{x/y} = \log\frac{e^2}{y}$$

$\Rightarrow \quad \log y = -2e^{x/y} + 2$

$\Rightarrow \quad y = e^{2-2e^{x/y}}$

Ans 17. A (4, 5, 1) B (0, –1, –1) C (3, 9, 4) D (–4, 4, 4)

$\therefore \quad \vec{AB} = \left(-4\hat{i} - 6\hat{j} - 2\hat{k}\right)$

$\vec{AC} = \left(-\hat{i} + 4\hat{j} + 3\hat{k}\right)$

$\vec{AD} = \left(-8\hat{i} + \hat{j} + 3\hat{k}\right)$

$$\therefore \quad \left[\vec{AB}\ \vec{AC}\ \vec{AD}\right] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

$$= \quad -4\begin{vmatrix} 4 & 3 \\ -1 & 3 \end{vmatrix} + 6\begin{vmatrix} -1 & 3 \\ -8 & 3 \end{vmatrix} - 2\begin{vmatrix} -1 & 4 \\ -8 & -1 \end{vmatrix}$$

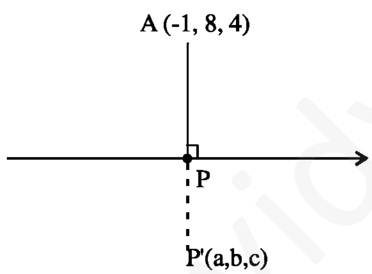
$= \quad -4\ (12 + 3) + 6\ (-3 + 24) - 2\ (1+32)$

$= \quad -60 + 126 - 66$

$= \quad 0$

$\therefore$ Four points A, B, C, D are coplannar.

Ans 18.



eq^n of Line BC $\Rightarrow \dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}$

$$\frac{x - 0}{2} = \frac{y + 1}{-2} = \frac{z - 3}{-4} = \lambda$$

general coordinates of P

P $(2\lambda, -2\lambda - 1, -4\lambda \times 3)$

D.R of AP $(2\lambda + 1, -2\lambda - 9, -4\lambda - 1)$

AP $\perp$ BC

$2\ (2\lambda + 1) - 2\ (-2\lambda - 9) - 4\ (-4\lambda - 1) = 0$

$4\lambda + 2 + 4\lambda + 18 + 16\lambda + 4 = 0$

$24 + 24\lambda = 0$

$\lambda = -1$

P (–2, 1, 7)

Coordinates of foot of $\perp$ (–2 , 1, 7)

Coordinaties of image of A is P' (a, b, c) is

$\frac{a-1}{2} = -2$, a = – 3

$\frac{b+8}{2} = 1$, b = – 6

$\frac{c+4}{2} = 7$, c = 10

P' (–3, –6, 10)

Ans 19. bag A = 4 white, 2 black

bag y = 3 white, 3 black

E_1 = first bag selected

E_2 = second bag selected

$P(E_1) = \frac{1}{2}$ $P(E_2) = \frac{1}{2}$

$P(A / E_1) = \frac{4}{6}\times\frac{2}{5}+\frac{2}{6}\times\frac{4}{5} = \frac{16}{30}$

$P(A / E_2) = \frac{3}{6}\times\frac{3}{5}+\frac{3}{6}\times\frac{3}{5} = \frac{18}{30}$

$P(E_2 / A) = \frac{P(E_2)\, P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\, P(A/E_2)}$

$P(E_2/A) = \frac{\frac{1}{2}\times\frac{18}{30}}{\frac{1}{2}\times\frac{16}{30}+\frac{1}{2}\times\frac{18}{30}}$

$= \frac{18}{16+18} = \frac{18}{34} = \frac{9}{17}$

OR

$P(\text{ win }) = \frac{3}{36} = \frac{1}{12}$

$P(\text{ lose }) = \frac{11}{12}$

$P(\text{ Awins }) = \frac{1}{12}+\frac{11}{12}\times\frac{11}{12}\times\frac{1}{12}+\left(\frac{11}{12}\right)^4\times\frac{1}{12}+ \ldots\ldots\ldots.$

$a = \frac{1}{12}$ $r = \frac{121}{144}$

by using formula of infinite G.P.

$$P\ (A wins) = \frac{\frac{1}{12}}{1-\frac{121}{144}} = \frac{12}{23}$$

Ans 20. X = larger of of three numbers

X = 3, 4, 5, 6

$P\ (x = 3) = 6 \times \frac{1}{6}\times\frac{1}{5}\times\frac{1}{4} = \frac{1}{20}$

