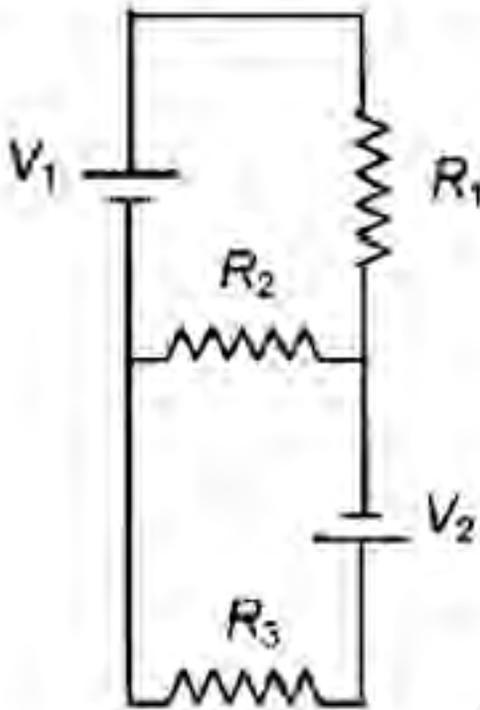


# JEE Advanced Paper-1 (Code-2)

## PART I : PHYSICS

SECTION -I : (One or More Than One Options Correct Type)

1. Two ideal batteries of *emf*  $V_1$  and  $V_2$  and three resistances  $R_1$ ,  $R_2$  and  $R_3$  are connected as shown in the figure. The current in resistance  $R_1$



- (A)  $V_1 = V_2$  and  $R_1 = R_2 = R_3$   
(B)  $V_1 = V_2$  and  $R_1 = 2R_2 = R_3$   
(C)  $V_1 = 2V_2$  and  $2R_1 = 2R_2 = R_3$   
(D)  $2V_1 = V_2$  and  $2R_1 = R_2 = R_3$

**Sol)**

Current through  $R_1$  will be zero if  $E_{\text{net}}$  of two batteries is zero

$$\text{i.e. } \frac{\frac{V_1}{R_1} - \frac{V_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = 0 \Rightarrow \frac{V_1}{R_1} = \frac{V_2}{R_2}$$

1 in option A,  $V_1 = V_2$  that gives  $R_1 = R_2$  whatever be the value of  $R_3$  to maintain image of loop  $V_1$  wrt  $V_2$ .

Hence, option A & B both are correct in options, C and D.

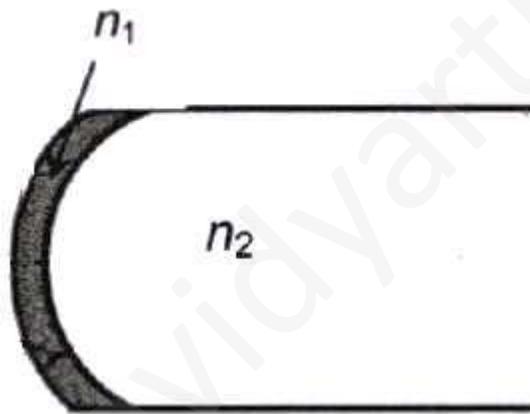
Using loop analysis method we find

$$\frac{V_1}{V_2} = 2, \text{ that satisfied, } \frac{R_3}{R_1} = 2$$

This relation is satisfied by option A&B

Hence, A,B and C are correct

2. A transparent thin film of uniform thickness and refractive index  $n_1 = 1.4$  is coated on the convex spherical surface of radius  $R$  at one end of a long solid glass cylinder of refractive index  $n_2 = 1.5$ , as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance  $f_1$  from the film, while rays of light traversing from glass to air get focused at distance  $f_2$  from the film. Then



- (A)  $|f_1| = 3R$
- (B)  $|f_1| = 2.8R$
- (C)  $|f_2| = 2R$
- (D)  $|f_2| = 1.4R$

