ANSWERS KEY

CHEMISTRY

1. b	2. d	3. d	4. c	5. d	6. d	7. d	8. c	9. d	10. b	11. b	12. b	13. d
14. b	15. b	16. b	17. b	18. c	19. b	20. c	21. d	22. d	23. c	24. b	25. c	26. b
27. b	28. d	29. b	30. b									

PHYSICS

1. b	2. a	3. a	4. d	5. d	6. a	7. a	8. b	9. b	10. d	11. a	12. a	13. b
14. c	15. d	16. c	17. b	18. b	19. d	20. d	21. c	22. b	23. d	24. c	25. a	26. b
27. c	28. a	29. c	30. c									

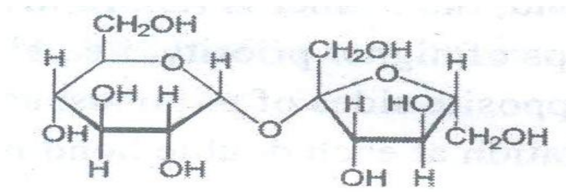
MATHEMATICS

1. a	2. b	3. c	4. b	5. a	6. c	7. a	8. b	9. a	10. d	11. d	12. b	13. a
14. c	15. a	16. b	17. c	18. b	19. c	20. c	21. a	22. c	23. d	24. a	25. b	26. a
27. d	28. c	29. d	30. d									

CHEMISTRY

Sol 1.

In sucrose, glucose and fructose are linked to each other by covalent linkage.



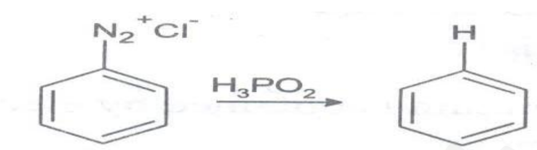
Cellobiose is a disaccharide, it consists of two glucose molecules linked by a ß (1 → 4) bond. Maltose also known as maltobiose or malt sugar, is a disaccharide formed from two units of glucose joined with an α (1→ 4) bond. Lactose is a disaccharide sugar that is found most notably in cow milk and is formed from linkage of galactose and glucose

Sol 2.

Camphor can undergo sublimation where as caffine cannot so it can be separated from the mixture by sublimation.

Sol 3.

The amino group of an aryl amine may be replaced by a 'H' upon reaction of its diazonium salt with H_3PO_2.



Sol 4.

For the preparation of iodoform, a compound containing CH_3CO- group a base and iodine is needed.

$C_6H_5COCH_3 + 3I_2 + 3NaOH \rightarrow C_6H_5COONa + CHI_3 + NaI + H_2O$

Sol 5.

Aldehydes give silver mirror test with Tollen's reagent which is $[Ag(NH_3)_2]^+$

Sol 6.

Inert pair effect is shown by T1.

Sol 7.

Nesseler's reagent is $K_2\,[HgI_4]$

Sol 8.

The IUPAC name of $Na_3\,[Co(NO_2)_6]$ is Sodium hexanitrocobaltate (III)

Sol 9.

Ziegler – Natta catalysts are used to polymerize terminal 1- alkenes. These consist of transition metal catalyst, like $TiCl_3$ and AI $(C_2H_5)_2$ Cl, or $TiCl_4$ with $Al(C_2H_5)_3$.

Sol 10.

P_4O_6 reacts with water to form phosphorous acid (H_3PO_3)

$P_4O_6 + 6H_2O \rightarrow 4H_3PO_3$

Sol 11.

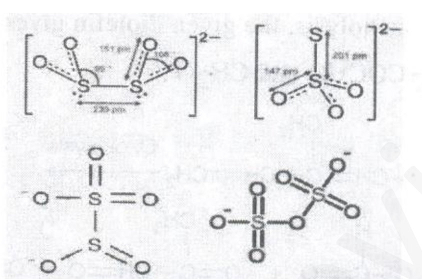
More energy is required to remove an electron from the completely filled or half filled subshells and also from the orbitals which are closer to the nucleus.

Sol 12.

The number and types of bonds between two carbon atoms in CaC_2 are one sigma and two pi(p).
$[:C \equiv C:]^{2-}$
Ca^{2+}

Sol 13.

$S_2O_7^{2-}$ has no S-S bond.



Sol 14.

1.5 N H_2O_2 contains 17 x 1.5 = 25.5 g of H_2O_2 According to the following equation.

$2H_2O_2 \rightarrow 2H_2O + O_2$

2X 34 g 22.4L

68 g of H_2O_2 = 22.4 L of O_2

25.5 g of H_2O_2 $= \frac{22.4 \ x \ 25.5}{68} = 8.4$ L

Volume strength of H_2O_2 is expressed as the volume of O_2 that a solution of H_2O_2 gives on decomposition by heat. Since 1.5 N H_2O_2 solution gives 8.4 L of oxygen at STP so its volume strength is 8.4.