$P\ (x = 4) = 18 \times \frac{1}{6}\times\frac{1}{5}\times\frac{1}{4} = \frac{3}{20}$

$P\ (x = 5) = 36 \times \frac{1}{6}\times\frac{1}{5}\times\frac{1}{4} = \frac{6}{20}$

$P\ (x = 6) = 60 \times \frac{1}{6}\times\frac{1}{5}\times\frac{1}{4} = \frac{10}{20}$

Xi	Pi	PiXi	$PiXi^2$
3	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{9}{20}$
4	$\frac{3}{20}$	$\frac{12}{20}$	$\frac{48}{20}$
5	$\frac{6}{20}$	$\frac{30}{20}$	$\frac{150}{20}$
6	$\frac{10}{20}$	$\frac{60}{20}$	$\frac{360}{20}$

Mean = $\Sigma P_i X_i^2 = \left(\frac{105}{20}\right)^2 = 5.25$

$\Sigma P_i X_i^2 = \frac{567}{20}$

$Var(X) = \Sigma P_i X_i^2 - (\Sigma P_i X_i)^2$

$= \frac{567}{20} - \left(\frac{105}{20}\right)^2 = 0.787$

Ans 21. (a, b) * (c, d) = (a + c, b + d)

(i) Commutative

(a, b) * (c, d) = (a+c, b+d)

(c, d) * (a, b) = (c+a, d+b)

for all, a, b, c, d $\in$ R

* is commulative on A

(ii) Associative : _______

(a, b), (e, d), (e, f) $\in$ A

{ (a, b) * (c, d) } * (e, f)

= (a + c, b+d) * (e, f)

= $((a+c)+e,\ (b+d)+f)$

= $(\ a+(c+e),\ b+(d+f)\ )$

= (a*b) * (c+d, d+f)

= (a * b) { (c, a) * (e, f) }

is associative on A

Let (x, y) be the identity element in A.
then,

(a, b) * (x, y) = (a, b) for all (a,b) $\in$ A

(a + x, b+y) = (a, b) for all (a, b) $\in$ A

a + x = a, b + y = b for all (a, b) $\in$ A

x = 0, y = 0

(0, 0) $\in$ A

(0, 0) is the identity element in A.

Let (a, b) be an investible element of A.

(a, b) * (c, d) = (0, 0) = (c, d) * (a, b)

(a+c, b+d) = (0, 0) = (c+a, d+b)

a + c = 0 b + d = 0

a = – c b = – d

c = – a d = – b

(a, b) is an invertible element of A, in such a case the inverse of (a, b) is (–a, –b)

Ans 22. $y = \dfrac{4\sin\theta}{2+\cos\theta} - \theta$

$$\frac{dy}{d\theta} = \frac{(2+\cos\theta)\,(4\cos\theta)+4\sin^2\theta}{(2+\cos\theta)^2} - 1$$

$$= \frac{8\cos\theta + 4\cos^2\theta + 4\sin^2\theta}{(2+\cos\theta)^2} - 1$$

$$= \frac{8\cos\theta + 4}{(2+\cos\theta)^2} - 1$$

$$= \frac{4\cos\theta - \cos^2\theta}{(2+\cos\theta)^2}$$

$$\frac{dy}{d\theta} = \frac{\cos\theta\,(4-\cos\theta)}{(2-\cos\theta)^2}$$

for increasing $\dfrac{dy}{d\theta} > 0$

$$\theta \in \left(0, \frac{\pi}{2}\right)$$

$0 \le \cos \le 1$

$(2 + \cos\theta)^2$ always greator than 0

So, $\frac{dy}{d\theta}$ is increaring on $\left[0,\frac{\pi}{2}\right]$

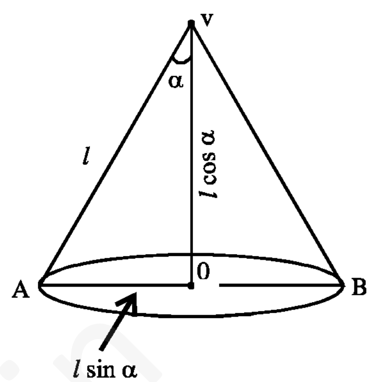
OR

Volume of cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (\ell \sin\alpha)^2(\ell\cos\alpha)$$

$$= \frac{1}{3}\pi \ell^3 \sin^2\alpha \ \cos\alpha$$



$$\frac{dv}{d\alpha} = \frac{\pi l^3}{3}[-\sin^3\alpha + 2\sin\alpha\cos\alpha \times \cos\alpha]$$

$$= \frac{\pi l^3 \sin\alpha}{3}(-\sin^2\alpha + 2\cos^2\alpha)$$

for maximum or minimum

$$\frac{dv}{d\alpha} = 0$$

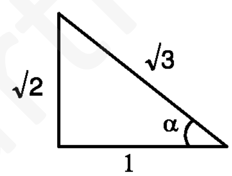
$$\frac{\pi l^3 \sin\alpha}{3}(-\sin^2\alpha + 2\cos^2\alpha) = 0$$

