Marking Scheme

CLASS: XII

Session: 2021-22

Mathematics (Code-041)

Term - 2

SECTION – A

1.	Find: $\int \frac{logx}{(1+logx)^2}dx$ Solution: $\int \frac{logx}{(1+logx)^2}dx = \int \frac{logx+1-1}{(1+logx)^2}dx = \int \frac{1}{1+logx}dx - \int \frac{1}{(1+logx)^2}dx$ $=\frac{1}{1+logx} \times x - \int \frac{-1}{(1+logx)^2} \times \frac{1}{x} \times xdx - \int \frac{1}{(1+logx)^2}dx = \frac{x}{1+logx} + c$ OR Find: $\int \frac{sin2x}{\sqrt{9-cos^4x}}dx$ Solution: Put $cos^2x = t \Rightarrow -2cosxsinxdx = dt \Rightarrow sin2xdx = -dt$ The given integral $= -\int \frac{dt}{\sqrt{3^2-t^2}} = -\sin^{-1}\frac{t}{3} + c = -\sin^{-1}\frac{cos^2x}{3} + c$	1/2 1+1/2 1 1
2.	Write the sum of the order and the degree of the following differential equation: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = 5$ Solution: Order = 2 Degree = 1 Sum = 3	 1 1/2 ½
3.	If $\hat{a}$ and $\hat{b}$ are unit vectors, then prove that $\lvert\hat{a}+\hat{b}\rvert = 2cos\frac{\theta}{2}$, where θ is the angle between them. Solution: $(\hat{a}+\hat{b}).(\hat{a}+\hat{b}) = \lvert\hat{a}\rvert^2 + \lvert\hat{b}\rvert^2 + 2(\hat{a}.\hat{b})$ $\lvert\hat{a}+\hat{b}\rvert^2 = 1 + 1 + 2cos\theta$ $= 2(1+cos\theta) = 4cos^2\frac{\theta}{2}$ $\therefore \lvert\hat{a}+\hat{b}\rvert = 2cos\frac{\theta}{2}$,	1 1/2 ½
4.	Find the direction cosines of the following line: $\frac{3-x}{-1} = \frac{2y-1}{2} = \frac{z}{4}$ Solution: The given line is $\frac{x-3}{1} = \frac{y-\frac{1}{2}}{1} = \frac{z}{4}$ Its direction ratios are <1, 1, 4> Its direction cosines are $\langle\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}\rangle$	1 1/2 ½

<table>
<tr><td>5.</td><td>A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement.
Solution: Let X be the random variable defined as the number of red balls.
Then X = 0, 1
P(X=0) = $\frac{3}{4}\times\frac{2}{3}=\frac{6}{12}=\frac{1}{2}$
P(X=1) =$\frac{1}{4}\times\frac{3}{3}+\frac{3}{4}\times\frac{1}{3}=\frac{6}{12}=\frac{1}{2}$
Probability Distribution Table:
<table><tr><td>X</td><td>0</td><td>1</td></tr><tr><td>P(X)</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td></tr></table></td><td>1/2
1/2
1/2
½</td></tr>
<tr><td>6.</td><td>Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack?
Solution: The required probability = P((The first is a red jack card and The second is a jack card) or (The first is a red non-jack card and The second is a jack card))
$=\frac{2}{52}\times\frac{3}{51}+\frac{24}{52}\times\frac{4}{51}=\frac{1}{26}$</td><td>1
1</td></tr>
<tr><td colspan="3"><u>SECTION – B</u></td></tr>
<tr><td>7.</td><td>Find: $\int\frac{x+1}{(x^2+1)x}dx$
Solution: Let $\frac{x+1}{(x^2+1)x}=\frac{Ax+B}{x^2+1}+\frac{C}{x}=\frac{(Ax+B)x+C(x^2+1)}{(x^2+1)x}$
$\Rightarrow x+1=(Ax+B)x+C(x^2+1)$ (An identity)
Equating the coefficients, we get
B = 1, C = 1, A + C = 0
Hence, A = -1, B = 1, C = 1
The given integral = $\int\frac{-x+1}{x^2+1}dx+\int\frac{1}{x}dx$
$=\frac{-1}{2}\int\frac{2x-2}{x^2+1}dx+\int\frac{1}{x}dx$
$=\frac{-1}{2}\int\frac{2x}{x^2+1}dx+\int\frac{1}{x^2+1}dx+\int\frac{1}{x}dx$
$=\frac{-1}{2}\log(x^2+1)+\tan^{-1}x+log\lvert x\rvert+c$</td><td>1/2
½
1/2
1+1/2</td></tr>
<tr><td>8.</td><td>Find the general solution of the following differential equation:
$x\frac{dy}{dx}=y-xsin(\frac{y}{x})$
Solution: We have the differential equation:
$\frac{dy}{dx}=\frac{y}{x}-sin(\frac{y}{x})$
The equation is a homogeneous differential equation.
Putting $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$
The differential equation becomes
$v+x\frac{dv}{dx}=v-sinv$
$\Rightarrow\frac{dv}{sinv}=-\frac{dx}{x}\Rightarrow cosecvdv=-\frac{dx}{x}$
Integrating both sides, we get</td><td>1
½</td></tr>
</table>

