Basic Mathematics (241)
Marking Scheme
2023-24

Section A

1) (b) xy^2	1
2) (c) 20	1
3) (b) ½	1
4) (d) No Solution	1
5) (d) 0,8	1
6) (c) 5 Unit	1
7) (a) $\Delta PQR \sim \Delta CAB$	1
8) (d) RHS	1
9) (b) 70˚	1
10) (b) ¾	1
11) (b) 45°	1
12) (a) $\sin^2 A$	1
13) (c) $\pi : 2$	1
14) (a) 7 cm	1
15) (d) $\frac{1}{6}$	1
16) (a) 15	1
17) (a) 3.5 cm	1
18) (b) 12-18	1
19) (a) Both assertion and reason are true and reason is the correct explanation of assertion.	1
20) (d) Assertion (A) is false but reason(R) is true.	1

SECTION B

21) $3x+2y = 8$

$6x - 4y = 9$

$a_1=3, \quad b_1=2, \quad c_1 = 8$

$a_2=6, \quad b_2=-4, \quad c_2 = 9$ 1

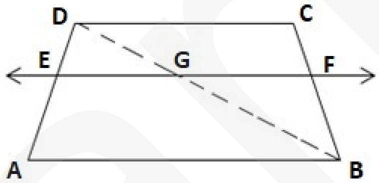
$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2} \qquad \frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2} \qquad \frac{c_1}{c_2} = \frac{8}{9}$ 1/2

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given pair of linear equations are consistent. 1/2

22) Given:-AB II CD II EF

To prove:- $\frac{AE}{ED} = \frac{BF}{FC}$



Construction:- Join BD to intersect EF at G. 1/2

Proof:- in Δ ABD

EG II AB (EF II AB)

$\frac{AE}{ED} = \frac{BG}{GD}$ (by BPT)____________(1) 1/2

In ΔDBC

GF II CD (EF II CD)

$\frac{BF}{FC} = \frac{BG}{GD}$ (by BPT)____________(2) 1/2

$from$ (1) & (2)

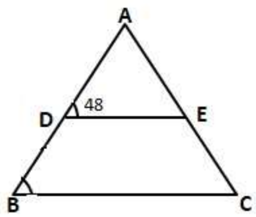
$\frac{AE}{ED} = \frac{BF}{FC}$ 1/2

OR

Given AD=6cm, DB=9cm

AE=8cm, EC=12cm, ∠ADE=48

To find:- ∠ABC=?



Proof:

In ΔABC

$\frac{AD}{DB} = \frac{6}{9} = \frac{2}{3}$ ……(1)

$\frac{AE}{EC} = \frac{8}{12} = \frac{2}{3}$ ……..(2)

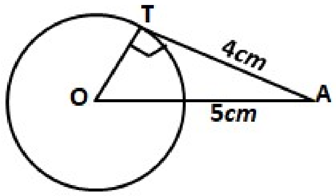
From (1) & (2) 1

$\frac{AD}{DB} = \frac{AE}{EC}$

DE II BC (Converse of BPT)

∠ADE=∠ABC (Corresponding angles)

⇒ ∠ABC=48˚ 1

23) In Δ OTA, ∠OTA = 90° By Pythagoras theorem $OA^2 = OT^2 + AT^2$ $(5)^2 = OT^2 + (4)^2$ $25-16 = OT^2$ $9 = OT^2$ OT=3cm radius of circle = 3cm.		1/2 1/2 1
24) $\sin^2 60^\circ + 2\tan 45^\circ - \cos^2 30^\circ$ $= \left(\frac{\sqrt{3}}{2}\right)^2 + 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$ $= \frac{3}{4} + 2 - \frac{3}{4}$ $= 2$		1 1
25) Area of the circle= sum of areas of 2 circles $\pi R^2 = \pi(40)^2 + \pi(9)^2$ $\pi R^2 = \pi x (40^2 + 9^2)$ $R^2 = 1600 + 81$ $R^2 = 1681$ $R = 41\ cm.$ $Diameter\ of\ given\ circle = 41 \times 2 = 82cm$ **OR** radius of circle = 10cm, $\theta = 90^\circ$ Area of minor segment = $\frac{\theta}{360^\circ}\pi r^2$ - Area of Δ $= \frac{\theta}{360^\circ} x\, \pi\, r^2 - \frac{1}{2} x\, b\, x\, h$ $= \frac{90^\circ}{360^\circ} x 3.14 \times 10 \times 10 - \frac{1}{2} \times 10 \times 10$ $= \frac{314}{4} - 50$ $= 78.5 - 50 = 28.5\ cm^2$ Area of minor segment = $28.5\ cm^2$		1/2 1/2 1/2 1/2 1/2 1/2 1/2 1/2

