ANSWERS XI

CHEMISTRY

1.d	2.a	3.b	4.d	5.d	6.c	7.c	8.a	9.c	10.b	11.b	12.c	13.a
14.d	15.b	16.c	17.a	18.b	19.c	20.a	21.d	22.c	23.b	24.d	25.a	26.c
27.d	28.a	29.d	30.c									

PHYSICS

1.d	2.b	3.c	4.b	5.c	6.c	7.b	8.d	9.b	10.c	11.c	12.a	13.d
14.b	15.a	16.a	17.a	18.b	19.a	20.a	21.d	22.c	23.c	24.d	25.c	26.d
27.a	28.b	29.a	30.b									

MATHEMATICS

1.d	2.a	3.b	4.b	5.d	6.c	7.d	8.b	9.a	10.a	11.b	12.b	13.a
14.c	15.b	16.d	17.c	18.a	19.d	20.c	21.a	22.d	23.b	24.d	25.a	26.d
27.b	28.c	29.d	30d									

HINTS AND EXPLANATIONS XI

CHEMISTRY

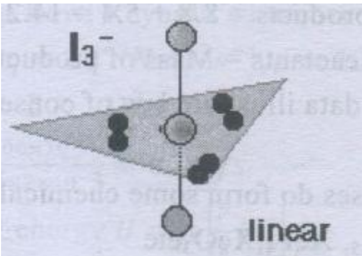
Sol.1

10% solution means 10g in 100 mL; 1 mol of glucose = 180 g

10 g of glucose is present in 100 mL 180 g of glucose will be present in $\frac{100 \times 180}{10}$ =1800 mL or 1.8 L

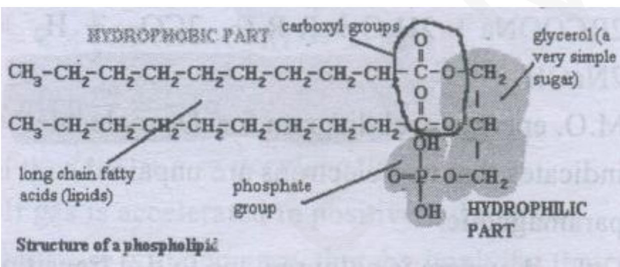
Sol.2

Formation of I_3 involves sp^3d hybridization.



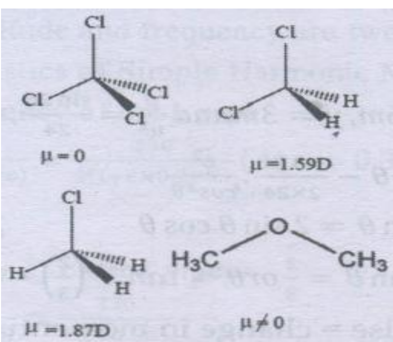
Sol.3

Structure of a phospholipod is shown below



Sol.4

Dipole moment of CCl_4 is zero.



Sol.5

Microwave has lowest frequency, γ-rays has highest frequency (lowest wavelength). Decreasing order of frequency: γ-rays > X-rays> visible> microwave.

Sol.6

In the given reaction oxidation state of Cr changes from +6→ +3, i.e., 3 electrons per Cr atom. For $Cr_2O^2{}_7$, the oxidation state changes by 6, therefore its equivalent wt = M/6

Sol.7

The formula of potassium dicyanobis(oxalate) nickelate (11) is $K_4[Ni(CN)_2(Ox)_2]$

Sol.8

$BaCrO_4 \rightleftharpoons Ba^{2+} + CrO_4^{2-}$

$K_{sp} = [Ba^{2+}][CrO_4^{2-}]$

$2.4 \times 10^{-10} = [Ba^{2+}] \times 6 \times 10^{-4}$

$[Ba^{2+}] = 0.4 \times 10^{-6} = 4 \times 10^{-7}$

Sol.9

Q = I x t = 1 x 60 = 60 C

96500 c delivers 1 mol or 6.023×10^{23} electrons at cathode

60 C will deliver electrons = $\frac{6.023 \times 10^{20}}{96500} \times 60 = 3.74 \times 10^{20}$

Sol.10

The given reaction $Cu_2O + FeS \rightarrow FeO + Cu_2S$takes place during smelting

Sol.11

pH of a buffer solution is given as

$pH = pKa + \log\frac{[salt]}{[acid]}$

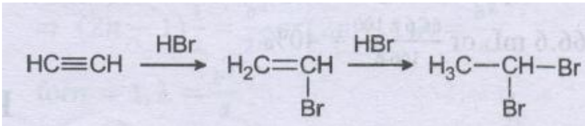
$= 4.75 + \log\frac{0.1}{0.1} = 4.75$

Sol.12

$2NO + O_2 \rightarrow 2NO_2$ (brown coloured gas)

Sol.13

Addition of HBr on acetylene gives ethylidene bromide



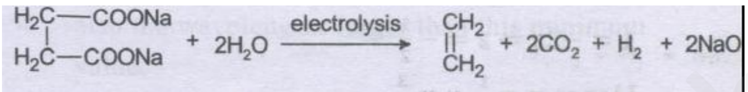
Sol.14

Number of electrons in the given species are :

$C1^{-} = 2,8,8$; $F^{-} = 2,8$; $Na^{+} = 2,8$; $Mg^{2+-} = 2,8$

Sol.15

Electrolysis of sodium salt of succinic cid gives ethylene.



