

ANSWERS KEY**CHEMISTRY**

1.c 2.d 3.d 4.b 5.c 6.b 7.d 8.c 9.d 10.d
11. c 12.b 13.b 14.b 15.c 16.a 17.a 18.b 19.c 20.b
21.b 22.c 23.c 24.a 25.a 26.c 27.c 28.b 29.d 30.b

PHYSICS

1.b 2.c 3.b 4.a 5.b 6.a 7.c 8.d 9.b 10.d
11.d 12.b 13.d 14.b 15.b 16.c 17.c 18.a 19.a 20.a
21.b 22.a 23.c 24.a 25.a 26.b 27.b 28.a 29.c 30.b

MATHEMATICS

1.c 2.c 3.c 4.a 5.d 6.b 7.c 8.a 9.b 10.d
11.a 12.a 13.c 14.b 15.b 16.a 17.a 18.b 19.a 20.a
21.a 22.b 23.d 24.c 25.b 26.a 27.a 28.b 29.c 30.c

HINTS AND EXPLANATIONS

CHEMISTRY

Sol. 1

In a face centered cubic arrangement each ion is surrounded by six oppositely charged ions. i.e., coordination number is six. Radius ratio for coordination number 6 is 0.414-0.732.

$$\text{When } \frac{r^+}{r^-} = 0.414; \frac{r^+}{241.5} = 0.414$$

$$r_+ = 0.414 \times 241.5 = 99.98 = 100$$

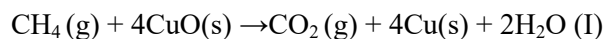
$$\text{when } \frac{r^+}{r^-} = 0.732; \frac{r^+}{241.5} = 0.732$$

$$r_+ = 0.732 \times 241.5 = 176.77$$

Sol.2

$$\log \frac{k_2}{k_1} = \frac{E_a}{2.303R} \left[\frac{1}{T_1} - \frac{1}{T_2} \right], \text{ since energy of activation is zero, rate constant should be the same.}$$

Sol.3



$$-7.49 \quad -4 \times 157.3 \quad \quad \quad -393.5 \quad -2 \times 285.9$$

$$-74.9 - 629.2 = -704.1 \quad -393.5 - 571.8 = -965.3$$

Enthalpy change of the reaction

$$= -965.3 - (-704.1) = -261.2$$

Sol.4

The addition of chlorine to the reaction mixture shifts the equilibrium backwards, thereby releasing more heat.

Sol. 5

Mole fraction of naphthalene = 0.1

Mole fraction of benzene = 0.9; amount of benzene = $0.9 \times 78 = 70.2$

$$\text{Molality of the solution} = 0.1 \times \frac{1000}{70.2} = 1.42m$$

Sol. 6

Polarisability of halide ions increases with the increase in size of the anion.

Sol.7

The gold number of a protective colloid is its minimum amount in milligrams which is just sufficient to prevent the coagulation of 10 ml of a gold sol on the addition of 1 mL of 10% sodium chloride solution. Amount in milligrams of 0.025g of starch is 25, so gold number is 25.

Sol.8

The total energy of a Bohr's orbit is proportional to $1/n^2$.

Sol. 9

$$K_p = K_c (RT)^{\Delta n}$$

$$\Delta n = 2-0=2, \text{ there, } K_p = k_c (RT)^2$$

Sol.10

EMF of cell = 1.229V

Sol. 11

$$0.00046100 = 4.6100 \times 10^{-4}$$

; number of significant figures =5

Sol. 12

For one nitrogen in the compound required % age of N = 14 10.5 g nitrogen is present in 100 g of compound 14 g nitrogen will be present in $\frac{100}{10.5} \times 14 = 133.3$

The minimum number molecular weight of the compound would be 133.3

Sol.13

$$\Delta T_b = 354.11 - 353.23 = 0.88$$

$$\Delta T_b = K_b \cdot \frac{w_2}{w_1} \times \frac{1000}{w_1}$$

$$0.88 = 2.53 \cdot \frac{1.8}{0.88} \times \frac{1000}{90} = 57.5$$

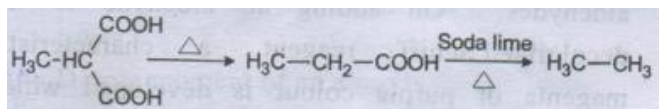
Sol.14

Option 4 is correct.