Ans. 2. A,C

$$1.4/v - 1/\infty = 1.4 - 1/R$$

$$V = 1.4R/0.4 = 7/2R$$

$$\therefore 1.5/f_1 - 1.4/3.5R = 1.5 - 1.4/R$$

$$\Rightarrow 1.5/f_1 = 0.1/R + 0.4/R$$

$$\Rightarrow 1.5/f_1 = 0.5/R \Rightarrow |f_1| = 3R$$

$$1.4/v - 1.5/\infty = 1.4 - 1.5/R$$

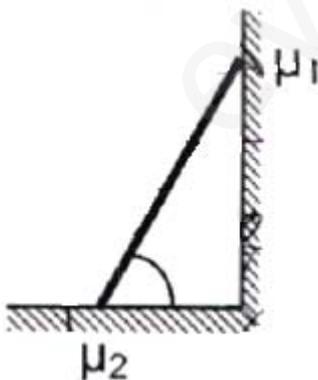
$$1.4/v = -0.1/R$$

$$\Rightarrow v = -14R$$

$$1/f_2 = -0.4/R - 0.1/R = -0.5/R$$

$$\Rightarrow f_2 = -2R \Rightarrow |f_2| = 2R$$

3. In the figure, a ladder of mass  $m$  is shown leaning against a wall. It is in static equilibrium making an angle  $\theta$  with the horizontal floor. The coefficient of friction between the wall and the ladder is  $\mu_1$  and that between the floor and the ladder is  $\mu_2$ . The normal reaction of the wall on the ladder is  $N_1$  and that of the floor is  $N_2$ . If the ladder is about to slip, then



(A)  $\mu_1 = 0$   $\mu_2 \neq 0$  and  $N_2 \tan \theta = mg/2$

(B)  $\mu_1 \neq 0$   $\mu_2 = 0$  and  $N_1 \tan \theta = mg/2$

(C)  $\mu_1 \neq 0$   $\mu_2 \neq 0$  and  $N_2 = mg/1 + \mu_1\mu_2$

(D)  $\mu_1 = 0$   $\mu_2 \neq 0$  and  $N_1 \tan \theta = mg/2$

Ans. 3. C,D Net force balance

$$N_1 = f_2 = \mu_2 N_2 \rightarrow \circ$$

$$N_2 + f_1 = mg$$

$$N_2 + \mu_1 N_1 = mg \rightarrow \circ$$

$$N_2 + \mu_1 \mu_2 N_2 = mg$$

$$\Rightarrow N_2 = mg/1 + \mu_1 \mu_2$$

Torque balance (about point of contact in ground)

$$mg \cos \theta \ell/2 = N_1 \sin \theta \ell + N_1 \mu_1 \ell \cos \theta$$

$$mg \cos \theta/2 = N_1 \sin \theta + \mu_1 N_1 \cos \theta$$

if,  $\mu_1 = 0$ , then  $N_1 \tan \theta = mg/2$

4. Heater of an electric kettle is made of a wire of length  $L$  and diameter  $d$ . It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length  $L$  and diameter  $2d$ . The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K ?

(A) 4 if wires are in parallel

(B) 2 if wires are in series

(C) 1 if wires are in series

(D) 0.5 if wires are in parallel

Ans. 4. B,D

$$L, d : \text{Rod 1}, \quad t = 4 \text{ min} : \Delta T = 40K$$

L,2d : Rod 2 & Rod 3 :  $\Delta T = 40K$

$$dQ/dT = KA dT/dx$$

$$\Delta Q/\Delta t = \Delta T/L/KA$$

Initial :  $L/KA = L/\pi d^2 K \rightarrow$  Rod1 ( $R_o$ ) (Thermal resistance )

$$\text{Rod : Rod 3 } \quad L/4\pi d^2 K = R_o/4$$

Rod1 :  $R_o$

Rod 2 & Rod 3 :  $R_o/4$

If series R net =  $R_o/2 \Rightarrow T = 2\text{min}$

If parallel R net =  $R_o/8 \Rightarrow T = 0.5\text{min}$

5. A light source, which emits two wavelengths  $\lambda_1 = 400 \text{ nm}$  and  $\lambda_2 = 600 \text{ nm}$ , is used in a Young's double slit experiment. If recorded fringe widths for  $\lambda_1$  and  $\lambda_2$  are  $\beta_1$  and  $\beta_2$  and the number of fringes for them within a distance  $y$  on one side of the central maximum are  $m_1$  and  $m_2$ , respectively, then

- (A)  $\beta_1 > \beta_2$
- (B)  $m_1 > m_2$
- (C) From the central maximum, 3<sup>rd</sup> maximum of  $\lambda_2$  overlaps with 5<sup>th</sup> minimum of  $\lambda_1$
- (D) The angular separation of fringes for  $\lambda_1$  is greater than  $\lambda_2$