Section C

26) Let us assume that $\sqrt{3}$ be a rational number

$\sqrt{3} = \frac{a}{b}$ where a and b are co-prime. 1

squaring both the sides

$\left(\sqrt{3}\right)^2 = \left(\frac{a}{b}\right)^2$ 1/2

$3=\frac{a^2}{b^2} \Rightarrow a^2=3b^2$

a^2 is divisible by 3 so a is also divisible by 3_________(1)

let a=3c for any integer *c*.

$(3c)^2=3b^2$ 1/2

$9c^2=3b^2$

$b^2=3c^2$

$since\ b^2$ is divisible by 3 so, b is also divisible by 3 _____(2)

From (1) & (2) we can say that 3 in a factor of a and b 1/2

which is contradicting the fact that a and b are co- prime.

Thus, our assumption that $\sqrt{3}$ is a rational number is wrong.

Hence, $\sqrt{3}$ is an irrational number. 1/2

27) $P(S)= 4S^2 -4S+1$

$4S^2 -2S-2S+1=0$

$2S(2S-1)-1(2S-1)=0$

$(2S-1)(2S-1)=0$

$S = ½ \quad S = ½$ 1

$a = 4 \quad b = -4 \quad c = 1 \quad \alpha = ½ \ \beta = ½$

$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$

$\frac{1}{2}+\frac{1}{2}=\frac{-4}{4}, \quad \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$ 1

$\frac{1+1}{2}=\frac{+4}{4}, \quad \frac{1}{4}=\frac{1}{4}$

$\frac{2}{2}=1$

$1 = 1$ 1

28) Let cost of one bat be Rs x

Let cost of one ball be Rs y 1/2

ATQ

$4x + 1y = 2050$___________(1)

$3x + 2y = 1600$___________(2) 1/2

$from\ (1) 4x + 1y = 2050$

$y = 2050 - 4x$ 1/2

$Substite\ \ value\ of\ y\ in\ (2)$

$3x + 2(2050 - 4x) = 1600$

3x + 4100 – 8x =1600

-5x $= -2500$

$x = 500$ 1/2

$Substiture\ value\ of\ x\ in\ (1)$

$4x + 1y = 2050$

$4(500) + y = 2050$

$2000 + y = 2050$

$y = 50$ 1/2

Hence

Cost of one bat = Rs. 500 1/2

Cost of one ball = Rs. 50

OR

Let the fixed charge for first 3 days= Rs. x

And additional charge after 3 days= Rs. y 1/2

ATQ

$x + 4y = 27$---------------(1)

$x + 2y = 21$ --------------(2) 1/2

Subtract eq^n (2) from (1)

$2y = 6$

$y = 3$ 1

Substitute value of y in (2)

$x + 2(3) = 21$

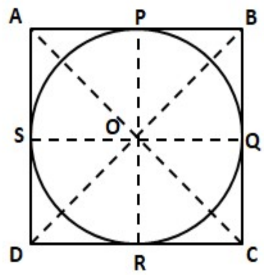
$x = 21 - 6$

$x = 15$ 1

Fixed charge= Rs. 15

Additional charge per day = Rs. 3

29) Given circle touching sides of ABCD at P,Q,R and S



To prove- AB+CD=AD+BC

Proof- 1

AP=AS-------(1) tangents from an external point

PB=BQ-------(2) to a circle are equal in length

DR=DS-------(3)

CR=CQ-------(4) 1

Adding eq^n (1),(2),(3) & (4)

AP+BP+DR+CR=AS+DS+BQ+CQ

AB+DC=AD+BC 1

30) $(cosec\theta - \cot\theta)^2 = \frac{1-cos\theta}{1+cos\theta}$

LHS=$(cosec\theta - cot\theta)^2$

$= \left(\frac{1}{sin\theta} - \frac{\cos\theta}{sin\theta}\right)^2$ 1/2

$= \left(\frac{1-\cos\theta}{sin\theta}\right)^2$ 1/2

$= \frac{(1-\cos\theta)^2}{sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{1-cos^2\theta}$ 1