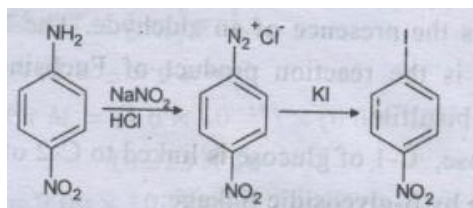
Sol.15

The formula for compound A suggests that it is a dicarboxylic acid. A β -keto acid or β -carboxyl acid readily undergoes decarboxylation on heating.

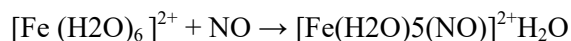
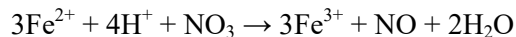
Therefore, A is $\text{CH}_3\text{CH}(\text{COOH})_2$



Sol.16

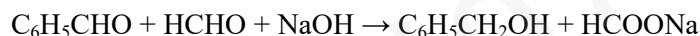


Sol.17



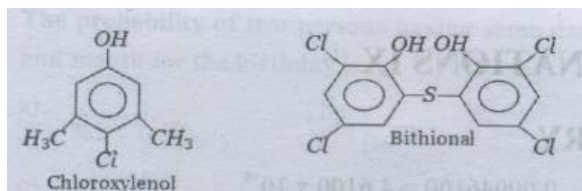
Sol. 18

It is a Crossed Cannizzarro reaction.



Sol. 19

Dettol is an antiseptic. It is a mixture of chloroxylenol and terpineol in a suitable solvent. Chloroxylenol has both antiseptic and disinfectant properties. **Bithional** is antiseptic which is generally added to medicated soaps to reduce the odour produced by bacterial decomposition of organic matter on the skin. **Iodine** is powerful antiseptic. It is used as a tincture of iodine which is 2-3% iodine solution of alcohol-water.

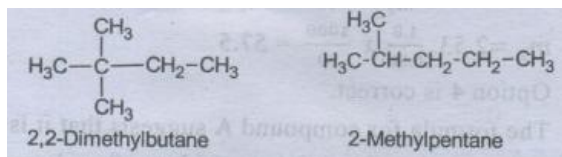
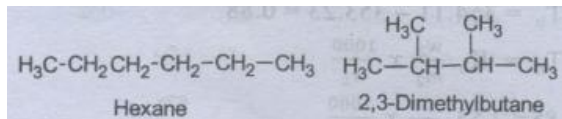


Sol. 20

HgS is not soluble in nitric acid

Sol.21

Hexane which contains only 2 different types of hydrogens will give two monochlorinated compounds. Out of the given choices only 2, 3- Dimethylbutane contains two types of hydrogens; Primary and secondary hydrogens.

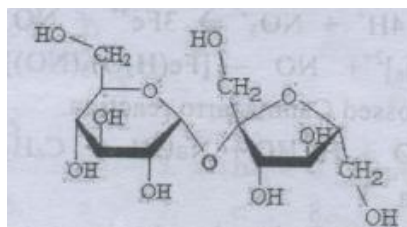


Sol.22

Schiff's reagent give pink colour with aldehydes only. It is a chemical test for the detection of aldehydes. On adding an aldehyde to the decolorized Schiff reagent a characteristic magenta or purple colour is developed which indicates the presence of an aldehyde. The Schiff reagent is the reaction product of Fuchsin and sodium bisulfite.

Sol.23

In sucrose, C-1 of glucose is linked to C-2 of fructose by β -glycosidic linkage.



Sol.24

Peptization is a process of passing of a precipitate into colloidal particles on adding suitable electrolyte. The electrolyte added is known as peptizing agent. A reddish brown coloured colloidal solution is obtained by adding small quantity of ferric chloride solution to the freshly precipitated ferric hydroxide.

Sol.25

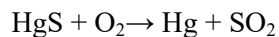
1.0 g of H_2 and 1.0 g of O_2 , contain different number of molecules at STP because their molecular weights are different.

Sol.26

Pb salts give gives a shining metallic bead in charcoal cavity test that marks paper.

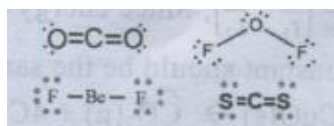
Sol. 27

The roasting of mercury sulphide in the air in metallurgical process produces mercury.



Sol. 28

OF_2 is a bent, non-linear molecule.

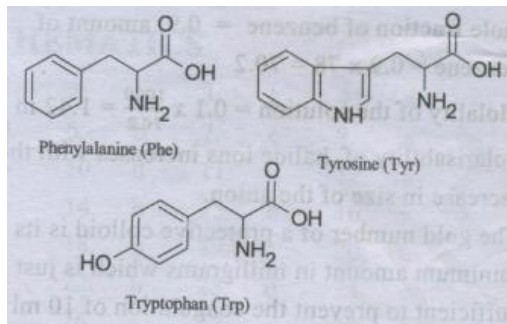


Sol. 29

A Ziegler- natta catalyst is a catalyst used in the synthesis of polymers of 1-alkenes (α -olefins). It uses organoaluminum compounds such as triethylaluminium $\text{Al}(\text{C}_2\text{H}_5)_3$.