Sol.16

All the elements of lanthanide and actinide series are not radioactive

Sol.17

Natural gas is a mixture of gaseous paraffins

Sol.18

Action of heat on mixture of anhydrous sodium propanoate and soda lime produces ethane.

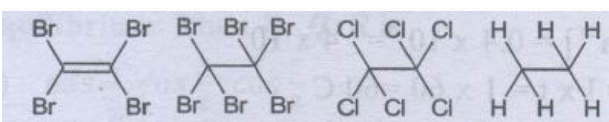
$$\mathrm{H_3C - CH_2 - COONa + Naoh \xrightarrow[heat]{cao} H_3C - CH_3 + \ Na_2CO}$$

Sol.19

The presence of electron withdrawing atoms or groups atv the a-carbon of acarboxylic acid increase its acidic. Fis more electronegative than both Br and Cl.

Sol.20

From the options given the rotaion about C= C in 1,1,2,2 – tetrabromoethylene is most stercically hindered.



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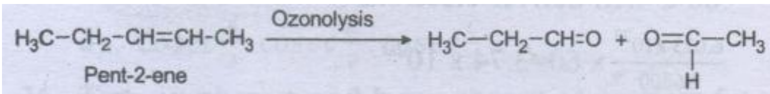
Sol.21

Pent-2-ene on ozonolysis gives CH_3CH_2CHO and CH_3CHO.



Sol.22

n −hexane can be prepared by Wurtz reaction of n- propyl bromide $CH_3CH_2CH_2Br + 2Na + BrCH_2CH_2CH_3 \rightarrow CH_3CH_2CH_2CH_2CH_2CH_3$

Sol.23

Fog is a colloidal system pf liquid in a gas

Sol.24

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ or $V_2 = \frac{V_1}{T_1} T_2 = \frac{Vx500}{300} = \frac{5V}{3}$;

If V = 100 mL then $V_2 = \frac{500}{300} = 166.6$ mL

Air escaped during heating = 66.6 mL or $\frac{66.6 \ x \ 100}{166.6} = 40\%$

Sol.25

Compounds A + $O_2 \rightarrow CO_2 + H_2O$ 3g? 8.8 g 5.4 g

8.8 g CO_2 = 0.2 mol; 5.4 g H_2O = 0.3 mol.

Number of moles of oxygen required to produce 0.2 mol of CO_2 and 0.3 moles of H_2O can be calculated from the balanced equation.

Compound A + $7/2 0_2 \rightarrow 2CO_2 + 3H_2O$

3g	0.35 mol	0.2 mol	0.3 mol
3g	11.2 g	8.8 g	5.4 g

Mass of reactants = 3 + 11.2 = 14.2 g

Mass of products = 8.8 + 5.4 = 14.2 g

Mass of reactants = Mass of products

Thus the data illustrate law of conservation of mass.

Sol.26

Noble gases do form some chemical compounds, e.g., XeF_5, Xef_4, XeO_3 etc

Sol.27

CaF_2 is ionic solid; H_2O and CI_2 are molecular solids ;SiC is a covalent solid.

Sol.28

$2RCOONa + 2H_2O \rightarrow R\text{-}R + 2CO_2 + H_2 + 2NaOH$

Sol.29

M. O. energy level diagram for O_2 molecule indicates that two electrons are unpaired so it is paramagnetic.

Sol.30

CuF_2, d^9 system is coloured due to d-d transitions

PHYSICS

Sol.1

$[T] \propto [\rho]^x [r]^y [s]^z$

$[T] \propto [ML^{-3}]^x [L]^y [MT^{-2}]^z$

$\Rightarrow 0 = x + z$

$0 = -3x + y$

$1 = 2z \Rightarrow z = -\frac{1}{2}$

Hence $x = \frac{1}{2}, y = \frac{3}{2}$

$\therefore T = \left[\frac{\rho r^3}{5}\right]^{1/2}$

Sol.2

Using $R = \frac{u^2 \sin 2\theta}{2g}$ we ge

$\frac{g}{u^2} = \frac{\sin 2\theta}{R}$

As range $R = 6 + 18 = 24$m

$\therefore \frac{g}{u^2} = \frac{\sin 2\theta}{24}$

Again $y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta}$

For $x = 6m, y = 3m and \frac{g}{u^2} = \frac{\sin 2\theta}{24}$, we get $3 = 6\tan\theta - \frac{\sin 2\theta}{2x24} \cdot \frac{6^2}{\cos^2\theta}$

Again sin θ = 2 sin θ cos θ

We get tan θ = $\frac{2}{3} or\ \theta = \tan^{-1}\left(\frac{2}{3}\right)$

Sol.3

As impulse = change in momentum

$\therefore$ Force of machine gun = $\frac{40}{1000} \times 1200 = 48/N$

Force of Man = $144N \Rightarrow No.\ of\ bullets = \frac{144}{48} = 3$

Sol.4

As $K \propto \frac{1}{L} \Rightarrow K' \times \frac{2L}{3} = KL$ or $K' = \frac{3}{2}K$

Sol.5

No horizontal force will act on the rod as the surface is smooth. The only vertical forces acting on it are its own weight and normal reaction. Therefore centre of mass should fall vertically downwards towards negative y-axis. The path will be a straight line.