Sol 15.

When gold is dissolved in aqua regia, $HAuCl_4$ (chloroauric acid) is formed

$Au\ (s) + 3NO_3^-\ (aq) + 6\ H^+\ (aq) \rightarrow Au^{3+}\ (aq) + 3\ NO_2\ (g) + 3H_2O\ (l)$

$Au^{3+}\ (aq) + 4\ Cl^-\ (aq) \rightarrow AuCl_4^-\ (aq)$.

Sol 16.

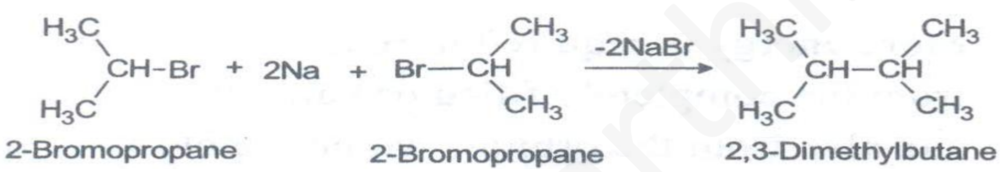
$NaHCO_3$ acts as an antacid.

Sol 17.

Straight chain hydrocarbons have higher boiling point than branched chain hydrocarbons. Higher hydrocarobons have higher boiling point than lower hydrocarbons

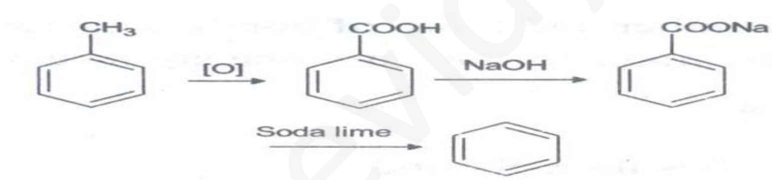
Sol 18.

When isopropyl is subjected to Wurtz reaction, 2, 3 – dimethylbutane is formed.



Sol 19.

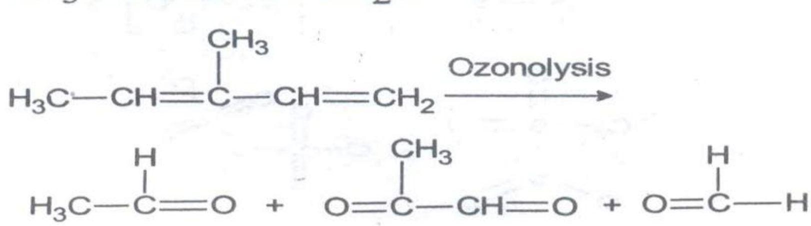
The product C is benzene as shown below.



Sol 20.

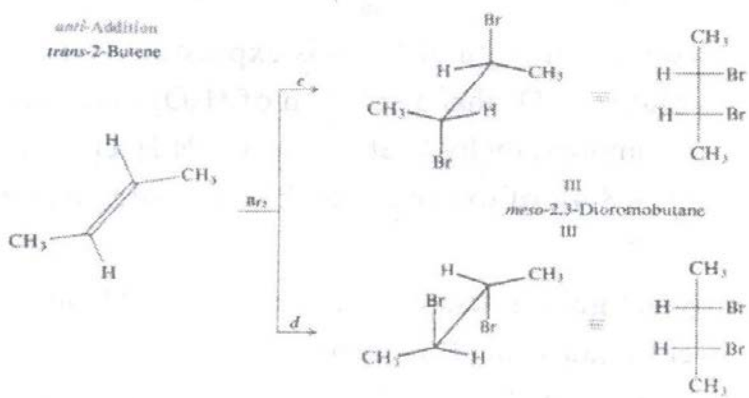
On ozonolysis, the given diolefin gives CH_3CHO, CH_3COCHO and CH_2O

CH_3COCHO and CH_2O



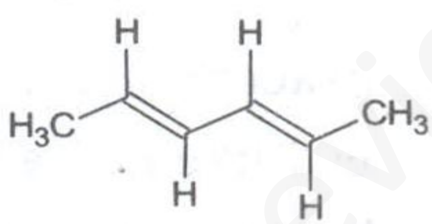
Sol 21.

The addition of bromine to alkenes is stereospecific; cis and trans alkenes react differently to give stereochemically different products. Cis-but-2-ene gives a pair of enanatiomers, i.e, a racemic mixture whereas trans but-2-ene gives a meso compound



Sol 22.