Sol.  $\lambda^1 = 400 \text{ nm}, \lambda^2 = 600 \text{ nm}$

Fringe width :  $\lambda D/d$

$$\lambda_2 > \lambda^1 \Rightarrow B^2 > B^1$$

$$m = dy/\lambda d \Rightarrow m \propto 1/\lambda$$

$$\Rightarrow m_1 > m_2$$

$$[\lambda_2 > \lambda_1]$$

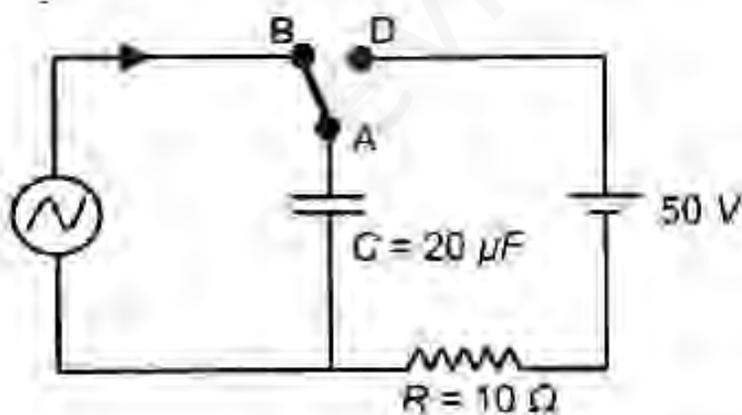
$$\text{Bright : } m\lambda = x d / D$$

Fringe

$$\text{Dark fring : } (2n + 1) \lambda / 2 = x d / D$$

$$\begin{aligned} \diamond 3\lambda_2 &= x d / D \\ \Rightarrow 3\lambda_2 &= 9\lambda_1 / 2 \\ \diamond 9\lambda_1 / 2 &= x d / D \\ 1800 &= 1800 \end{aligned}$$

6. At time  $t = 0$ , terminal A in the circuit shown in the figure is connected to B by a key and an alternating current  $I(t) = I_0 \cos(\omega t)$ , with  $I_0 = 1A$  and  $\omega = 500 \text{ rad s}^{-1}$  starts flowing in it with the initial direction shown in the figure. At  $t = 7\pi/6\omega$ , the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If  $C = 20 \mu F$ ,  $R = 10 \Omega$  and the battery is ideal with emf of 50V, identify the correct statement (s).



- (A) Magnitude of the maximum charge on the capacitor before  $t = 7\pi/6\omega$  is  $1 \times 10^{-3} C$ .
- (B) The current in the left part of the circuit just before  $t = 7\pi/6\omega$  is clockwise.
- (C) Immediately after A is connected to D, the current in R is 10A.

$$(D) \quad Q = 2 \times 10^{-3} C$$

Sol.  $q = i_0/w \sin \omega t$ ;  $\epsilon_0 = i_0/w \omega C$

$$i_0/w = 2mc = 2 \times 10^{-3} C$$

$$Q (t = 7\pi/6\omega) = -1 \times 10^{-3} C$$

$$Q/C - IR + V = 0$$

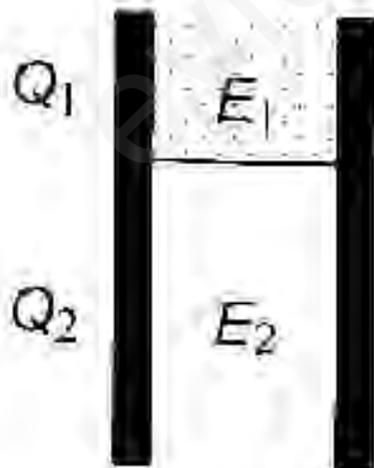
$$50 - 10i + 50 = 0 \Rightarrow i = 10 A$$

$$Q_i (t = 7\pi/6\omega) = -1 \times 10^{-3} C$$

$$Q_i (t = \infty) = -1 \times 10^{-3} C$$

$$\text{Charge from battery} = 2 \times 10^{-3} C$$

7. A parallel plate capacitor has a dielectric slab of dielectric constant  $K$  between its plates that covers  $1/3$  of the area of its plates, as shown in the figure. The total capacitance of the capacitor is  $C$  while that of the portion with dielectric in between is  $C_1$ . When the capacitor is charged, the plate area covered by the dielectric gets charge  $Q_1$  and the rest of the area gets charge  $Q_2$ . The electric field in the dielectric is  $E_1$  and that in the other portion is  $E_2$ . Choose the correct option/options, ignoring edge effects.