$\sin\alpha \neq 0$

$2\cos^2\alpha = \sin^2\alpha$

$\tan^2\alpha = 2$

$\tan\alpha = \sqrt{2}$



$\cos\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \cos^{-1}\frac{1}{\sqrt{3}}$

again diff. w.r.t. α, we get

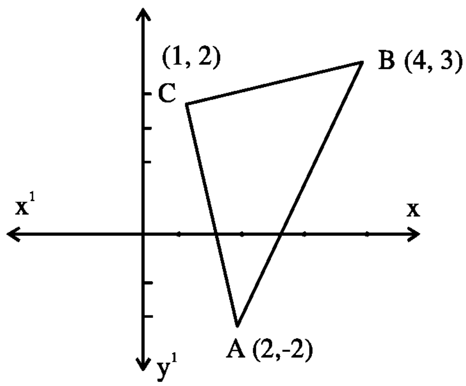
$$\frac{d^2v}{d\alpha^2} = \frac{1}{3}\pi l^3 \cos^2\alpha\,(2 - 7\tan^2\alpha)$$

at $\cos\alpha = \frac{1}{\sqrt{3}}$

$$\frac{d^2v}{d\alpha^2} < 0$$

V is maximum when $\cos\alpha = \frac{1}{\sqrt{3}}$ or $\alpha = \cos^{-1}\frac{1}{\sqrt{3}}$

Ans 23.



Equation of line AB : -

$$y + 2 = \frac{3+2}{2}(x-2)$$

$\Rightarrow$ $2y = 5x - 14$

Equation of line BC : -

$$y - 3 = \frac{1}{3}(x-4)$$

$\Rightarrow$ $3y = x + 5$

Equation of line CA : -

$(y - 2) = -4\ (x-1)$

$4x + y = 6$

$\therefore$ ar (ΔABC)

$$= \int_{-2}^{3} \frac{2y+14}{5}\ dy - \int_{2}^{3} 3y - 5\ dy - \int_{-2}^{2} \frac{6-y}{4}\ dy$$

$$= \frac{75}{5} - \frac{5}{2} - \frac{24}{4}$$

$$= \frac{300-120-50}{20} = \frac{130}{20}$$

$$= \frac{13}{2} \text{ sq.units}$$

Ans 24.

$$\vec{r} \cdot \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) - 4 = 0$$

$$\vec{r} \cdot \left(-2\hat{i} + \hat{j} + \hat{k}\right) + 5 = 0$$

$$\vec{r} \cdot \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) + \lambda \left\{\vec{r} \cdot \left(-2\hat{i} + \hat{j} + \hat{k}\right)\right\} - 4 + 5\lambda = 0$$

$$\Rightarrow \quad \vec{r} \cdot \left[(1-2\lambda)\ \hat{i} + (-2+\lambda)\ \hat{j} + (3+\lambda)\ \hat{k}\right] - 4 + 5\lambda = 0$$

$$\Rightarrow \quad (1 - 2\lambda)x + (-2 + \lambda)y + (3 + \lambda)z = -5\lambda + 4$$

$$\Rightarrow \quad \frac{x}{\frac{-5\lambda+4}{1-2\lambda}} + \frac{y}{\frac{-5\lambda+4}{-2+\lambda}} + \frac{z}{\frac{-5\lambda+4}{3+\lambda}} = 1$$

$$\therefore \quad \frac{-5\lambda+4}{1-2\lambda} = \frac{-5\lambda+4}{-2+\lambda}$$