	$log\|cosecv - cotv\| = -log\|x\| + logK, K > 0$ (Here, $logK$ is an arbitrary constant.) $\Rightarrow log\|(cosecv - cotv)x\| = logK$ $\Rightarrow \|(cosecv - cotv)x\| = K$ $\Rightarrow (cosecv - cotv)x = \pm K$ $\Rightarrow \left(cosec\frac{y}{x} - cot\frac{y}{x}\right)x = C$, which is the required general solution. OR Find the particular solution of the following differential equation, given that y = 0 when x = $\frac{\pi}{4}$: $\frac{dy}{dx} + ycotx = \frac{2}{1+sinx}$ Solution: The differential equation is a linear differential equation I F = $e^{\int cotxdx} = e^{logsinx} = sinx$ The general solution is given by $ysinx = \int 2\frac{sinx}{1+sinx}dx$ $\Rightarrow ysinx = 2\int \frac{sinx+1-1}{1+sinx}dx = 2\int [1 - \frac{1}{1+sinx}]dx$ $\Rightarrow ysinx = 2\int [1 - \frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)}]dx$ $\Rightarrow ysinx = 2\int [1 - \frac{1}{2cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}]dx$ $\Rightarrow ysinx = 2\int [1 - \frac{1}{2}sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)]dx$ $\Rightarrow ysinx = 2[x + \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)] + c$ Given that y = 0, when x = $\frac{\pi}{4}$, Hence, 0 = 2[$\frac{\pi}{4} + tan\frac{\pi}{8}$] + c $\Rightarrow c = -\frac{\pi}{2} - 2tan\frac{\pi}{8}$ Hence, the particular solution is $y = cosecx[2\left\{x + \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\} - \left(\frac{\pi}{2} + 2tan\frac{\pi}{8}\right)]$	1 ½ 1 ½ 1 ½
9.	If $\vec{a} \neq \vec{0}, \vec{a}.\vec{b} = \vec{a}.\vec{c}, \vec{a}\times\vec{b} = \vec{a}\times\vec{c}$, then show that $\vec{b} = \vec{c}$. Solution: We have $\vec{a}.\left(\vec{b} - \vec{c}\right) = 0$ $\Rightarrow \left(\vec{b} - \vec{c}\right) = \vec{0}$ or $\vec{a} \perp \left(\vec{b} - \vec{c}\right)$ $\Rightarrow \vec{b} = \vec{c}$ or $\vec{a} \perp \left(\vec{b} - \vec{c}\right)$ Also, $\vec{a} \times \left(\vec{b} - \vec{c}\right) = \vec{0}$ $\Rightarrow \left(\vec{b} - \vec{c}\right) = \vec{0}$ or $\vec{a} \parallel \left(\vec{b} - \vec{c}\right)$ $\Rightarrow \vec{b} = \vec{c}$ or $\vec{a} \parallel \left(\vec{b} - \vec{c}\right)$ $\vec{a}$ *can not be both perpendicular to* $\left(\vec{b} - \vec{c}\right)$ *and parallel to* $\left(\vec{b} - \vec{c}\right)$ Hence, $\vec{b} = \vec{c}$.	1 1 1
10.	Find the shortest distance between the following lines: $\vec{r} = \left(\hat{\imath} + \hat{\jmath} - \hat{k}\right) + s\left(2\hat{\imath} + \hat{\jmath} + \hat{k}\right)$ $\vec{r} = \left(\hat{\imath} + \hat{\jmath} + 2\hat{k}\right) + t\left(4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right)$	