$$(A) \quad E_1/E_2 = 1$$

$$(B) \quad E_1/E_2 = 1/K$$

$$(A) \quad Q_1/Q_2 = 3/K$$

(A)  $C/C_1 = 2 + K/K$

Sol) As portion  $\frac{A}{3}$  is filled with dielectric

$$\text{So, } C_1 = \frac{KE.A}{d} C = \frac{KE.A}{3d} + \frac{K.2A}{3d}$$

$$\frac{C}{C_1} = \frac{2+K}{K}$$

So, about electric field after completely charged.

$$E = \frac{\Delta V}{d}$$

Potential difference is same, that gives the same electric fields at two end so,  $\frac{E_1}{E_2}$

and, charged ratio  $\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{3}{K}$

8. One end of a taut string of length  $3m$  along the  $x$  axis is fixed at  $x = 0$ . The speed of the waves in the string is  $100 \text{ ms}^{-1}$ . The other end of the string is vibrating in the  $y$  direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is(are)

- (A)  $y(t) = A \sin \pi x/6 \cos 50\pi t/3$
- (B)  $y(t) = A \sin \pi x/3 \cos 100\pi t/3$
- (C)  $y(t) = A \sin 5\pi x/6 \cos 250\pi t/3$
- (D)  $y(t) = A \sin 5\pi x/2 \cos 250\pi t$

Sol. 8

$$L = 3m$$

$$V = 100 \text{ms}^{-1}$$

For standing wave;

$$Lw/\pi V = n + \frac{1}{2}$$

$$3w/\pi 100 = n + \frac{1}{2}$$

$$3w / 100\pi = n + \frac{1}{2}$$

9. A student is performing an experiment using a resonance column and a tuning fork of frequency  $244 \text{ s}^{-1}$ . He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is  $(0.350 \pm 0.005) \text{ m}$ , the gas in the tube is

(Useful information:  $\sqrt{167RT} = 640 \text{ J}^{1/2} \text{ mole}^{-1/2}$ ;  $\sqrt{140RT} = 590 \text{ J}^{1/2} \text{ mole}^{-1/2}$ . The molar masses  $M$  in grams are given in the options. Take the values of  $\sqrt{10/M}$  for each gas as given there.)

- (A) Neon ( $M = 20, \sqrt{10/20} = 7/10$ )
- (B) Nitrogen ( $M = 28, \sqrt{10/28} = 3/5$ )
- (C) Oxygen ( $M = 32, \sqrt{10/32} = 9/16$ )
- (D) Argon ( $M = 36, \sqrt{10/36} = 17/32$ )

Sol)  $f_{\text{exp}}$  and  $\Delta f$

$$\text{So } f = f_{\text{exp}} + \Delta f$$

$$(A) F = \frac{v}{4L} = 320$$

$$(B) \frac{\Delta f}{f} = -\frac{\Delta l}{l} \quad (\Delta f)_{\text{max}} = \frac{\Delta l}{l} F = \pm 0.4$$

Calculating frequency for all the options

$$\text{i.e. } f = 242$$

10. Let  $E_1(r)$ ,  $E_2(r)$  and  $E_3(r)$  be the respective electric fields at a distance  $r$  from a point charge  $Q$ , an infinitely long wire with constant linear charge density  $\lambda$ , and an infinite plane with uniform surface charge density  $\sigma$ . If  $E_1(r_0) = E_2(r_0) = E_3(r_0)$  at a given distance  $r_0$ , then

- (A)  $Q = 4\sigma\pi r_0^2$
- (B)  $r_0 = \lambda/2\pi\sigma$

$$(C) \quad E_1(r_0/2) = 2E_2(r_0/2)$$

$$(D) \quad E_2(r_0/2) = 4E_3(r_0/2)$$

Sol.  $E_1(r = r_0) = Q/4\pi\epsilon_0 r_0^2$  ;

$$E_2(r = r_0) = \lambda/2\pi\epsilon_0 r_0$$

$$E_3(r = r_0) = \sigma/2\epsilon_0$$

$$E_1(r = r_0/2)/4 = E_2(r = r_0/2)/2 = E_3(r = r_0/2)$$

$$E_1 = 2E_2 = E_3 \quad [\text{at } r = r_0/2]$$

**SECTION -2 : (One Integer Value Correct Type)**

11. Two parallel wires in the plane of the paper are distance  $X_0$  apart. A point charge is moving with speed  $u$  between the wires in the same plane at a distance  $X_1$  from one of the wires. When the wires carry current of magnitude  $I$  in the same direction, the radius of curvature of the path of the point charge is  $R_1$ . In contrast, if the currents  $I$  in the two wires have directions opposite to each other, the radius of curvature of the path is  $R_2$ . If  $X_0/X_1 = 3$ , the value of  $R_1/R_2$  is