$= \frac{(1-\cos\theta)^2}{(1-cos\theta)(1+cos\theta)}$

$= \frac{1-cos\theta}{1+cos\theta}$ =RHS 1

LHS = RHS, Hence Proved

OR

secA $(1 - sinA)(\sec A + tanA)$=*1*

LHS=$\frac{1}{cosA}(1 - sinA)\left(\frac{1}{cosA} + \frac{sinA}{cosA}\right)$ 1

= $\frac{(1-\sin A)}{\cos A}\frac{(1+\sin A)}{\cos A}$

= $\frac{(1-\sin A)\ (1+\sin A)}{cos^2 A}$

= $\frac{1-sin^2 A}{cos^2 A}$ (1-$sin^2 A = cos^2 A$) 1

= $\frac{cos^2 A}{cos^2 A}$

= 1 = RHS 1

LHS=RHS. Hence Proved

31) (i) Red balls= 6 , Black balls = 4 , White balls = x

P(white ball) = $\frac{x}{10+x} = \frac{1}{3}$ 1

⇒ 3x = 10 + x ⇒ x= 5 white balls 1/2

(ii) Let y red balls be removed, black balls = 4, white balls = 5

P(white balls)= $\frac{5}{(6-y)+4+5} = \frac{1}{2}$ 1

$\Rightarrow \frac{5}{15-y} = \frac{1}{2} \Rightarrow 10 = 15 - y \Rightarrow y = 5$ ½

So 5 balls should be removed.

Section D

32) Let the speed of train be $x\ km/hr$ 1/2

distance= 360 km

Speed = $\frac{distance}{time}$

Time = $\frac{360}{x}$ 1/2

New speed = $(x + 5)km/hr$

Time = $\frac{D}{5}$

$x + 5 = \frac{360}{\left(\frac{360}{x}-1\right)}$ 1

$(x + 5)\left(\frac{360}{x} - 1\right) = 360$

$(x+5)(360-x)=360x$

$-x^2-5x+1800=0$

$x^2+5x-1800=0$ 1

$x^2+45x-40x-1800=0$

$x(x+45)-40(x+45)=0$

$(x+45)(x-40)=0$ 1

$x+45=0$, $x-40=0$

$x=-45$, $x=40$

Speed cannot be negative

Speed of train =40km/hr 1

OR

Let the speed of the stream=xkm/hr 1/2

Speed of boat= 18 km/hr

Upstream speed= $(18-x)km/hr$

Downstream speed=$(18+x)km/hr$ 1/2

Time taken (upstream)=$\frac{24}{(18-x)}$

Time taken (downstream)=$\frac{24}{(18+x)}$

ATQ

$$\frac{24}{(18-x)}=\frac{24}{(18+x)}+1$$ 1

$$\frac{24}{(18-x)}-\frac{24}{(18+x)}=1$$

$24(18+x)-24(18-x)=(18-x)(18+x)$

$24(18+x-18+x)=(18)^2-x^2$

$24(2x)=324-x^2$

$48x-324+x^2=0$

$x^2+48x-324=0$ 1

$x^2-6x+54x-324=0$

$x(x-6)+54(x-6)=0$

$(x-6)(x+54)=0$ 1

$x-6=0$, $x+54=0$

$x=6$, $x=-54$

Speed cannot be negative 1

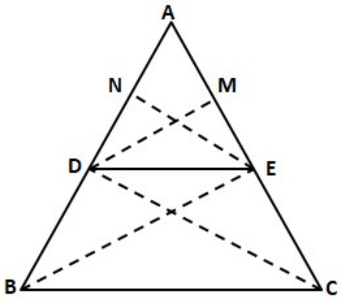
Speed of stream=$6km/hr$

33) Given ΔABC , DE || BC

To prove $\frac{AD}{DB}=\frac{AE}{EC}$

Construction: join BE and CD 1/2

Draw DM ⊥ AC and EN ⊥ AB



Proof: $Area\ of\ \Delta ADE=\frac{1}{2}$x b x h

= $\frac{1}{2}$x AD x EN-----------------------(1)

Area $(\Delta DBE)=\frac{1}{2}$x DB x EN--------(2)

Divide eq^n (1) by (2)