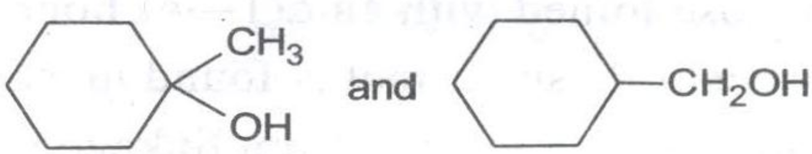
The (E) and (Z) system is based on the assignment of a priority to the substituents on each end of the double bond. If the highest priority atoms or groups are on the opposite sides of pi bond, the isomer is (E) and if these are one the same sides of the pi bond, the isomer is (Z). In the given diene the groups of higher priority, i.e., alkyl group are on the opposite sides of pi bonds, so the configuration at each double bond is E.



(2*E*,4*E*)-hexa-2,4-diene

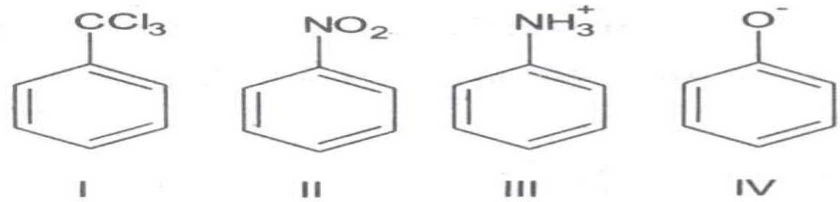
Sol 23.

A and B in the given reactions are



Sol 24.

In options I-III, CCI3$_3$, NO_2 and NH_3^+ are electron withdrawing groups and meta directing so electrophile in these cases will attack meta positions; in IV it attacks ortho and para positions

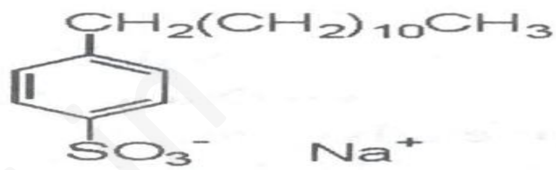


Sol 25.

Pure aluminium is obtained by electrolysis of fused alumina.

Sol 26.

Sodium p-dodecylbenzene suplhonate is an anionic detergent.



Sol 27.

K^+ is detected by flame test, it gives violet pink color,

Sol 28.

IR radiation are responsible for global warming and UV radiation are responsible for ozone depletion.

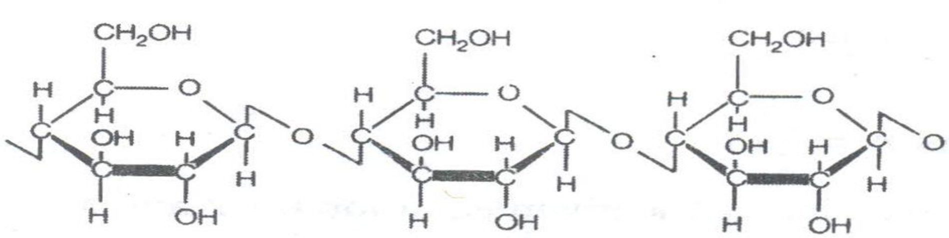
Sol 29.

Ammonium dichromate is used in some fireworks. The green colored powder blown in the air when ammonium dichromate is used in some fireworks is due to formation of Cr_2O_3

$(NH_4)_2\,Cr_2O_7 \rightarrow Cr_2O_3 + 4H_2O + N_2$

Sol 30.

Complete hydrolysis of cellulose gives D-glucose. It is a polysaccharide consisting of a linear chain of several hundred to over ten thousand ß $(1 \rightarrow 4)$ linked D-glucose units



PHYSICS

Sol 1.

Mass of salt $= \frac{4 \text{ x } 6}{100} = \frac{24}{100} = 0.24$ kg After evaporation 0.24 kg salt remains in 5 kg water

$\therefore$ Remaining salt $= \frac{0.24}{5}$ x 100

$= 4.8\%$

Sol 2.

$(-4)\,\vec{P} = 4P\,(-\vec{P})$

$\therefore$ The direction is reversed and magnitude is quadrupled

Sol 3.

P has a higher momentum. Therefore on exchange of packet from P, Q will be gainer.

Sol 4.

As kinetic energy $\propto u^2$

$\therefore$ The curve is a parabola

Sol 5.

Using conservation of Angular Momentum

$I_1\,\omega_1 = I_2\,\omega_2$

$I_2 = \frac{I_1\omega_1}{\omega_2}$

$= I \text{ x } \frac{20}{10} = 2I$

Sol 6.