Sol.  $mv^3/R = qvB$

$$R = mv/Bq$$

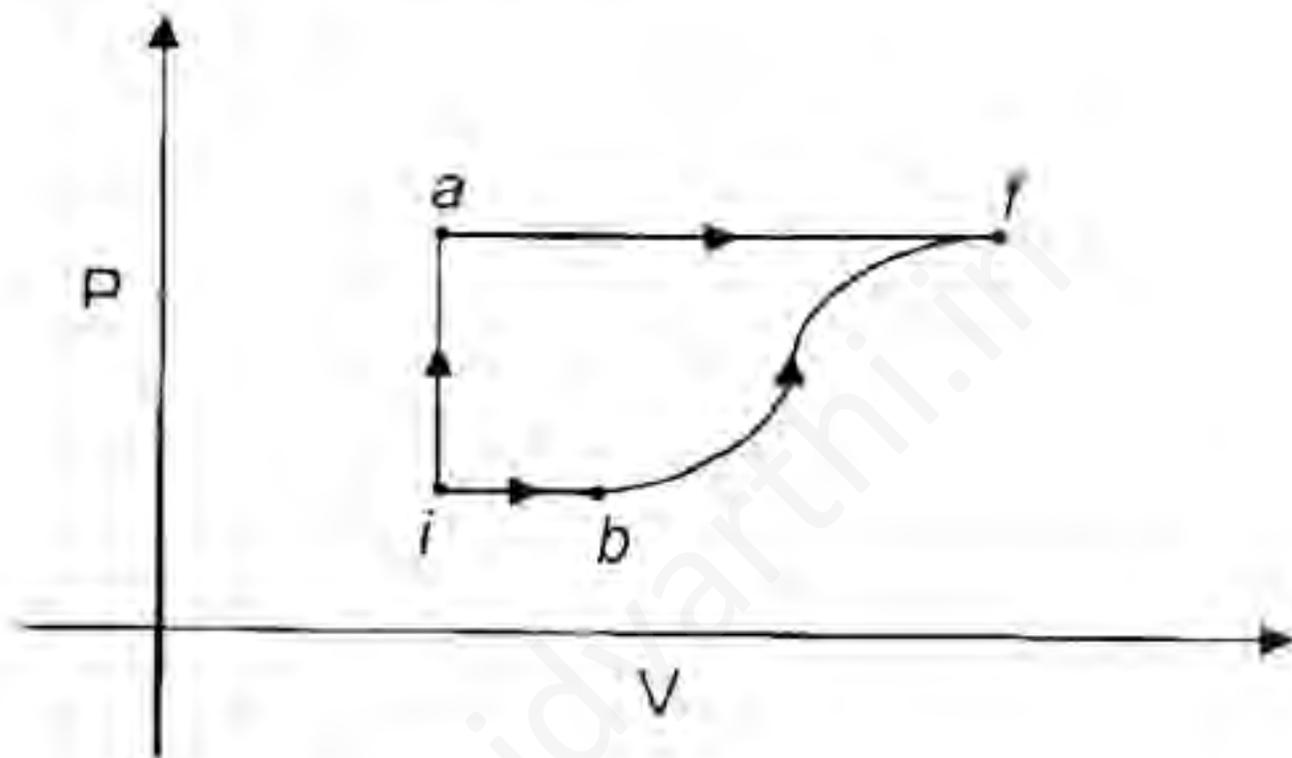
$$R_1/R_2 = B_2/B_1$$

$$B_1 = \mu_0 I/2\pi X_0/3 - \mu_0 I/2\pi 2X_0/3 = 3\mu_0 I/2\pi X_0 \times 1/2 = 3\mu_0 I/4\pi X_0$$

$$B_2 = \mu_0 I/2\pi X_0/3 + \mu_0 I/2\pi 2X_0/3 = 3\mu_0 I/2\pi X_0 \times 3/2 = 9\mu_0 I/4\pi X_0$$

$$R_1/R_2 = 3$$

12. A thermodynamic system is taken from an initial state  $i$  with internal energy  $U_i = 100\text{J}$  to the final state  $f$  along two different paths  $iaf$  and  $ibf$ , as schematically shown in the figure. The work done by the system along the paths  $af$ ,  $ib$  and  $bf$  are  $W_{af} = 200\text{J}$ ,  $W_{ib} = 50\text{J}$  and  $W_{bf} = 100\text{J}$  respectively. The heat supplied to the system along the path  $iaf$ ,  $ib$  and  $bf$  are  $Q_{iaf}$ ,  $Q_{ib}$  and  $Q_{bf}$  respectively. If the internal energy of the system in the state  $b$  is  $U_b = 200\text{J}$  and  $Q_{iaf} = 500\text{J}$ , the ratio  $Q_{bf}/Q_{ib}$  is