	Solution: Here, the lines are parallel. The shortest distance = $\frac{\lvert(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}\rvert}{\lvert\vec{b}\rvert}$ $= \frac{\lvert(3\hat{k})\times(2\hat{\imath}+\hat{\jmath}+\hat{k})\rvert}{\sqrt{4+1+1}}$ $(3\hat{k})\times(2\hat{\imath}+\hat{\jmath}+\hat{k}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = -3\hat{\imath}+6\hat{\jmath}$ Hence, the required shortest distance = $\frac{3\sqrt{5}}{\sqrt{6}}$ units OR Find the vector and the cartesian equations of the plane containing the point $\hat{\imath}+2\hat{\jmath}-\hat{k}$ and parallel to the lines $\vec{r}=(\hat{\imath}+2\hat{\jmath}+2\hat{k})+s(2\hat{\imath}-3\hat{\jmath}+2\hat{k})$ and $\vec{r}=(3\hat{\imath}+\hat{\jmath}-2\hat{k})+t(\hat{\imath}-3\hat{\jmath}+\hat{k})$ Solution: Since, the plane is parallel to the given lines, the cross product of the vectors $2\hat{\imath}-3\hat{\jmath}+2\hat{k}$ and $\hat{\imath}-3\hat{\jmath}+\hat{k}$ will be a normal to the plane $(2\hat{\imath}-3\hat{\jmath}+2\hat{k})\times(\hat{\imath}-3\hat{\jmath}+\hat{k}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & 2 \\ 1 & -3 & 1 \end{vmatrix} = 3\hat{\imath}-3\hat{k}$ The vector equation of the plane is $\vec{r}.(3\hat{\imath}-3\hat{k}) =(\hat{\imath}+2\hat{\jmath}-\hat{k}).(3\hat{\imath}-3\widehat{k})$ or, $\vec{r}.(\hat{\imath}-\hat{k})=2$ and the cartesian equation of the plane is x – z – 2 = 0	1+1/2 1 ½ 1 1 1
	SECTION – C	
11.	Evaluate: $\int_{-1}^{2}\lvert x^3-3x^2+2x\rvert dx$ Solution: The given definite integral = $\int_{-1}^{2}\lvert x(x-1)(x-2)\rvert dx$ $= \int_{-1}^{0}\lvert x(x-1)(x-2)\rvert dx + \int_{0}^{1}\lvert x(x-1)(x-2)\rvert dx + \int_{1}^{2}\lvert x(x-1)(x-2)\rvert dx$ $= -\int_{-1}^{0}(x^3-3x^2+2x)dx + \int_{0}^{1}(x^3-3x^2+2x)dx - \int_{1}^{2}(x^3-3x^2+2x)dx$ $= -[\frac{x^4}{4}-x^3+x^2]_{-1}^{0} + [\frac{x^4}{4}-x^3+x^2]_{0}^{1} - [\frac{x^4}{4}-x^3+x^2]_{1}^{2}$ $= \frac{9}{4}+\frac{1}{4}+\frac{1}{4}=\frac{11}{4}$	1+1/2 1/2 2

12. Using integration, find the area of the region in the first quadrant enclosed by the line x + y = 2, the parabola $y^2 = x$ and the x-axis.

Solution: Solving x + y = 2 and $y^2 = x$ simultaneously, we get the points of intersection as (1, 1) and (4, -2). 1