Gravitational force is two body interaction and is independent of presence of other bodies whereas nuclear force is multi body interaction.

$\therefore$ Resultant gravitational force due to number of bodies, $\vec{F} = \overrightarrow{F_2} + \overrightarrow{F_3} +$

Sol 7.

For a perfectly plastic body there is no restoring force, so stress is zero.

$\therefore$ Young's modulus is also zero.

Sol 8.

Efficiency, $\eta = 1 - \frac{T_2}{T_1} \Rightarrow \eta_1 = 1 - \frac{273}{473} = \frac{200}{473}$

and $\eta_2 = 1 - \frac{73}{70} = \frac{200}{273} \Rightarrow \frac{\eta_1}{\eta_2} = \frac{273}{473} = \frac{1}{1.73}$

Sol 9.

Using PV^y = constant

$P_1 V_1^y = P_2 V_2^y$ Here $\gamma = \frac{5}{3}$

$\therefore \frac{P_2}{P_1} = \frac{V_1^\gamma}{V_2^\gamma} = \frac{V_1^{5/3}}{(V_1/8)^{5/3}} = 2^5$

Sol 10.

$V_m > V_d$

Sol 11.

Apparent frequency will be greater than the frequency of source.

Sol 12.

As $W = \frac{1}{2}\frac{q^2}{C} \Rightarrow W = \frac{1}{2} \text{x} \frac{(8 \, x \, 10^{18})^2}{100 \, x \, 10^{-6}} = 32 \text{ x } 10^{-32}$ J

Sol 13.

Resistance of a part $= \frac{R}{4}$

$\therefore$Resistance of combination $= \frac{1}{4} \, x \frac{R}{4} = \frac{R}{16}$

Sol 14.

As $V = \omega r = \frac{2\pi}{7} x \, r \Rightarrow = T = \frac{2\pi r}{V} = \frac{2\pi m}{qB} = \frac{Km}{q}$

Now $m_\propto = 4m_p \; and \; q_\propto = 2q_p$

$\therefore T_p = k\frac{m_p}{q_p}$

and $T_\propto = k\frac{m_\propto}{q_\propto} = k\frac{4m_p}{2q_p} = 2k\frac{m_p}{q_p}$

$\Rightarrow T_\propto = 2T_p$ or $T_p = \frac{1}{2} T_\propto$

Sol 15.

Given $B = 6 \text{ x } \frac{\mu_0}{4\pi}\frac{i}{r}(\sin \propto_1 + \sin \propto_2)$

$= 6\ \frac{\mu_0}{4\pi} \cdot \frac{l\ (2 \sin 30°)}{\left(\frac{\sqrt{3}}{2}t\right)}$

$= \frac{\sqrt{3}\mu_0 i}{\pi l}$

as $r = \frac{\sqrt{3}}{2} I$

Sol 16.

Considering the given equation for V and I $E_0 = 100V$, $I_0 = 100$ A and $Q = \frac{\pi}{3}$

$\therefore E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{100}{\sqrt{2}}$

and $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{100}{\sqrt{2}}$

Power dissipated in the circuit will be

$P = E_{rms}\ I_{rms} \cos \phi$

$= \frac{100}{\sqrt{2}} \text{ x } \frac{100}{\sqrt{2}} \text{ x } \cos\frac{\pi}{2}$

$= \frac{100x\ 100}{2} \text{ x } \frac{1}{2} = 2500$ W

Sol 17.

Both the statements are independently true.

Sol 18.

Average energy density of electric field

$U_E = \frac{1}{2} \epsilon_0 E_0^2$ Average energy density of magnetic field

$U_B = \frac{B_0^2}{2\mu_0}$

Now $B_0 = \frac{E_0}{C}$ and $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\therefore U_B = \frac{E_0^2}{2\mu_0 c^2} = \frac{E_0^2}{2\mu_0 \text{x} \frac{1}{\mu_0 \varepsilon_0}}$

$\Rightarrow U_B = \frac{1}{2} \varepsilon_0 E_0^2 = U_E$

Sol 22.

Using Q.E = $\frac{4\pi}{3}$ r^3 ρg

⇒ Q^1 x $\frac{v^1}{l}$ = $\frac{4\pi}{3}$ (2r)^3 ρg

Dividing $\frac{QV}{Q'V'}$ = $\frac{1}{8}$⇒ Q^1 = $\frac{QV}{V'}$ x 8

or Q' = $\frac{Q\ x\ 800\ x\ 8}{3200}$ = 2Q

Sol 23.