Sol:- Applying First law of thermodynamic  $iaf$ :

$$\text{So, } Q_{iaf} = 500\text{J} = (U_f - U_i) + W_{iaf}$$

$$W_{ia} = 0, \Delta v = 0$$

$$W_{af} = p(v_f - v_i) = 200\text{J}$$

$$\text{So, } 50 = (v_f - 100) + 200$$

$$\rightarrow u_f = 400\text{J}$$

$$\text{Now difference of } \Delta U_{bf} = U_b - U_f$$

$$\text{So } Q_{bf} = (U_f - U_b) = W_{bf}$$

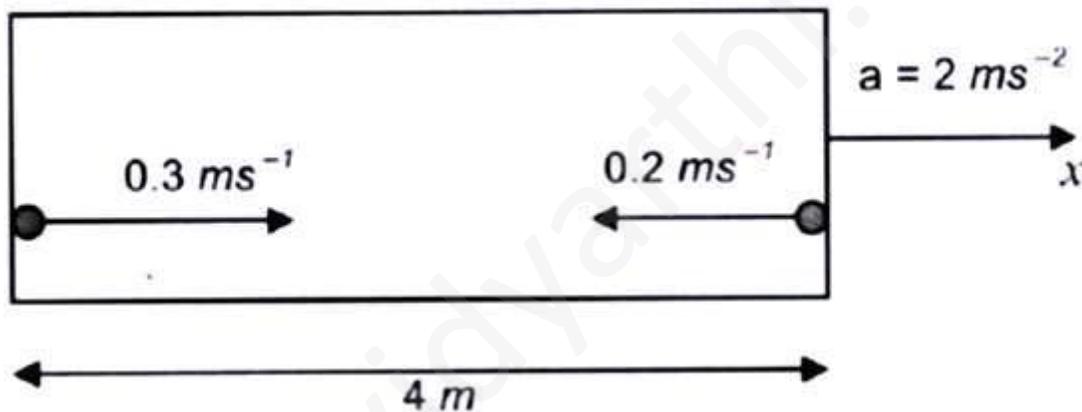
$$200 + 100 = 300$$

$$\text{Similarly } Q_{ib} = U_b - U_i + W_{ib}$$

$$= 200 - 100 + 50 = 150\text{J}$$

So, Ratio of  $Q_{bf}$  and  $Q_{ib}$  is 2

13. A rocket is moving in a gravity free space with a constant acceleration of  $2 \text{ ms}^{-2}$  along + x direction (see figure). The length of a chamber inside the rocket is  $4 \text{ m}$ . A ball is thrown from the left end of the chamber in + x direction with a speed of  $0.3 \text{ ms}^{-1}$  relative to the rocket. At the same time, another ball is thrown in -x direction with a speed of  $0.2 \text{ ms}^{-1}$  from its right end relative to the rocket. The time in seconds when the two balls hit each other is



Sol:- Constant acceleration,  $a = 2 \text{ ms}^{-2}$

Both the ball in projected towards each other,

Now in reference of rocket is motion ball will follow the velocity of rocket for a moment

As the ball collides with the wall again and again, and distance calculated is

$$S = 1/2at^2$$

$$t = 2(0.3)/2 = 0.3 \text{ sec}$$

if second ball want to with the first ball, then distance travelled by second ball is

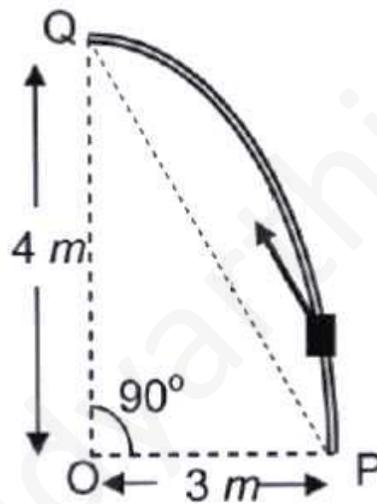
$$S^1 = u_2t + 1/2a t^2$$

$$4 = 0.2t + 1/2*2*t^2$$

Surfing the above, can we get

$$t = 2\text{sec as } > 0$$

14. Consider an elliptically shaped rail PQ in the vertical plane with  $OP = 3\text{ m}$  and  $OQ = 4\text{ m}$ . A block of mass  $1\text{ kg}$  is pulled along the rail from P to Q with a force of  $18\text{ N}$ , which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is  $(n \times 10)\text{ Joules}$ . The value of  $n$  is (take acceleration due to gravity  $= 10\text{ ms}^{-2}$ )