$\frac{\text{ar }\Delta ADE}{\text{ar }\Delta DBE}=\frac{\frac{1}{2}\text{X } AD \text{ X } EN}{\frac{1}{2}\text{X } DB \text{ X } EN}=\frac{AD}{DB}$-----------(3) 1 area ΔADE = $\frac{1}{2}$ x AE x DM -------(4)

area ΔDEC = $\frac{1}{2}$ x EC x DM -------(5)

Divide eq^n (4) by (5)

$\frac{\text{ar } \Delta ADE}{\text{ar } \Delta DEC} = \frac{\frac{1}{2} X\, AE\, X\, DM}{\frac{1}{2} X\, EC\, X\, DM} = \frac{AE}{EC}$-----------(6) 1

ΔBDE and ΔDEC are on the same base DE and between same parallel lines BC and DE

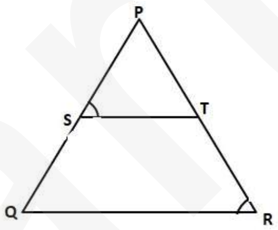
$\therefore area\ (\Delta DBE) = ar\ (\ DEC)$

hence

$\frac{ar(\ \Delta ADE)}{ar\ (\Delta DBE} = \frac{ar(\ \Delta ADE)}{ar\ (\Delta DEC)}$ [LHS of (3) =RHS of (6)]

$\frac{AD}{DB} = \frac{AE}{EC}$ [RHS of (3) = RHS of (6) 1/2

Since $\frac{PS}{SQ} = \frac{PT}{TR}$ $\therefore ST \parallel QR$ $(by\ converse\ of\ BPT)$



∠PST = ∠PQR (Corresponding angles) 1

But ∠PST = ∠PRQ (given)

∠PQR = ∠PRQ

PR = PQ (sides opposite to equal angles are equal

Hence ΔPQR is isosceles. 1

34) Diameter of cylinder and hemisphere = 5mm radius, (r) = $\frac{5}{2}$

Total length = 14mm

Height of cylinder = 14 - 5 = 9mm 1

CSA of cylinder = 2πrh

= 2 x $\frac{22}{7}$ x $\frac{5}{2}$ x 9

= $\frac{990}{7}$ mm^2 1

CSA of hemispheres = 2πr^2

= 2x $\frac{22}{7}$ x $\left(\frac{5}{2}\right)^2$

= $\frac{275}{7}$ mm^2 1

CSA of 2 hemispheres = 2 x $\frac{275}{7}$

= $\frac{550}{7}$ mm^2 1

Total area of capsule = $\frac{990}{7} + \frac{550}{7}$

= $\frac{1540}{7}$

= 220 mm^2 1

OR

Diameter of cylinder = 2.8 cm

$radius$ of cylinder = $\frac{2.8}{2}$ = 1.4 cm

$radius$ of cylinder = $radius$ of hemisphere = 1.4 cm

Height of cylinder = 5-2.8 1

= 2.2 cm

Volume of 1 Gulab jamun = vol. of cylinder + 2 x vol. of hemisphere

= $\pi r^2 h + 2 \times \frac{2}{3} \pi r^3$ 1

$\frac{22}{7} \times (1.4)^2 \times 2.2 + 2 \times \frac{2}{3} \times \frac{22}{7} \times (1.4)^3$

$= 13.55 + 11.50$

$= 25.05\ cm^3$ 1

$volume\ of\ 45\ Gulab\ jamun = 45 \times 25.05$

$syrup\ in\ 45\ Gulab\ jamun = 30\% \times 45 \times 25.05$

$= \frac{30}{100} \times 45 \times 25.05$ 1

= 338.175 cm^3

≈ 338 cm^3

35)

Life time (in hours)	Number of lamps(f)	Mid x	d	fd
1500-2000	14	1750	-1500	-21000
2000-2500	56	2250	-1000	-56000
2500-3000	60	2750	-500	-30000
3000-3500	86	3250	0	0
3500-4000	74	3750	500	37000
4000-4500	62	4250	1000	62000
4500-5000	48	4750	1500	72000
	400			64000

2

Mean = $a + \frac{\Sigma fd}{\Sigma f}$ 1/2

a = 3250 1/2

Mean $= 3250 + \frac{64000}{400}$ 1

$= 3250 + 160$

$= 3410$

Average life of lamp is 3410 hr 1

Section E

36) a_6 =16000 a_9 = 22600

a+5d=16000-------(1)

a+8d=22600 --------(2)

substitute a = 1600 -5d from (1)