Sol.30

When heated with conc. HNO_3 acid, proteins containing an aromatic ring give yellow ppt. On adding NaOH , the ppt turns orange. This is known as Xanthoprotein test.



PHYSICS

Sol.1

As magnetic moment = current \times area \Rightarrow [Magnetic Moment] = $[\text{AL}^2]$

Sol. 2

If v is the velocity of projection then kinetic energy = $\frac{1}{2}mv^2$ velocity of body at the highest point,
 $v_x = v\sin\theta$

\therefore K.E. of the body at the highest point is $\frac{1}{2}m(v\sin\theta)^2 = \frac{1}{2}mv^2 \sin^2\theta = (\text{K.E.})\sin^2\theta$ For $\theta=45^\circ$

$$\text{K.E.} = (\text{K.E.})\sin^2 45^\circ = \frac{\text{K.E.}}{2}$$

Sol.3

$$\text{As } \frac{mv^2}{r} = \mu R = \mu mg$$

$$\Rightarrow v^2 = \mu rg = 0.6 \times 150 \times 10 = 900$$

$$\Rightarrow v = \frac{30m}{s}$$

Sol.4

The momentum of third fragment is given by

$$P_3 = \sqrt{p_1^2 + P_2^2}$$

$$\text{or } p_3 = \sqrt{(mv)^2 + (mv)^2} = \sqrt{2}mv$$

Final kinetic energy of system

$$\frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{p_3^2}{2(2m)}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{3}{2}mv^2$$

Since initial kinetic energy is zero, therefore energy released = $\frac{3}{2}mv^2$

Sol.5

$$\text{Given } X_{cm} = \frac{1}{M} \int_0^1 x \, dm = \frac{1}{M} \int_0^1 x (\mu dx)$$

$$= \frac{1}{M} \int_0^1 x (ax) dx = \frac{a}{M} \int_0^1 x^2 dx = \frac{al^3}{3M}$$

$$\text{Also } M = \int_0^1 \mu dx \int_0^1 ax dx = \frac{al^2}{2}$$

$$\Rightarrow X_{cm} = \frac{al^3 \times 2}{3(al^2)} = \frac{2}{3}l$$

Sol.6

Both the statements are self explanatory.

Sol.7

As stress = $\frac{F}{A} \therefore$ Ultimate shear stress < F/A

$\Rightarrow F >$ shear stress \times Area as Area = $2\pi \frac{d}{2} yt$ Or $F = \pi d \sigma t$

Sol. 8

Here $\frac{\Delta Q}{\Delta t} \in \sigma AT^4$, also $\Delta Q = m C \Delta T$

Dividing

$$\therefore \frac{\Delta T}{\Delta t} = \frac{\epsilon \sigma AT^4}{mC} \quad \text{Now Area, } A = \pi r^2 = \pi \left(\frac{3m}{4\pi\delta}\right)^{2/3}$$

$$\therefore m = \frac{4}{3} \pi r^3 \delta$$

$$\frac{\Delta T}{\Delta t} = \frac{\epsilon \sigma T^4}{mC} \left[\pi \left(\frac{3m}{4\pi\delta}\right)^{2/3} \right]$$

$$= k \left(\frac{1}{3}\right)^{1/3} \quad \text{where } k \text{ is a constant}$$

$$\Rightarrow \frac{\frac{\Delta T_1}{\Delta t_1}}{\frac{\Delta T_2}{\Delta t_2}} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

Sol.9

$$\text{Using } V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{rms} \text{ (at } 120^0) = V = \sqrt{\frac{3R}{M} \times 120}$$

$$V_{rms} \text{ (at } 480^0) = \sqrt{\frac{3R}{M} 480}$$

$$V_{rms} = \sqrt{\frac{480}{120}} = 2$$

$$\Rightarrow V_{rms} = 2V$$

Sol.10

$$\text{As } f' = \frac{300+V}{300} f$$

$$\text{And } f'' = \frac{300-V}{300} f \Rightarrow f' - f'' = \frac{2}{100} f$$

$$\text{i.e. } \frac{300+V}{300} - \frac{300-V}{300} = \frac{2}{100} \text{ or } 2V = 6 \Rightarrow V=3\text{m/s}$$

Sol.11

$$\text{As } I = \frac{1}{2} \rho A^2 \omega^2 c^2$$

$$I = \frac{1}{2} \times (1.3) \times (10^{-8})^2 (2\pi \times 2000)^2 (340)$$

$$= 3.5 \times 10^{-6} \text{ w/m}^2$$

Sol.12

\vec{E} at all point on the y-axis is along \hat{i} . \vec{E} at all points on x-axis cannot have the same direction.