U^{235} is the fertile material

Sol 24.

Another photon must be emitted in Lymann Series

Sol 25.

At 0 K, there will be no free electrons

Sol 26.

Using d = $\sqrt{2rh}$

= $\sqrt{2\ x\ 6.4\ x\ 10^3\ x\ 160\ x\ 10^{-3}}$

= 45 km

⇒ Range = 2d = 2 x 45 = 90 km

And area covered = πd^2 = 3.14 x (45)^2

= 6359 km^2

Sol 27.

The tube is open initially at the both ends and then it is closed

$\therefore f_0 = \frac{v}{2l_0}$ and $f_c = \frac{v}{4l_c}$

Given that tube is half dipped in water

We have $l_c = \frac{l_0}{2}$

$\Rightarrow f_C = \frac{v}{4\left(\frac{l_0}{2}\right)} = \frac{v}{2l_0} = f_0 = f$

Sol 28.

The maximum number of electrons in an orbit are given by $2n^2$. If n > 4 is not allowed the number of maximum electron that can be in first four orbits are

$2(1)^2 + 2(2)^2 + 2(3)^2 + 2(4)^2 = 60$

⇒ The possible electrons are 60.

Sol 29.

The resistivity of conductors increases with increases in temperature whereas the resistivity of semiconductor decreases with increase in temperature. Both statements are self explanatory.

Sol 30.

Using law of conservation of mechanical energy

Decrease in kinetic energy = Increase in Potential Energy

$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{(Ze)(2e)}{r_{min}} = 5 \text{ MeV} = 5 \times 1.6 \times 10^{-13}$

$\therefore r_{min} = \frac{1}{4\pi\ \epsilon_0}\frac{2ze^2}{5\ x\ 1.6\ x\ 10^{-13}}$

$= \frac{(9\ x\ 10^9) x\ (2) x\ (92) x\ (1.6\ x\ 10^{-19})^2}{5\ x\ 1.6\ x\ 10^{-13}}$

$= 5.3 \times 10^{-14}$ m $= 5.3 \times 10^{-12}$ cm

$\Rightarrow r_{min} = 10^{-12}$ cm

MATHEMATICS

Sol 1.

Given that |z – iRe (z)| = |z|

⇒ |x + iy – ix| = |x + iy|

⇒ |x + I (y – x)| = |x + iy|

$\Rightarrow x^2 + (y - x)^2 = x^2 + y^2$

$\Rightarrow x^2 - 2xy = 0$

⇒ x (x – 2y) = 0

⇒ x = 2y ⇒ Re (z) 2lm (z)

Sol 2.

Given equation is $3^{log\,3\ (x^2-6x+8)}$= -2 (x – 2)

⇒ $x^2 - 6x + 8 = -2x + 4$

⇒ $x^2 - 4x + 4 = 0$

⇒ $(x - 2)^2 = 0$

⇒ x = 2, 2

∵ a^{-1} , b^{-1} , c^{-1} are in A.P.

∴ a, b, c are in H.P.

For any three numbers a^{201} , b^{201} , c^{201}

A.M. > G.M.

$\Rightarrow \frac{a^{201}+c^{201}}{c} > \left(\sqrt{ac}\right)^{201} > b^{201} (\therefore \sqrt{ac} > b)$

⇒ $a^{201} + c^{201}\ 2b^{201} > 0$

⇒ $2b^{201} - a^{201} - c^{201} > 0$

⇒ $2b^{201} - a^{201} - c^{201} < 0$

Given equation is

$x^2 - kx + 2b^{201} - a^{201} - c^{201} = 0$

∴ Product of roots $= \frac{2b^{201} - a^{201} - c^{201}}{1} < 0$

∴ Product of roots < 0

Sol 4.

5!, 6!, 7! . . . . 100! Each is divisible by 15, We know 1! + 2! + 3! = 33 and 15 x 2 + 3

Hence required remainder = 3

Sol 5.

The coefficient of x^2 in the expansion of $(1 + ax)^5$

is ${}^5c_2\ a^2 = 40$

⇒ $10\ a^2 = 40 \Rightarrow a^2 = 4 \Rightarrow a = + 2$

Sol 6.

Given $f(x) = \begin{vmatrix} sinx & cosx \\ 2\cos 2x & cos2x \end{vmatrix}$

Therefore $f^{'}(x) = \begin{vmatrix} cosx & cosx \\ 2cos2x & cos2x \end{vmatrix} + \begin{vmatrix} sin & -sin \\ sin2x & -2sin2x \end{vmatrix}$

$\Rightarrow f^{'}\left(\frac{\pi}{2}\right) = \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 1 & -2 \end{vmatrix}$

$= 0 + \left(\frac{1}{\sqrt{2}}\right)\begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix}$

$= f^{'}\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ (- 2 + 1) $= -\frac{1}{\sqrt{2}}$

Sol 7.