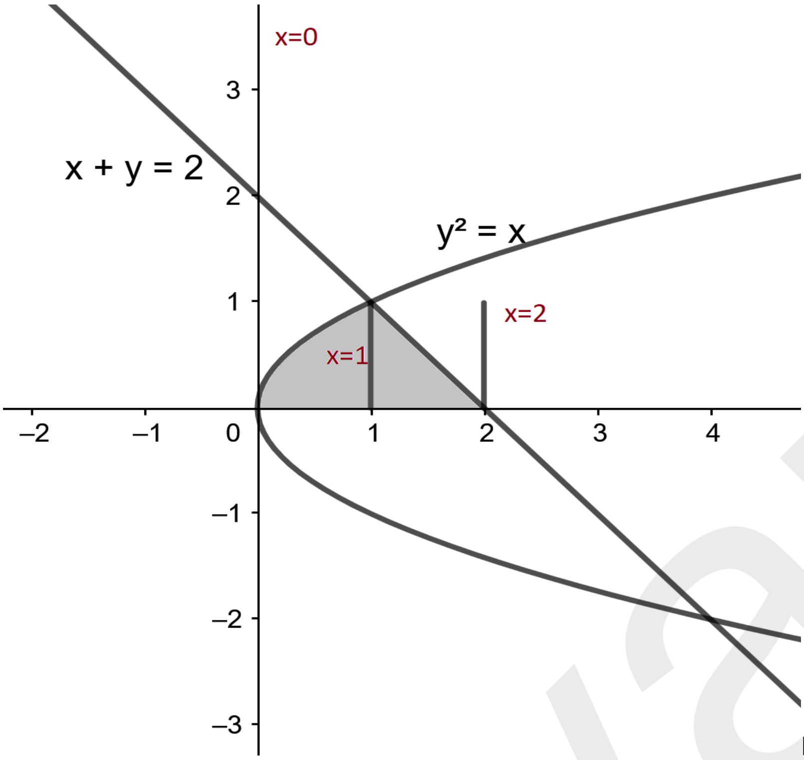

Fig 1 1

The required area = the shaded area = $\int_0^1 \sqrt{x}\,dx + \int_1^2 (2-x)dx$ 1

$= \frac{2}{3}[x^{\frac{3}{2}}]_0^1 + [2x - \frac{x^2}{2}]_1^2$

$= \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$ square units 1

OR

Using integration, find the area of the region: $\{(x, y): 0 \le y \le \sqrt{3}x, x^2 + y^2 \le 4\}$

Solution: Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$, we get the points of intersection as $(1, \sqrt{3})$ and $(-1, -\sqrt{3})$ 1

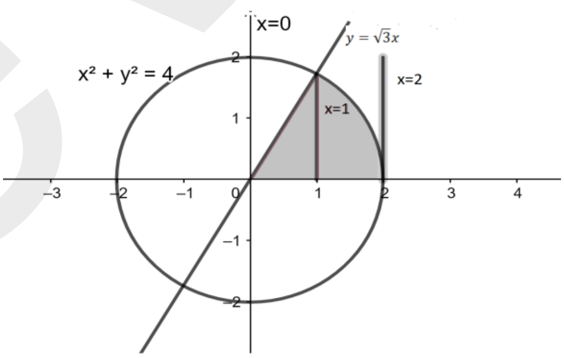

Fig 2 1

	The required area = the shaded area = $\int_0^1 \sqrt{3}x\,dx + \int_1^2 \sqrt{4-x^2}dx$ $= \frac{\sqrt{3}}{2}[x^2]_0^1 + \frac{1}{2}[x\sqrt{4-x^2} + 4\sin^{-1}\frac{x}{2}]_1^2$ $= \frac{\sqrt{3}}{2} + \frac{1}{2}[2\pi - \sqrt{3} - 2\frac{\pi}{3}]$ $= \frac{2\pi}{3}$ square units	1 1
13.	Find the foot of the perpendicular from the point (1, 2, 0) upon the plane x – 3y + 2z = 9. Hence, find the distance of the point (1, 2, 0) from the given plane. Solution: The equation of the line perpendicular to the plane and passing through the point (1, 2, 0) is $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z}{2}$ The coordinates of the foot of the perpendicular are $(\mu+1, -3\mu+2, 2\mu)$ for some μ These coordinates will satisfy the equation of the plane. Hence, we have $\mu + 1 - 3(-3\mu+2) + 2(2\mu) = 9$ $\Rightarrow \mu = 1$ The foot of the perpendicular is (2, -1, 2). Hence, the required distance = $\sqrt{(1-2)^2 + (2+1)^2 + (0-2)^2} = \sqrt{14}\ units$	1 ½ 1 ½ 1

14.	**CASE-BASED/DATA-BASED**	
	 Fig 3 An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone. **Based on the given information, answer the following questions.**	
	(i)what is the probability that a new policyholder will have an accident within a year of purchasing a policy?	
	(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?	
	Solution: Let E_1 = The policy holder is accident prone. E_2 = The policy holder is not accident prone. E = The new policy holder has an accident within a year of purchasing a policy. (i) $P(E) = P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)$ $= \frac{20}{100} \times \frac{6}{10} + \frac{80}{100} \times \frac{2}{10} = \frac{7}{25}$ (ii) By Bayes' Theorem, $P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E)}$ $= \frac{\frac{20}{100} \times \frac{6}{10}}{\frac{280}{1000}} = \frac{3}{7}$	1 1 1 1