Sol.13

Resistance of wire becomes x^2 times and not x times.

Sol.14

Dipole moment of an atom is

$$M = I \times A = \frac{e}{T} \pi r^2$$

$$\Rightarrow M = e \times v \times \pi r^2$$

$$\text{Or } M = (1.6 \times 10^{-19}) \times (6.6 \times 10^{15}) \times 3.142 \times (0.523 \times 10^{-10})^2$$

$$= 9.06 \times 10^{-24} \text{ Am}^2$$

Sol.15

Using $\tau = n/AB \sin \Theta$, here Θ is angle between magnetic induction and normal to the surface to loop.

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 0.02 \times 0.02 \sin 60 = 1.732 \times 10^{-4}$$

$$\Rightarrow \tau = 1 \times 0.1 \times 1.732 \times 10^{-4} \times 5 \times 10^{-2} \sin 90$$

$$= 8.66 \times 10^{-7} \text{ Nm}$$

Sol.16

As per Lenz's law induced emf opposes the cause producing it.

Sol.17

Using $E \propto N$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} \Rightarrow E_s = \frac{N_s}{N_p} E_p$$

$$\Rightarrow E_s = \frac{5}{1} \times 200 = 1000V$$

Sol.18

$$\text{As } B = \frac{E}{c} = \frac{9.3}{3 \times 10^8} = 3.1 \times 10^{-8} T$$

Sol.19

As normal to mirror 2 is parallel to mirror 1, the reflected ray is parallel to the ray incident on mirror 1 and is in opposite direction.

Sol.20

$$\text{Using } \beta = \frac{\lambda D}{d} \text{ and } \theta = \frac{\beta}{D}$$

$$\text{i.e. } \beta = D\theta \Rightarrow D\theta = \frac{\lambda D}{d} \text{ or } \theta = \frac{\lambda}{d}$$

$$\text{with lens } \theta = \frac{\beta'}{f} \text{ or } \frac{\beta'}{f} = \frac{\lambda}{d}$$

$$\Rightarrow \beta' = \frac{\lambda f}{d} = \frac{5890 \times 10^{-10} \times 1}{0.2 \times 10^{-3}} = 3mm$$

Sol.21

For coherence to be observed, same frequency is the essential requirement. Different wavelengths give different frequencies. The lights of different intensities can give coherence, however the contrast of fringes may be poor.

Sol. 22

Both the statements are self explanatory.

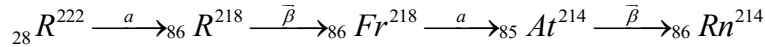
Sol.23

$$\text{Given that Energy in a day} = (1 \times 10^6) (24 \times 60 \times 60) J$$

$$\text{Energy per fission} = 3.2 \times 10^{-11} J \text{ Mass of uranium required in a day}$$

$$\frac{1 \times 10^6 (24 \times 60 \times 60) (235 \times 1.67 \times 10^{-27})}{3.2 \times 10^{-11}}$$

$$= 1.05 \times 10^{-3} \text{ kg} = 1.05g$$

Sol.24


There are 4 α decays and 2 β decays

Sol.25

The width of depletion region increases with increase in reverse voltage in a diode.

Sol. 26

$$\text{Using } \mu = \frac{A_m}{A_c} \Rightarrow A_m = \mu \times A_c = \frac{75}{100} \times 12 = 9V$$

Sol. 27

If $(b-a) \geq r$, radius of the circular path of particle. The particle cannot enter the region $x > b$

To enter in the region $x > b$

$$r > (b-a) \Rightarrow \frac{mV}{Bq} > (b-a) \Rightarrow V > \frac{q(b-a)}{m} B$$

Sol. 28

$$\text{As } y = kt^2, \frac{d^2y}{dt^2} = 2k \text{ as } k = 1 \frac{m}{s^2}$$

$$\text{Or } y = 2 \text{ m/s}^2$$

$$T_1 = 2\pi \sqrt{\frac{l}{g}} \text{ and } T_2 = 2\pi \sqrt{\frac{1}{g+ay}} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{g+ay}{g} = \frac{10+2}{10} = \frac{6}{5}$$