Given that $A^3 + 3A^2 + 3A^2 + 5A - I = 0$

$\Rightarrow A^{-1}(A^3 + 3A^2 + 5A - I) = A^{1}0$

$\Rightarrow a^{-1} A^3 + 3A^{-1} A^2 + 5A^{-1} A - A^{-1} I = 0$

$\Rightarrow A^2 + 3A + 5I = A^{-1}$

Sol 8.

Given that $a = \log_3 2$, $b = \log_5 3$, $c = \log_7 5$ Therefore $a = \frac{log\,2}{log\,3}$, $b = \frac{log\,3}{log\,5}$, $c = \frac{log\,5}{log\,7}$

$\Rightarrow abc = \frac{log\,2}{log\,7}$ and $ab = \frac{log\,2}{log\,5}$

$\log_{210} 60 = \frac{log\ 60}{log\ 210} = \frac{\log(2^2 x\ 3\ x\ 5)}{\log(2\ x\ 3\ x\ 5x\ 7)}$

$= \frac{2\log\ 2 + log\,3 + \log\ 5}{log\,2 + \log 3 + \ \log\ +\ \log 7}$

$= \frac{2 + \frac{log\,3}{log\,2} + \frac{log\,5}{log\,2}}{1 + \frac{log\,3}{log\,2} + \frac{log\,5}{log\,2} + \frac{log\,7}{\log\ 2}}$

$= \frac{2 + \frac{1}{a} + \frac{1}{ab}}{1 + \frac{1}{a} + \frac{1}{ab} + \frac{1}{ab\,c}}$

$= \frac{2ab + b + 1}{abc + bc + c + 1}$

Sol 9.

Odd numbers on dice are 1, 3, 5

The probability that an odd numbers appear in a throw $= \frac{3}{6} = \frac{1}{2}$

If the dice is thrown (2n + 1) times, then the probability that faces with odd number appear odd number of times = P {that an odd number appear once or thrice or five times. . . . . or (2n + 1) time}

$$= {}^{2n+1}c_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{2n} + {}^{2n+1}c_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{2n-2} + {}^{2n+1}c_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{2n-4} + \ldots\ldots\ldots\ldots + {}^{2n+1}c_{2n+1}\left(\frac{1}{2}\right)^{2n+1}\left(\frac{1}{2}\right)^0$$

$$= \frac{1}{2^{2n+1}}\{{}^{2n+1}c_1 + {}^{2n+1}c_3 + {}^{2n+1}c_5 + \ldots\ldots\ldots\ldots + {}^{2n+1}c_{2n+1}\}$$

$$= \frac{2^{2n}}{2^{2n+1}} = \frac{1}{2}$$

Sol 10.

P $(\bar{A})$ = 0.4 and P $(\bar{B})$ = 0.3

P (A) = .6 and P (B) = 0.7

The probability tht at least one of them fails = 1 – P (A∩B)

= 1 – (0.6) (0.7) = 0.58

Sol 11.

For L.H.L., Put x = 2 – h, where 0 < h < 1

As – 1 < h<0, then 1 <2 – h <2. Therefore [2-h] = 1

L.H.L. = $Lt_{x\to 2-}[x] = Lt_{h\to 0+}[2-h] = Lt_{x\to 0+}1 = 1$

For R.H.L., Put x = 2+h, where 0 < h< 1 then 2<2 + h< 3 Therefore [2+h] = 2

R.H.L. = $Lt_{x\to 2+}[x] = Lt_{h\to 0+}[2+h] = Lt_{x\to 0+}2 = 2$Now L.H.L. ≠ R.H.L.

Therefore $Lt_{x\to 2}[x]$ does not exist.

Sol 12.

$$Lt_{x\to\infty}\frac{n^p\cos n!}{n+2} = Lt_{n\to\infty}\frac{n^p\cos n!}{n\left(1+\frac{2}{n}\right)}$$

$$Lt_{x\to\infty}\frac{\cos n!}{n^{1-p}\left(1+\frac{2}{n}\right)}$$

= (∵ 0 < cos n! < 1)

Sol 13.

$f'(x) = Lt_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= Lt_{h \to 0} \frac{f(x) + f(h) - f(x)}{h}$

(because f (x + h) = f(x) + f (h)

$= L_{h \to 0} \frac{f(h)}{h}$

$= Lt_{h \to 0} \frac{h^3 g(h)}{h}$ $\quad (\because f(x) = x^3 g(x))$

$= Lt_{h \to 0} h^2 g(h) = 0$

Sol 14.