$\Rightarrow \quad 1 - 2\lambda = -2 + \lambda$

$\Rightarrow \quad -3\lambda = -3$

$\Rightarrow \quad \lambda = 1$

$\therefore$ Equation of the required plane

$-x - y + 4z = -1$

$x + y - 4z - 1 = 0$

Vector eq^n of the required Plane

$\Rightarrow \quad \vec{r} \cdot (\hat{i} + \hat{j} - 4\hat{k}) - 1 = 0$

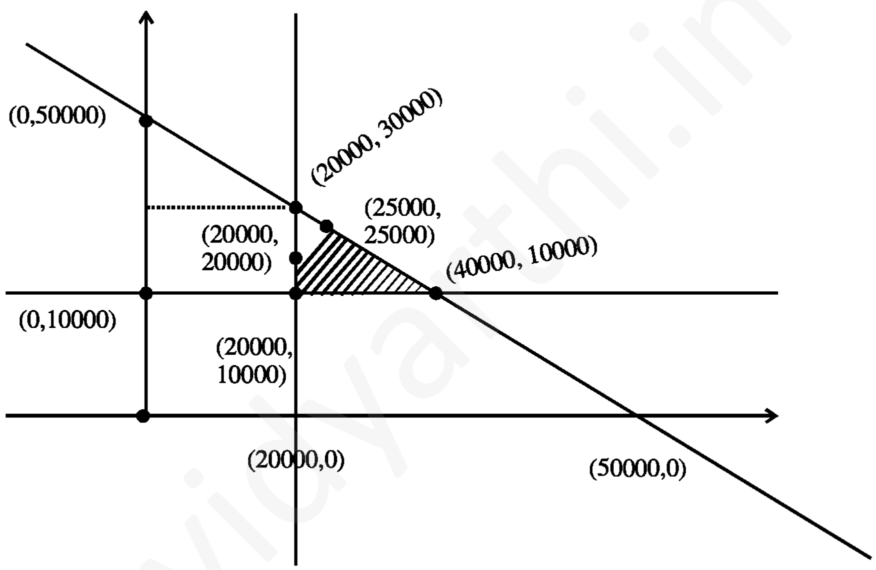
Ans 25. $Z = 0.1x + 0.09y$

$x + y \le 50000$

$x \ge 20000$

$y \ge 10000$

$y \le x$



	$z = 0.1x + 0.09y$	
P_1 (20000, 10000)	2900	
P_2 (40000, 10000)	4900	
P_3 (25000, 25000)	4750	
P_4 (20000, 20000)	3800	

When A invest 40000 & B invest 10000

Ans 26. $$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz\,(x+y+z)^3$$

L.H.S.

Multipiying R_1, R_2 and R_3 by z, x, y respectively

$$= \frac{1}{xyz}\begin{vmatrix} z(x+y)^2 & z^2x & z^2y \\ x^2z & x(z+y)^2 & x^2y \\ y^2z & xy^2 & y(z+x)^2 \end{vmatrix}$$

take common z, x, y from C_1, C_2, & C_3

$$= \frac{xyz}{xyz}\begin{vmatrix} (x+y)^2 & z^2 & z^2 \\ x^2 & (z+y)^2 & x^2 \\ y^2 & y^2 & (z+x)^2 \end{vmatrix}$$

$C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$

$$= \frac{c+4}{2} = 7$$

taking common x+y+z from C_1 & C_2

$$= (x+y+z)^2\begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ y-z-x & y-z-x & (z+x)^2 \end{vmatrix}$$

$R_3 \to R_3 - (R_1 + R_2)$

$$= (x+y+z)^2\begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ -2x & -2z & 2xz \end{vmatrix}$$

$C_1 \to zC_1$, $C_2 \to xC_3$

$$= \frac{(x+y+z)^2}{xz}\begin{vmatrix} z(x+y-z) & 0 & z^2 \\ 0 & x(z+y-x) & x^2 \\ -2xz & -2xz & 2xz \end{vmatrix}$$

$C_1 \to C_1 + C_3$ $\qquad C_2 \to C_2 + C_3$

$$= \frac{(x+y+z)^2}{xz}\begin{vmatrix} z(x+y) & z^2 & z^2 \\ x^2 & x(z+y) & x^2 \\ 0 & 0 & 2xz \end{vmatrix}$$

taking z and x common from R_1 & R_2

$$= \frac{(x+y+z)^2}{xz} \times zx \begin{vmatrix} x+y & z & z \\ x & z+y & x \\ 0 & 0 & 2xz \end{vmatrix}$$

expanasion along R_3

$= \quad (x+y+z)^2 \times 2xz \left((x+y)(z+y) - xz\right)$

$= \quad (x+y+z)^2 \times 2xz\ (xz + xy + yz + y^2 - xz)$

$= \quad (x+y+z)^2 \times 2xz\ (xy + yz + y^2)$

$= \quad 2xyz\ (x + y + z)^3$

OR

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A^2 = AA \quad = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2.A \quad = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$\therefore \quad A^3 - 6A^2 + 7A + KI_3 = 0$

$$\Rightarrow \quad \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$\Rightarrow \quad k = 2$