Sol.29

$$W_{\text{apparent}} = W_{\text{actual}} - U_{\text{pthrust}}$$

$$U_{\text{pthrust, F} = v_s \delta L' g} \Rightarrow \frac{F'}{F} = \frac{V_s^1 \delta_L'}{V_s \delta_L} = \frac{(1+\gamma s \Delta \theta)}{(1+\gamma L \Delta \theta)}$$

$$\text{As } \gamma < \gamma_L \Rightarrow F' < F \text{ or } W_{\text{apparent}} > W_{\text{actual}}$$

Sol. 30

Using Wien's Displacement Law $\lambda_m T = \text{constant}$

$$T = \frac{1}{\lambda_m}$$

$$\therefore \frac{T_{\text{sun}}}{T_{\text{nort } h_{\text{star}}}} = \frac{(\lambda_m)_{\text{nort } h_{\text{star}}}}{(\lambda_m)_{\text{sun}}} = \frac{350}{510} = 069$$

MATHEMATICS

Sol. 1

$$\alpha + \beta + \gamma = \frac{\pi}{2} \Rightarrow \alpha + \gamma = \frac{\pi}{2} - \beta$$

$$\cot(\alpha + \gamma) = \cot\left(\frac{\pi}{2} - \beta\right)$$

$$\frac{\cot \alpha \cot \gamma - 1}{\cot \alpha + \cot \gamma} = \tan \beta$$

$$\Rightarrow \cot \alpha \cot \gamma - 1 = \tan \beta (\cot \alpha + \cot \gamma)$$

Given that $\cot \alpha, \cot \beta, \cot \gamma$ are in A.P. therefore, $\cot \alpha \cot \gamma - 1 = 2 \tan \beta \cot \beta \Rightarrow \cot \alpha \cot \gamma = 3$

Sol.2

Given that $\cos 2x + a \sin x = 2a - 7$ posses a solution

$$1 - 2 \sin^2 x - a \sin x = 2a - 7$$

$$\Rightarrow 2 \sin^2 x - a \sin x + (2a - 8) = 0$$

$$\Rightarrow \sin x = \frac{a \pm \sqrt{a^2 - 8(2a - 8)}}{4}$$

$$\Rightarrow \sin x = \frac{a-2}{2}, 2 \quad [\because -1 \leq \sin x \leq 1]$$

$$\Rightarrow -1 \leq \frac{a-2}{2} \leq 1 \quad \Rightarrow 0 \leq a \leq 4$$

Number of integral values of a are 5

Sol.3

$$\text{Given } \sin^{-1} x + \tan^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1} x = \cos^{-1} x = \tan^{-1} \frac{\sqrt{1-x^2}}{x}$$

$$\Rightarrow x = \frac{\sqrt{1-x^2}}{x}$$

$$\Rightarrow x^2 = \sqrt{1-x^2} \Rightarrow x^4 = (1-x^2)$$

$$\Rightarrow x^4 + x^2 - 1 = 0 \quad \Rightarrow x^2 = -\frac{1 \pm \sqrt{5}}{2}$$

$$x^2 = \frac{\sqrt{5}-1}{2} \quad [\text{because } x^2 > 0]$$

$$\therefore 2x^2 + 1 = \sqrt{5}$$

Sol.4

The point P is on the line parallel to $x - y = 3$ and passing through A(2, 1).

$$\frac{x-2}{1/\sqrt{2}} = \frac{y-1}{1/\sqrt{2}} \pm 4 \quad (\text{because distance AP} = 4)$$

$$\therefore P \text{ is either } (2\sqrt{2} + 2, 2\sqrt{2} + 1) \text{ or } (-2\sqrt{2} + 2, -2\sqrt{2} + 1)$$

\therefore P lies in third quadrant

$$\therefore P \text{ is } (-2\sqrt{2} + 2, -2\sqrt{2} + 1)$$

Sol.5

Centre of circle is (0, 0) and radius = 3. If line $x + y \tan \theta = \cos \theta$ touches the circle, then perpendicular distance from (0, 0) to the line is equal to radius of circle $\frac{|0+0-\cos \theta|}{\sqrt{1^2+\tan^2 \theta}} = a$

$$\therefore \cos^2 \theta = a \text{ which is impossible}$$

Sol.6

Let PQ be a focal chord and $P = (at^2, 2at) Q = \left(\frac{a}{t^2}, -\frac{2a}{t}\right)$

$$\therefore PQ = a \left(t + \frac{1}{t}\right)^2 \text{ and } \tan \alpha = \frac{2}{t - \frac{1}{t}}$$