16000-5d + 8d = 22600

3d = 22600-16000

3d=6600

d = $\frac{6600}{3}$ = 2200

a = 16000-5(2200)

a = 16000-11000

a = 5000

(i) a_n = 29200, a = 5000, d = 2200

$a_n = a + (n-1)d$

29200 = 5000 + (n – 1)2200 1/2

29200-5000 = 2200n-2200

24200+2200=2200n

26400=2200n

n= $\frac{264}{22}$

n=12 1/2

in 12th year the production was Rs 29200

(ii) n=8, a=5000, d=2200

$a_n = a + (n-1)d$ 1/2

= 5000+(8-1)2200 1/2

= 5000+7 x 2200

= 5000+15400 1/2

= 20400

The production during 8th year is = 20400 1/2

OR

n = 3, a = 5000, d = 2200

$s_n = \frac{n}{2}$ [2a + (n-1)d] 1/2

$= \frac{3}{2}$ [2(5000) + (3-1) 2200]

$S_3 = \frac{3}{2}$ (10000 + 2 x 2200) 1/2

$=\frac{3}{2}$ (10000 + 4400) 1/2

= 3 x 7200

= 21600 1/2

The production during first 3 year is 21600

(iii) a_4 = a+3d

= 5000 + 3 (2200)

= 5000 + 6600

= 11600 1/2

a_7 = a+6d

= 5000 + 6 x 2200

=5000 + 13200

= 18200

$a_7 - a_4$ = 18200-11600 = 6600 1/2

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37) coordinates of A (2, 3) Alia's house

coordinates of B (2, 1) Shagun's house

coordinates of C (4,1) Library

(i) AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2-2)^2 + (1-3)^2}$ 1/2

$= \sqrt{(0^2 + (-2)^2}$

$AB = \sqrt{0+4} = \sqrt{4}$ = 2 units 1/2

$Alia's\ house\ from\ shagun's\ house\ is\ 2\ units$

(ii) C(4,1), B (2,1)

CB = $\sqrt{(2-4)^2 + (1-1)^2}$ 1/2

$= \sqrt{(-2)^2 + 0^2}$

$= \sqrt{4+0} = \sqrt{4}$ = 2 unit 1/2

(iii) 0(0,0), B(2,1)

OB = $\sqrt{(2-0)^2 + (1-0)^2}$

$= \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$ units 1

Distance between Alia's house and Shagun's house, AB = 2 units

Distance between Library and Shagun's house, CB = 2 units 1/2

OB is greater than AB and CB, 1/2

For shagun, school [O] is farther than Alia's house [A] and Library [C]

OR

C (4, 1), A(2, 3)

CA = $\sqrt{(2-4)^2 + (3-1)^2}$

= $\sqrt{(-2)^2 + 2^2}$+ = $\sqrt{4+4}$ = $\sqrt{8}$

= $2\sqrt{2}$ units $AC^2 = 8$ 1

Distance between Alia's house and Shagun's house, AB = 2 units

Distance between Library and Shagun's house, CB = 2 units 1/2

$AB^2 + BC^2 = 2^2 + 2^2 = 4 + 4 = 8 = AC^2$ ½

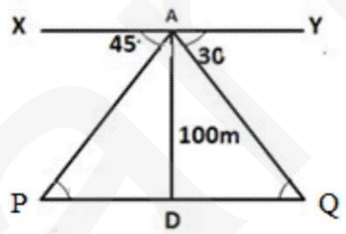
Therefore A, B and C form an isosceles right triangle.

38)

(i) XY ‖ PQ and AP is transversal.

∠APD = ∠PAX (alternative interior angles) 1/2

∠APD=45° 1/2



(ii) Since XY || PQ and AQ is a transversal

so alternate interior angles are equal

hence ∠YAQ = ∠AQD=30° 1/2

(iii) In ΔADP, $\theta = 45^\circ$

$\tan\theta = \frac{P}{B}$

$\tan 45^\circ = \frac{100}{PD}$ 1/2

PD=100 m

1/2

Boat P is 100 m from the light house 1

OR

In ΔADQ, $\theta = 30^\circ$

$\tan\theta = \frac{P}{B}$ 1/2

$\tan 30 = \frac{100}{DQ}$

$\frac{1}{\sqrt{3}} = \frac{100}{DQ}$ 1/2

DQ = $100\sqrt{3}$ m 1