Given that $x^y = y^x$

$\therefore$ y log x = x log y Differentiating both sides w.r.t. x

$y\frac{1}{x} + logx\frac{dy}{dx} = x\frac{1}{y}\frac{dy}{dx} + 1.logy \Rightarrow \left(\log x - \frac{x}{y}\right)\frac{dy}{dx} = \log y - \frac{y}{x}$

$\Rightarrow \left(\frac{ylog - x}{y}\right)\frac{dy}{dx} = \left(\frac{xlogy - y}{x}\right) \Rightarrow \frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$

$\left(\frac{dy}{dx}\right)_{(1,2)} = \frac{2(1\,log\,2 - 2)}{1(2\,log\,1 - 1)} = \frac{2(log\,2 - 2)}{(0-1)}$

= - 2 (log 2 – 2)

Sol 15.

Given that

$y = \sqrt{x + \sqrt{x + \sqrt{x \ldots\ldots \infty}}}$

$\therefore y = \sqrt{x + y}$

$y^2 = x + y$ Differentiating both sides

$2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$(2y - 1)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y - 1}$

Sol 16.

Given $f\ (x) = \sin\left(\frac{\pi}{x}\right)$ is increasing

$\therefore f^{'}(x) = \frac{-\pi}{x^2}\cos\left(\frac{\pi}{x}\right) < 0 \therefore \cos\left(\frac{\pi}{x}\right) > 0$

$\Rightarrow 2n\pi - \frac{\pi}{2} < \frac{\pi}{x} < 2n\pi + \frac{\pi}{2}$

$\Rightarrow (4n - 1)\frac{\pi}{2} < \frac{\pi}{x} < (4n + 1)\frac{\pi}{2}$

$\Rightarrow \frac{2}{(4n-1)^{\pi}} > \frac{x}{\pi} > \frac{2}{(4n+1)\pi}$

$\Rightarrow \frac{2}{4n+1} < x < \frac{2}{4n-1}$

$\therefore n \in \left(\frac{2}{4n+1}, \frac{2}{4n-1}\right) \forall n \ \epsilon\ N$

Sol 17.

The given function is

$f\ (x) = \int_2^x e^{-t^2}\,(4 - t^2)dt$

$f'(x) = e^{-x^2}(4 - x^2)$

$f'(x) = 0 \Rightarrow 4 - x^2 = 0$

$\Rightarrow x = \pm\ 2$

Sol 18.

$I = \int \frac{5+4\ sinx}{(4+5\sin\ x\ )^2}dx$

$I = \int \frac{\frac{5+4\ sinx}{\cos^2 x}}{\left(\frac{4+5\sin\ x}{cos}\right)^2}\ dx$

$I = \int \frac{5\sec^2 x + 4\ secx\ \ tanx}{(4\ secx + 5\tan\ x)^2}\ dx$

Put 4 secx + 5 tanx = t

$\Rightarrow$ (4secx tanx + 5sec^2 x) dx = dt

$I = \int \frac{dt}{t^2} = -\frac{1}{t} + \ c$

$I = -\frac{1}{4secx + 5\tan\ x} + \ c$

Sol 19.

$\mathrm{I} = \int_0^\infty e^{ax^2}\ dx$

Put $\sqrt{a}\,x\ = \mathrm{t} \Rightarrow \sqrt{a}\mathrm{dx} = \mathrm{dt} \Rightarrow \mathrm{dx} = \frac{dt}{\sqrt{a}}$

$\mathrm{I} = \int_0^\infty e^{t^2} \frac{dt}{\sqrt{a}}$

$= \frac{1}{\sqrt{a}} \int_0^\infty e^{t^2}\ dt$

$= \frac{1}{\sqrt{a}} \int_0^\infty e^{t^2}\ dx = \frac{1}{\sqrt{a}}\ b\ = \frac{b}{\sqrt{a}}$

Sol 20.

Sinx < x (∵ x > 0)

$\Rightarrow \frac{sinx}{x} < 1$

$\int_0^{\pi/2} \frac{sinx}{x}\ dx < \int_0^{\pi/2} 1dx = \frac{\pi}{2}$

$\therefore \int_0^{\pi/2} \frac{sinx}{x}\ dx < \frac{\pi}{2}$

Sol 21.