$$\therefore \cot \alpha = \frac{1}{4} \left(t - \frac{1}{t}\right)$$

$$\Rightarrow \cot^2 \alpha + 1 = \frac{1}{4} \left(t + \frac{1}{t}\right)^2$$

$$\Rightarrow \frac{PQ}{4a} = \frac{a \left(t + \frac{1}{t}\right)^2}{4a}$$

$$\frac{1}{4} \left(t + \frac{1}{t}\right)^2 = \operatorname{cosec}^2 \alpha$$

Sol.7

Given $f(x) = e^x$

$\therefore x_1, x_2, x_3$ from G.P. Therefore

$$x_1 + x_3 = 2x_2 \Rightarrow e^{x_1+x_3} = e^{2x_2}$$

$$\Rightarrow e^{x_1} \cdot e^{x_3} = (e^{x_2})^2 \Rightarrow f(x_1), f(x_3) = (f(x_2))^2$$

Therefore $f(x_1), f(x_2), f(x_3)$ are in G.P.

Sol.8

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{(\cos x)^{\frac{1}{2}} - (\cos x)^{\frac{1}{3}}}{\sin^2 x} &= \lim_{x \rightarrow 0} \frac{\left(\frac{1}{2}\right)(\cos x)^{-\frac{1}{2}}(-\sin x) - \left(\frac{1}{3}\right)(\cos x)^{-\frac{2}{3}}(-\sin x)}{2 \sin x \cos x} \\ &= \lim_{x \rightarrow 0} \frac{-\frac{1}{2}(\cos x)^{-\frac{1}{2}} + \frac{1}{3}(\cos x)^{-\frac{2}{3}}}{2 \cos x} = \frac{1}{2} \left(\frac{-1}{2} + \frac{1}{3} \right) = -\frac{1}{12} \end{aligned}$$

Sol.9

Given that $f(x) = |\cos 2x|$

$$\begin{aligned} f' \left(\frac{\pi}{4} \right) &= \lim_{h \rightarrow 0} \frac{f \left(\frac{\pi}{4} + h \right) - f \left(\frac{\pi}{4} \right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{|\cos 2 \left(\frac{\pi}{4} + h \right)| - |\cos 2 \left(\frac{\pi}{4} \right)|}{h} = \lim_{h \rightarrow 0} \frac{|\cos \left(\frac{\pi}{4} + 2h \right)| - |\cos \frac{\pi}{2}|}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin 2h}{h} = 2 \end{aligned}$$

Sol.10

Given that $y = \tan^{-1} \sqrt{1 - x^2}$

At $x = 4$, $\sqrt{1 - x^2}$ becomes imaginary

$\therefore y$ does not exist.

Sol.11

Given $y^2 = x(2 - x)^2$

Differentiating both sides w. r. t. x , we get

$$2y \frac{dy}{dx} = 3x^2 - 8x + 4 \Rightarrow \left(\frac{dy}{dx} \right)_{(1,1)} = -\frac{1}{2} \text{ Equation to tangent at } (1,1) \text{ is}$$

$$y - 1 = \left(-\frac{1}{2} \right) (x - 1) \Rightarrow y = -\frac{x}{2} + \frac{3}{2}$$

$$\Rightarrow y^2 = \frac{1}{4} (3 - x)^2$$

$$\therefore x(2 - x)^2 = \frac{1}{4} (3 - x)^2 \Rightarrow (x - 1)^2 (4x - 9) = 0$$

$$\Rightarrow x = \frac{9}{4}$$

$$x = \frac{9}{4} \Rightarrow y = \frac{3}{8}$$

Sol.12

$$\begin{aligned}
 I &= \int \frac{\sin x}{\sin 4x} dx = \int \frac{\sin x}{2 \sin 2x \cos 2x} dx \\
 &= \int \frac{\sin x}{4 \sin x \cos x \cos 2x} dx \\
 &= \frac{1}{4} \int \frac{\cos x}{(1-\sin^2 x)(1-2\sin^2 x)} dx \text{ Put } \sin x = t \Rightarrow \cos x dx = dt \\
 &= \frac{1}{4} \int \frac{dt}{(1-t^2)(1-2t^2)} = \frac{1}{4} \int \left(\frac{2}{1-2t^2} - \frac{1}{1-t^2} \right) dt \\
 &= \frac{1}{4} \left\{ \frac{2}{2\sqrt{2}} \log \left| \frac{1+\sqrt{2}t}{1-\sqrt{2}t} \right| - \frac{1}{2} \log \left| \frac{1+t}{1-t} \right| \right\} + c \\
 &= \frac{1}{4\sqrt{2}} \log \left| \frac{1+\sqrt{2} \sin x}{1-\sqrt{2} \sin x} \right| - \frac{1}{8} \log \left| \frac{1+\sin x}{1-\sin x} \right| + c \\
 &= \frac{1}{8} \log \left| \frac{\sin x - 1}{\sin x + 1} \right| - \frac{1}{4\sqrt{2}} \log \left| \frac{\sqrt{2} \sin x - 1}{\sqrt{2} \sin x + 1} \right| + c
 \end{aligned}$$

