SAMPLE TEST PAPER SET – B

MATHEMATICS

CBSE EXAMINATIONS 2012-13

Max. Marks: 100 **Time: 180 Minutes**

SECTION – A

Q01. If $f:\mathrm{R}\to\mathrm{R}$ be defined by $f(x)=\left(7-x^5\right)^{1/5}$, then find $fof(x)$.

Q02. Evaluate: $\int \frac{1}{\left[\sqrt{1-x^2}\left(16-\sin^{-1}x\right)^{1/2}\right]}dx$.

Q03. Write one of the range of $\operatorname{cosec}^{-1}x$ other than its principal branch.

Q04. In the matrix equation $\begin{pmatrix}11 & 16\\ 7 & 10\end{pmatrix}=\begin{pmatrix}2 & 3\\ 1 & 2\end{pmatrix}\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$, apply $C_2\to C_2-C_1$ on both the sides.

Q05. Evaluate: $\begin{vmatrix}a+ib & c+id\\ -c+id & a-ib\end{vmatrix}$.

Q06. If $A=\begin{bmatrix}1\\2\\3\end{bmatrix}$ then, find AA'.

Q07. If $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ then, find the angle between $\vec{a}$ and $\vec{b}$.

Q08. Evaluate: $\int_0^{3/2}[x]dx$, where $[x]$ represents a greatest integer function.

Q09. If '*' is a binary operation defined on R and if $a*b=\frac{ab}{2}$, write the value for (4∗2)∗6.

Q10. For a vector equiangular with the coordinate axis, write its direction cosines.

SECTION – B

Q11. Show that: $\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)=\pi=2\left(\tan^{-1}(1)+\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)\right)$.

OR Prove that: $\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]=\frac{2b}{a}$.

Q12. Using properties of determinants, evaluate: $\begin{vmatrix}(x-2)^2 & (x-1)^2 & x^2\\ (x-1)^2 & x^2 & (x+1)^2\\ x^2 & (x+1)^2 & (x+2)^2\end{vmatrix}$.

Q13. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a^3\left(x^3-y^3\right)$ then, show that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$.

OR If $y=x\log\left(\frac{x}{a+bx}\right)$, then show that $x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$.

Q14. Prove that the sum of intercepts of the tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ upon the coordinate axes is of constant length.

Q15. If $x^p.y^q=(x+y)^{p+q}$ then, prove that $\frac{dy}{dx}=\frac{y}{x}$. Hence show that $\frac{d^2y}{dx^2}=0$.

Q16. Evaluate: $\int_0^1\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$. **OR** Evaluate: $\int_0^1\cot^{-1}\left(1-x+x^2\right)dx$.

Q17. Solve: $y\sin x\dfrac{dy}{dx}=\cos x\left(\sin x-\dfrac{y^2}{2}\right),\ y\left(\dfrac{\pi}{2}\right)=1$.

Q18. Find a point on the line $\dfrac{x+2}{3}=\dfrac{y+1}{2}=\dfrac{z-3}{2}$ at a distance of $3\sqrt{2}\,units$ from the point $(1,2,3)$.

Q19. **a)** Let $f:\mathrm{R}\to\mathrm{R}$ be given by $f(x)=\dfrac{x^2+4x+30}{x^2-8x+18}$. Is f a one- one function?

b) Find the range of $f(x)=\dfrac{|x-3|}{x-3}$.

Q20. Decompose the vector $6\hat{i}-3\hat{j}-6\hat{k}$ into the vectors which respectively are parallel and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$.

OR If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$ then, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$.

Q21. Find $\mathrm{P}(|x-4|\le 2)$ if x follows a Binomial Distribution with the mean 4 and variance 2.

Q22. Solve: $\left(\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}}-\dfrac{y}{\sqrt{x}}\right)\dfrac{dx}{dy}=1,\ x\neq 0$. **OR** Solve: $\dfrac{dy}{dx}+x\sin 2y=x^3\cos^2 y$.

SECTION – C

Q23. A point P is given on the circumference of a circle of radius *r*. A chord QR is parallel to the tangent line at P. Find the maximum area of the triangle PQR.

Q24. Solve the following system of equations using matrix:

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2;\ x,y,z\neq 0.$$

OR Find the inverse of $\begin{bmatrix}1&3&-2\\-3&0&-1\\2&1&0\end{bmatrix}$ using elementary transformations.

Q25. Using integration, find area of the triangle formed by positive *x*-axis and the tangent and the normal to the curve $x^2+y^2=4$ at $(1,\sqrt{3})$.

Q26. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 bus drivers. The probability of an accident involving a scooter, a car and a bus are respectively 0.01, 0.03 and 0.15. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Q27. Find the distance of the point $\mathrm{P}(-2,3,-4)$ from the line $\dfrac{x+2}{3}=\dfrac{2y+3}{4}=\dfrac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$.

OR Find the distance of the point $\mathrm{P}(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to the line $\dfrac{x}{2}=\dfrac{y}{3}=\dfrac{z}{-6}$.

Q28. There are two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14kg of nitrogen and 14kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Q29. Evaluate: $\displaystyle\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$.

ANSWERS OF SAMPLE TEST PAPER SET – B

Q01. x **Q02.** $-2\sqrt{16-\sin^{-1}x}+k$ **Q03.** $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]-\{\pi\}$ **Q04.** $\begin{bmatrix}11 & 5\\ 7 & 3\end{bmatrix}=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}1 & 1\\ 3 & 1\end{bmatrix}$

Q05. $a^2+b^2+c^2+d^2$ **Q06.** $\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 9\end{bmatrix}$ **Q07.** $\frac{\pi}{3}$ **Q08.** $\frac{1}{2}$ **Q09.** 12 **Q10.** $\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}$

Q12. -8 **Q16.** 0 **OR** $\frac{\pi}{2}-\log 2$ **Q17.** $y^2=\sin x$ **Q18.** $(-2,-1,3),\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$

Q19.a) No **(b)** $\{-1,1\}$ **Q20.** $-\hat{i}-\hat{j}-\hat{k},\ 7\hat{i}-2\hat{j}-5\hat{k}$ **OR** $\frac{1}{3}\left(5\hat{i}+2\hat{j}+2\hat{k}\right)$

Q21. $\frac{119}{128}$ **Q22.** $y=\left(2\sqrt{x}+k\right)e^{-2\sqrt{x}}$ **OR** $2\tan y=x^2-1+ke^{-x^2}$ **Q23.** $\frac{3\sqrt{3}}{4}r^2\ sq.units$

Q24. $x=2,y=3,z=5$ **OR** $\begin{bmatrix}1 & -2 & -3\\ -2 & 4 & 7\\ -3 & 5 & 9\end{bmatrix}$ **Q25.** $2\sqrt{3}\ sq.units$ **Q26.** $\frac{1}{52}$

Q27. $\frac{17}{2}units$ **OR** $1\ unit$ **Q28.** Fertilizer F_1: $100kg$; fertilizer F_2: $80kg$; Minimum cost: Rs.1000

Q29. $\sqrt{1-x}\left(\sqrt{x}-2\right)-\sin^{-1}\sqrt{x}+k$