MODEL TEST PAPER - I

Time: 3 hours **Maximum Marks: 100**

General Instructions:

(i) All questions are compulsory.

(ii) This questions paper contains 29 questions.

(iii) Questions nos. 1-4 in section A are very short answer type questions carrying 1 mark each

(iv) Question nos. 5 -12 in section B are short answer type questions carrying 2 marks each

(v) Questions non. 13 -23 in section C are long answer-I type questions carrying 4 marks each

(vi) Question 24 -29 in section D are long answer –II type questions carrying 6 marks each.

SECTION A

1. Differentiate f(x) = $\frac{x^3 + x^2 + 1}{x}$ with respect to x.

2. Find the component statements for the compound statement : Number seven is prime and odd

3. Solve for x : $x^2 + 3x + 9 = 0$.

4. If A = {1, 2, 3, 4, 5, 6}, B = {2, 4, 6, 8}, then find A - B

SECTION B

5. (a) Write the contra positive of the statement : “If a triangle is equilateral then it is isosceles.”

(b) Write the negation of the statement "All triangles are not equilateral triangles.."

6. Let A and B be two sets containing 3 and 6 elements respectively. Find the maximum and number of elements in $A \cup B$.

7. Find the coordinate of the point R which divide the joint of the points P(0, 0, 0) and Q(4, –1, –2) in the ratio 1 : 2 externally and verify that P is the mid point of RQ.

8. Find the derivative of $f(x) = \frac{\cos x}{1+\sin x}$ w.r.t. '*x*'

9. If $z_1 = z - i$, $z_2 = -2 + i$ then find the value of $\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$

10. Find the range of the real function $f(x) = 1 - |x-2|$.

11. Using binomial theorem prove that $6^n - 5n - 1$ is divisible by 25, $\forall\ n \in N$.

12. If the letters of the word "ALGORITHM" are arranged at random in a row, what is the probability that the letters G, O and R must remain together?

SECTION C

13. Find the general solution of the equation (sin 2x – sin 4x + sin 6x = 0)

14. Find the equation of the circle which passes through the points (2, – 2), (3, 4) and has its centre on the line 2x + 2y = 7

OR

Find the equation of the hyperbola whose foci are $\left(\pm 3\sqrt{5}, 0\right)$ and the length of length of lat us rectum is 8 units

15. Find the sixth term of the expansion $\left(y^{\frac{1}{2}} + x^{\frac{1}{3}}\right)^n$, if the binomial coefficient of the third from the end is 45

16. Three squares of a chess board are selected at random Find the probability of selecting two squares of one colour and the other of a different colour. What is the importance of games in life?

17. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

18. In a plane there are 27 straight lines, of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no line passes through both points A and B and no two are parallel. Find the number of points of intersection of the straight lines.

19. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? Justify. If this is described by the relation g(x) = ax + b then what value should be assigned to a and b?

20. If A = {2, 3, 4, 5, 6, 7, 8, 9}. Let R be a relation on A defined by {(x, y) : x $\in$ A, y $\in$ A and x divides y}.

    (a) Draw arrow diagram of R

    (b) Find : (i) R in roster form (ii) Domain of R (iii) Range of R

21. Find the square root of $2-2\sqrt{3}i$.

OR

If $a+ib=\frac{c+i}{c-i}$.; a, b, c $\in$ R then show that $a^2 + b^2 = 1$ and $\frac{b}{a}=\frac{2c}{c^2-1}$

22. Solve the following system of linear inequalities graphically :

    $X - 2y \leq 3$; $3x + 4y \geq 12$; $x \geq 0$; $y \geq 1$

23. Evaluate : $\lim_{x \to 0} \frac{\sin x - \tan x}{x^3}$

Find the derivative of x sin x with respect to x from first principle of derivative.

SECTION D

24. Find the mean, variance and standard deviation for the following data :

Class-Interval	Frequency
30-40	3
40-50	7
50-60	12
60-70	15
70-80	8
80-90	3
90-100	2

25. Find the direction in which a straight line must be drawn through the point (–1, 2) so that its point of intersection with the line

x + y = 4 may be at a distance of 3 units from this point

OR

The hypotenuse of an isosceles right angled triangle has its ends at the points (1, 3) and (–4, 1) find the equation of the legs (perpendicular sides) of the triangle.

26. Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an AP and the ratio of 7^{th} and (m –1) the numbers is 5 : 9 Find the value of m.

OR

Let S be the sum, P the product and R the sum of reciprocals of n terms of a GP Prove that $P^2 R^n = S^n$.

27. In a town of 10000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C. 5% of families buy newspaper A and B, 3% of families buy newspaper B and C and 4% of families buy newspaper A and C. If 12% of families buy all the three newspaper that find.

(a) the number of families which buy newspaper A only.

(b) the number of families which buy none of the newspapers A, B and C.

28. Prove that $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

OR

If $x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right)$ Prove that $xy + yz + zx = 0$

29. Using principle of mathematical induction for all n $\in$ N, prove that

$1.3 + 2.3^2 + 3.3^3 + \ldots\ldots + n.3^n = \frac{(2n \quad 1)\ 3^{n\ 1} \quad 3}{4}$

ANSWER OF MODEL TEST PAPER - I

1. $f'(x) = 2x + 1 - \frac{1}{x^2}$

2. p : Numbers seven is prime

   q : Numbers seven is odd.

3. $x \quad \frac{3 \quad 3\sqrt{3}i}{2}$

4. $A - B = \{1, 3.5\}$

5. (a) If a triangle is not isosceles them it is not equilateral.

   (b) All triangles are equilateral triangles.

6. 6

7. R(–4, 1, 2)

8. $f'(x) = \frac{-1}{1 + \sin x}$

9 $\mathrm{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right) = \frac{-2}{5}$

10. (– ∞, 1) 12. $\frac{1}{12}$ 13. $x = n\pi \pm \frac{\pi}{6}$

14. $x \quad \frac{5}{2}^{\ 2} \quad y \quad 1^{\ 2} \quad \frac{37}{4}$ or $\frac{x^2}{25} \quad \frac{y^2}{20} \quad 1$

15. $T_6 \quad 252 y^{\frac{5}{2}} x^{5}{}^{3}$

16. $\frac{16}{21}$ Games keep is fit and healthy.

17. 33810 18. 220 19. a = 2, b = –1

20. (b) (i) R = $(2,2),(2,4),(2,6),(2,8)(3,3),(3,6),(3,9),$
$(4,4),(4,8),(5,5),(6,6),(7,7),(8,8),(9,9)$

(ii) D(R) = A (iii) Range (R) = A

21. $\pm\left(\sqrt{3}-i\right)$ 23. $x\cos x+\sin x$

24. Mean = 62; variance = 201 s.d. = $\sqrt{201}=14.17$

25. m = 0; required line is parallel to x –axis.

OR

7y + 3x – 24 = 0; 3y – 7x – 2 = 0 and 7x – 3y + 31 = 0; 3x + 7y + 5 = 0

26. m = 14

27. (a) 3300 families (b) 4000 families.