Sol.13

Let $I_1 = \int_0^1 \frac{e^x}{x+1} dx = a$ and $I_2 = \int_{b-1}^b \frac{e^{-x}}{x-b-1} dx$

Put $x = b - y \Rightarrow dx = -dy$

$$\begin{aligned}
 I_2 &= - \int_1^0 \frac{e^{-b+y}}{-y-1} dy \\
 &= -e^{-b} \int \frac{e^y dy}{y+1} = -e^{-b} a
 \end{aligned}$$

Sol.14

Let $\lim_{n \rightarrow \infty} \frac{(n!)^{\frac{1}{n}}}{n} = a \Rightarrow a = \lim_{n \rightarrow \infty} \left(\frac{n!}{n^n} \right)^{\frac{1}{n}} \Rightarrow a = \text{Lt}_{n \rightarrow \infty} \left(\frac{1}{n} \cdot \frac{2}{n} \cdot \frac{3}{n} \cdot \frac{4}{n} \dots \frac{n}{n} \right)^{\frac{1}{n}}$ Taking log on both sides, we get

$$\log a = \text{Lt}_{n \rightarrow \infty} \frac{1}{n} \left[\log \frac{1}{n} + \log \frac{2}{n} + \log \frac{3}{n} + \dots + \log \frac{n}{n} \right]$$

$$= \text{Lt}_{n \rightarrow \infty} \sum_{r=1}^n \log \left(\frac{r}{n} \right)^{\frac{1}{n}}$$

$$= \int_0^1 \log x dx = [x \log x - x]_0^1$$

$$= -1 \therefore a = e^{-1}$$

Sol.15

Given that $\vec{a}, \vec{b}, \vec{c}$ are linear dependent vectors.

$$\therefore |\vec{a}\vec{b}\vec{c}|=0$$

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

Using $c_2 \rightarrow c_1, c_3 \rightarrow c_3 - c_1$ transformation

$$\begin{vmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 1 & \alpha - 1 & \beta - 1 \end{vmatrix} = 0$$

$$\Rightarrow -(\beta - 1) = 0 \quad \Rightarrow \beta = 1$$

Given $|c| = \sqrt{6}$

$$\therefore \sqrt{1 + \alpha^2 + \beta^2} = \sqrt{6}$$

$$\Rightarrow 1 + \alpha^2 + \beta^2 = 6$$

$$\Rightarrow 1 + \alpha^2 + 1 = 6$$

$$\Rightarrow \alpha^2 = 4$$

$$\Rightarrow \alpha = \pm 2$$

Sol.16

Given that $(a + ib)(c + id) = (a^2 + b^2)\hat{i}$

$$ac + iad + ibc - bd = (a^2 + b^2)\hat{i}$$

$$(ac - bd) + i(ad + bc) = (a^2 + b^2)\hat{i}$$

$$\Rightarrow ac - bd = 0 \text{ \& } ad + bc = a^2 + b^2$$

From $ad + bc = a^2 + b^2$, it is obvious that $a = d$ and $b = c$

Sol. 17

Given that

$$f(x, y, z) = x^2 + 4y^2 + 3z^2 - 2x^2 - 12y - 6z + 15 = (x - 1)^2 + (2y - 3)^2 + 3(z - 1)^2 + 2$$

\therefore Least value of $f(x, y, z)$ is obtained at $x=1, y=3/2, z=1$

\therefore Least value of $f(x, y, z) = 2$

Sol. 18

$1 + (1 + 3) + (1 + 3 + 5) \dots$ To n terms

$= 1 + 4 + 9 + \dots$ To n terms

$= 1^2 + 2^2 + 3^2 + \dots$ to n terms

$$= \frac{n(n+1)(2n+1)}{6}$$

Sol. 19

Let a, ar, ar² be in G. P.

Let $T_1 = a + ar$, $T_2 = ar + ar$, $T_3 = ar^2 + ar$

$$\frac{2T_1T_3}{T_1 + T_3} = \frac{2(a + ar)(ar^2 + ar)}{(a + ar) + (ar^2 + ar)}$$

$$= \frac{2a^2(1+r)ar(1+r)}{a(1+r+ar+r^2)}$$

$$= \frac{2a^2r(1+r)^2}{a(1+r)^2} = 2ar = T_2$$

$\therefore T_1, T_2, T_3$ are in H.P.