Sol :- Force is acting parallelly to PQ line, so, after drawing FBD of the block

$$W_{\text{force}} + G_{\text{gravity}} = \Delta\text{KE}$$

$$F_x \cdot dx + (-mg \cdot h) = \Delta\text{KE}$$

$$F \cos\theta \cdot \sum_{i=A}^B di - mgh - \Delta\text{KE}$$

$$\rightarrow (18 \cdot AB) - mgh = \Delta\text{KE}$$

$$\rightarrow 18 \cdot 5 - 40 = 10n$$

$$n = 5$$

15. During Searle's experiment, zero of the Vernier scale lies between  $3.20 \times 10^{-2}m$  and  $3.25 \times 10^{-2}m$  of the main scale. The 20<sup>th</sup> division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between  $3.20 \times 10^{-2}m$  and  $3.25 \times 10^{-2}m$  of the main scale but now the 45<sup>th</sup> division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is  $8 \times 10^{-7}m^2$ . The least count of the Vernier scale is  $1.0 \times 10^{-5}m$ . The maximum percentage error in the Young's modulus of the wire is

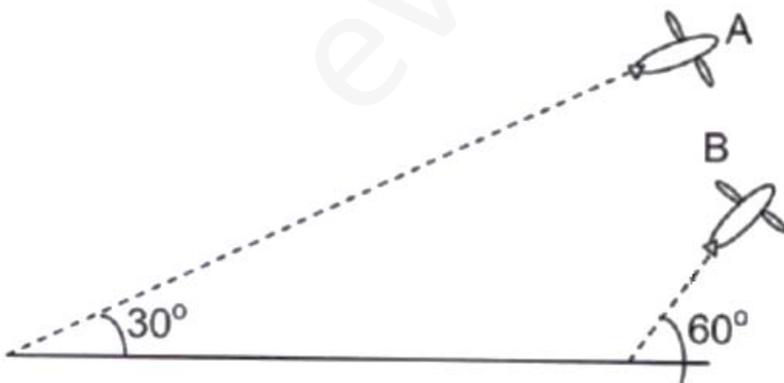
Sol. since, young's module

$$Y = F/A, l/\Delta l = mg/A, l/\Delta l$$

We are interested to find error in ' $\Delta l$ ' using verniercolliper experiment,

$$\text{Total reading} = \text{MSR} + \text{L C} \times n$$

16. Airplanes A and B are flying with constant velocity in the same vertical plane at angles  $30^\circ$  and  $60^\circ$  with respect to the horizontal respectively as shown in figure. The speed of A is  $100\sqrt{3}ms^{-1}$ . At time  $t = 0$  s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at  $t = t_0$ , A just escapes being hit by B,  $t_0$  in seconds is



Sol. Resolving, component of velocity  $V_B$  in the parallel and perpendicular direction of  $V_A$

Horizontal component, of  $(V_B)_H = v_A$

Also, angle between  $v_A$  and  $v_B$  is  $30^\circ$

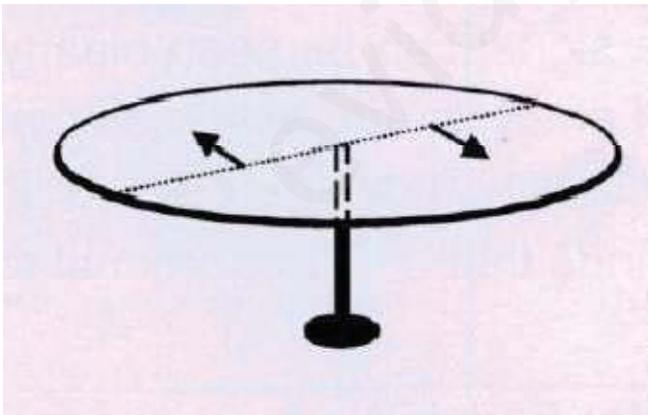
$$v_B \cos 30^\circ = 100\sqrt{3}$$

$$v_B = 200 \text{ms}^{-1}$$

$$\text{Also, } v_B \sin 30^\circ = 200 \times 1/2 = 100 \text{ms}^{-1}$$

$$\text{And, } t = \text{d separation}/v_B = 500/100 = 5 \text{ sec}$$

17. A horizontal circular platform of radius  $0.5 \text{ m}$  and mass  $0.45 \text{ kg}$  is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass  $0.05 \text{ kg}$  are attached to the platform at a distance  $0.25 \text{ m}$  from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of  $9 \text{ ms}^{-1}$  with respect to the ground. The rotational speed of the platform in  $\text{rad s}^{-1}$  after the balls leave the platform is