The given function is

$y^3\, x^3$ dx = (ydx – xdy)

x^4 dx $= \frac{x}{y}\left(\frac{ydx - xdy}{y^2}\right)$

x^4 dx $= \left(\frac{x}{y}\right) d\ \left(\frac{x}{y}\right)$ On integrating we get

$\frac{x^5}{5} = \frac{\left(\frac{x}{y}\right)^2}{2} + \mathrm{c}\ \Rightarrow \frac{x^5}{5} - \frac{x^2}{2y^2}\ = \mathrm{c}$

Sol 22.

The image of A (a,b) on x = y is B (b, a) and the image of B (b, a) on x = - y line is C (- α , - b)

The mid point of AC is $(\frac{a-a}{2}, \frac{b-b}{2})$

The mid point of AC is (0, 0).

Sol 23.

Coefficient of x^2 + Coefficient of $y^2 = 0$

Sol 24.

The equation of the circle is

$r^2 = 1 - 2\,r\cos\theta + 3r\cos\theta$

Put $x = r\cos\theta$, $y = \sin\theta$

$x^2 + y^2 = 1 - 2x + 3y$

$x^2 + y^2 + 2x - 3y - 1 = 0$

Therefore $g = 1$ $f = \frac{-3}{2}$ $c = -1$

Centre (- g, - f) i.e. centre $\left(-1, \frac{3}{2}\right)$

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{1 + \frac{9}{4} + 1} = = \sqrt{\frac{17}{2}}$

Sol 25.

The diameter are conjugate if

$m_1 m_2 = -\frac{b^2}{a^2}$ (i)

Equation of pair of conjugate diameters is $4x^2 + xy - 5y^2 = 0$

$4x^2 + xy - 5y^2 = 0$

$(x - y)(4x + 5y) = 0$

Thus the slope of conjure diameters are $1, \frac{-5}{4}$

$\therefore m_1 = 1, m_2 = -\frac{4}{5}$

Put values of m_1, m_2 in (i) we get

$(1)\left(\frac{-4}{5}\right) = -\frac{b^2}{a^2}$

$\Rightarrow \frac{4}{5} = \frac{b^2}{a^2}$

$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{5}} = \frac{1}{\sqrt{5}}$

Sol 26.

The given equation of line is

$\frac{x-1}{3} = \frac{y-1}{4} = z$ Any point on given line is (3r + 1, 4r + 1, r) Its distance from (1, 1, 0) is

$(3r)^2 + (4r)^2 + r^2 = (3\sqrt{26})^2$

$26r^2 = 9 \times 26$

$r^2 = 9 \Rightarrow r = \pm 3$

Coordinates of points are (10, 13, 3) and (-8m – 11 , 3).

Sol 27.

Given that sinα = cosß

$\Rightarrow \sin\alpha = \sin\left(\frac{\pi}{2} - ß\right) \Rightarrow \sin\alpha - \sin\left(\frac{\pi}{2} - ß\right) = 3$

$\Rightarrow 2\cos\left(\frac{\alpha + \frac{\pi}{2} - ß}{2}\right)\sin\left(\frac{\alpha - \frac{\pi}{2} + ß}{2}\right) = 0$ (i)

Also cosα= sinß $\Rightarrow \cos\alpha = \cos\left(\frac{\pi}{2} - ß\right)$

$\Rightarrow \cos\alpha - \cos\left(\frac{\pi}{2} - ß\right) = 0$

$\Rightarrow 2\sin\left(\frac{\alpha + \frac{\pi}{2} - ß}{2}\right)\sin\left(\frac{\alpha - \frac{\pi}{2} + ß}{2}\right) = 0$ (ii)

From (i) and (ii), we get

$\sin\left(\frac{\alpha - \frac{\pi}{2} + ß}{4}\right) = 0$

Sol 28.

Given that sinx + cosx = 1

$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin x \sin\frac{\pi}{4} + \cos x \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(x + \frac{\pi}{4}\right) = \sin\frac{\pi}{4}$

$\Rightarrow x + \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4},\ n \in I$

Sol 29.

$5\vec{a} + 6\,\vec{b} + 7\vec{c} = 0$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar.

Sol 30.

$\vec{r} = l\,(\vec{b} \times \vec{c}\,) + m\,(\vec{c} \times \vec{a}) \; + n\,(\vec{a} \times \vec{b})$

$\vec{r}.\,\vec{a} = l\,[\vec{a}, \vec{b}, \vec{c}]$

$\vec{r}.\,\vec{a} = l$ $\quad\because [\vec{a}, \vec{b}, \vec{c}] = 1$

$l = 3$ $\quad\because \vec{r}.\,\vec{a} = 3$

Similarly m = 5, n = 7.

Therefore $\vec{r} = 3\,(\vec{b} \times \vec{c}) + 5(\vec{c}\; x\; \vec{a}) + 7\,(\vec{a} \times \vec{b})$