Sol.20

$$180 = 2^2 \cdot 3^2 \cdot 5^1$$

Sum of division of 180

$$= (1 + 2 + 2^2) (1 + 3 + 3^2) (1 + 5)$$

$$= 7 \times 13 \times 6 = 546$$

Sol. 21

The required number of ways are

$$3! \left\{ 1 - \frac{1}{1!} + \frac{2}{2!} - \frac{3}{3!} \right\}$$

$$= 3! \left\{ \frac{2}{2!} - \frac{3}{3!} \right\} = 3 - 1 = 2$$

Sol.22

The required number of four digits number are

$$= 3 \times 4 \times 4 \times 4 = 192$$

Sol.23

$$\begin{aligned} \left(x^3 + 1 + \frac{1}{x^3}\right)^n &= \left[1 + \left(x^3 + \frac{1}{x^3}\right)\right]^n \\ &= {}^n C_0 + {}^n C_1 \left(x^3 + \frac{1}{x^3}\right) + {}^n C_2 \left(x^3 + \frac{1}{x^3}\right)^2 \dots + {}^n C_n \left(x^3 + \frac{1}{x^3}\right)^n \end{aligned}$$

Here, all the terms all positive and will contain powers

$$(x^3)^0, (x^3)^1, (x^3)^2 \dots, (x^3)^n, (x^3)^{-1}, (x^3)^{-2}, \dots, (x^3)^{-n}$$

Hence number of terms = $2n + 1$

Sol.24

$$\begin{aligned} \Delta &= \begin{vmatrix} al & bm & cn \\ cm & an & bl \\ bn & cl & am \end{vmatrix} \\ &= lmn(a^3 + b^3 + c^3) - abc(l^3 + m^3 + n^3) \end{aligned}$$

Given that $a + b + c = 0 = 1 + m + n$

$$\therefore a^3 + b^3 + c^3 = 3abc$$

Therefore $\Delta = lmn(3abc) - abc(3lmn) = 0$

Sol. 25

The required no. of ways = $2^{n-1} - 1$

Sol.26

Total number of cases = $(365)^2$ Favourable number of cases = ${}^{361} C_1 = 365$

$$\text{Required probability} = \frac{365}{(365)^2} = \frac{1}{365}$$

Sol.27

$$\text{Required probability} = \frac{2!6!6!}{12!} = \frac{1}{462}$$

Sol. 28

Given $A \cup \{2, 4, 6\} = \{2, 4, 6, 8, 10\} \therefore$ smallest $A = \{8, 10\}$

Largest $A = \{2, 4, 6, 8, 10\}$

$$n(A) = m \Rightarrow 2 \leq m \leq 5$$

Sol.29

a), (b), (d) are correct (c) is incorrect

Sol.30

$$\begin{aligned}
 S &= \frac{1}{2} \left(\frac{1}{6}\right)^2 + \left(\frac{2}{3}\right) \left(\frac{1}{6}\right)^3 + \left(\frac{3}{4}\right) \left(\frac{1}{6}\right)^4 + \dots \\
 &= \left(1 - \frac{1}{2}\right) \left(\frac{1}{6}\right)^2 + \left(1 - \frac{1}{3}\right) \left(\frac{1}{6}\right)^3 + \left(1 - \frac{1}{4}\right) \left(\frac{1}{6}\right)^4 + \dots \\
 \Rightarrow S &= \left[\left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^4 + \dots \right] \\
 &\quad - \left[\frac{1}{2} \left(\frac{1}{6}\right)^2 + \frac{1}{3} \left(\frac{1}{6}\right)^3 + \frac{1}{4} \left(\frac{1}{6}\right)^4 + \dots \right] \\
 \Rightarrow S &= \left[\left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^4 \right] \\
 &\quad - \left[\left\{ \frac{1}{6} + \frac{1}{2} \left(\frac{1}{6}\right)^2 + \frac{1}{3} \left(\frac{1}{6}\right)^3 + \dots \right\} - \frac{1}{6} \right] \\
 \Rightarrow S &= \frac{\left(\frac{1}{6}\right)^2}{1 - \frac{1}{6}} + \log \left[1 - \frac{1}{6} \right] + \frac{1}{6} \\
 \Rightarrow S &= \frac{1}{30} + \frac{1}{6} + \log \frac{5}{6} \\
 \Rightarrow S &= \frac{6}{30} + \log \frac{5}{6} \cdot \text{mxx}
 \end{aligned}$$