Sol. Conserve angular momentum of Gun w.r.t rotating circular platform

$$2(m_g v_g r) = I_p \omega$$

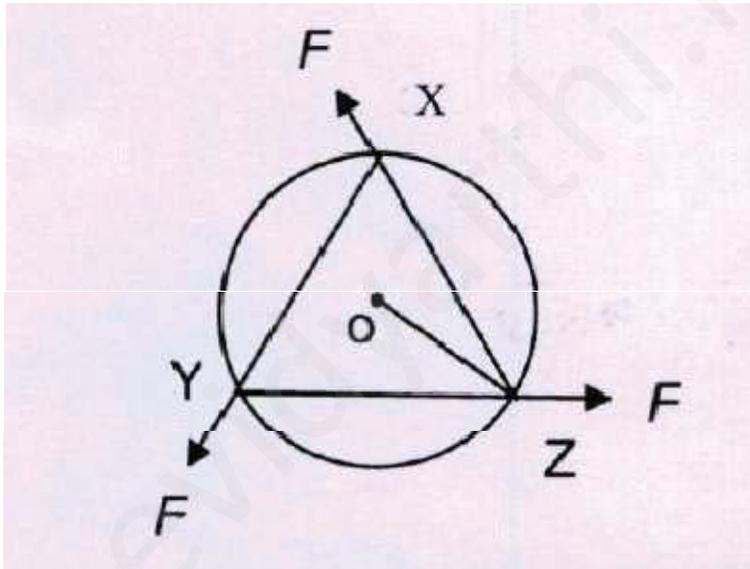
On substituting the value, we get

$$2(0.05 \times 9 \times 0.25) = \frac{1}{2} \times 0.45(0.5)^2 \times \omega$$

Solving, we get

$$\Omega = 4 \text{ rad sec}^{-1}$$

18. A uniform circular disc of mass  $1.5 \text{ kg}$  and radius  $0.5 \text{ m}$  is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude  $F = 0.5 \text{ N}$  are applied simultaneously along the three sides of an equilateral triangle  $XYZ$  with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in  $\text{rad s}^{-1}$  is



**Sol.** Calculating the component of applied force along tangent to the radius calculating force along

$$T = I\alpha$$

$$3.F \cdot R \sin 30^\circ = MR^2 \cdot \alpha$$

$$\text{Solving, } \alpha = 2 \text{ rad/sec}^2$$

$$\text{Hence, } \omega = 0 + \alpha t$$

$$\omega = 2 \times 1 = 2 \text{ rad/sec}$$

19. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990  $\Omega$  resistance, it can be converted into a voltmeter of range 0 - 30 V. If connected to a  $\frac{2n}{249} \Omega$  resistance, it becomes an ammeter of range 0 - 1.5 A. The value of  $n$  is

Sol. Galvanometer into voltmeter

$$I = I_g = \frac{V}{R + G}$$

$$0.006 = \frac{30}{4990 + G}$$

On solving,  $G = 1 \Omega$

Now, Galvanometer into ammeter

$$I_g G = (I - I_g) R$$

$$0.006 \times 1 = (1.5 - 0.006) \frac{2n}{249}$$

On solving, we get,  $n = 5$

20. To find the distance  $d$  over which a signal can be seen clearly in foggy conditions, a railway engineer uses dimensional analysis and assumes that the distance depends on the mass density  $\rho$  of the fog, intensity (power/area)  $S$  of the light from the signal and its frequency  $f$ . The engineer finds that  $d$  is proportional to  $S^{1/n}$ . The value of  $n$  is

Sol. distance

$$d \propto (\rho)^a (S)^b (f)^c \dots\dots(i)$$

$$[L] = K [ML^{-3}]^a [MT^{-3}]^b [T^{-1}]^c$$

$$[L] = k [M^{a+b} L^{-3b} T^{-3b-c}]$$

On comparing

$$a + b = 0, -3a = 1, -3b - c = 0$$

$$b=1/3, a=-1/3, C=-1$$

substitally

$b=1/3$  in eqn(1), we get

$$d \times (s)^{1/3} \times (s)^{1/x}$$

$$n=3$$

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