Sol.6

Total workdone = 4 × Potential energy along sides + 2×Potential energy along diagonals

$= 4 \times \left[-\frac{Gm_2m_2}{0.2}\right] + 2\left[-\frac{Gm_1m_2}{0.2\sqrt{2}}\right]$

$= 4 \times \left[-\frac{(6.67\times10^{-11})(0.1)^2}{0.2}\right] + 2\left[-\frac{(6.67\times10^{-11})(0.1)^2}{0.2\sqrt{2}}\right]$

$= -1.33 \times 10^{-11} - 0.47 \times 10^{-11} = -1.8 \times 10^{-11} J$

Sol.7

As we know $V_c = R\frac{n}{\rho D}, R = Reynold'snumber$

For laminar flow, Reynold's number

$R = 2000, \eta = 10^{-3}Nm^{-2}s^{-1}, \rho = 10^3 kgm^{-3}$

$D = 2cm = 2 \times 10^{-2}m$

$\Rightarrow V_c = \frac{2000\times10^{-3}}{10^3\times2\times10^{-2}} = 0.1m/s$

Sol.8

As internal energy $U = n\left[\frac{F}{2}RT\right]$

$F = degreesof\ freedom$

Given $U = V_0 + U_{Ar}$

$= 2 \times \frac{5}{2}RT + 4 \times \frac{3}{2}RT = 11RT$

Sol.9

Given $\frac{d\delta}{dx} = -\delta a$

δ = densiy, a = acceleration

If gas is accelerated in positive x direction, pressure will decrease, thereby implying that pressure is lower on the front side.

Sol.10

The amplitude and frequency are two independent characteristics of simple Harmonic Motion.

Sol.11

The frequency of open pipe

$n_1 = \frac{v}{2(l+2e)} = \frac{330}{2(l+2\times0.3\times d)} (Ase = 0.3d) \ = \frac{330}{2(l+0.6d)}$

Frequency of closed pipe

$n^2 = \frac{v}{4(l+e)} = \frac{330}{4(l+0.3d)}$

$\Rightarrow \frac{n_2}{n_1} = \frac{2(l+0.6d)}{4(l+0.3d)} = \frac{1(l+0.6d)}{2(l+0.3d)}$

Sol.12

From the relation

$kinetic\ Energy = \ q\Delta V$, we get

$K.E. = 2 \times 1.6 \times 10 \times^{-19} (70 - 50)J = 40eV$

Sol.13

Making use of relation

$V = IR \Rightarrow \frac{V}{R}$

Here $V = 15V$ and $R = 4 + 6 + 10 + 2 = 22\Omega$

$\therefore I = \frac{15}{22} — = 0.6A i.e.r = 5\Omega$

Sol.14

Magnetic field at the middle of solenoid

$B = \mu_0 nI = \mu_0 \frac{N}{L} I = 4\pi \times 10^{-7} \times \frac{500}{0.4} \times 3 = 4.713 \times 10^{-3} T$

Magnetic dipole moment of the coil

$M = NIA = NI\pi r^2 = 10 \times 0.4 \times 3.142 \times (0.01)^2 = 1.26 \times 10^{-3} Am^2$

Torque acting on the coil

$\tau = MB \sin\theta As\theta = 90^0$

$\tau = MB = 1.26 \times 10^{-3} \times 4.713 \times 10^{-3} = 5.94 \times 10\text{^} - 6Nm$

Sol.15

Here the proton has no acceleration so $E = 0, B = 0$

Sol.16

The input and output powers should be same for 100% efficiency.

Sol.17

As we know that $E = \frac{LdI}{dt} \Rightarrow L = \frac{Edt}{dI} = \frac{20\times0.05}{18.2} = 62.5 \times 10^{-3} H = 62.5mH$

Sol.18

As $\lambda = \frac{C}{V} = \frac{3\times10^{B}}{2\times10^{10}} = 1.5 \times 10^{-2}$

Sol.19

Path difference for a point

$= (d^2 + b^2)^{1/2} - d = \frac{b^2}{2d}$

Path difference for a dark bonds= $(2n-1)\frac{\lambda}{2}$

$\Rightarrow (2n-1)\frac{\lambda}{2} = \frac{b^2}{2d} or\ (2n-1)\lambda = \frac{b^2}{d}$

for$n = 1, \lambda = \frac{b^2}{d}$

Sol.20

As $P = \frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2} = \frac{1}{0.5} - \frac{1}{1} = 1.0D$

Sol.21

The length of telescope tube would increase by an amount equal to $4f$

Sol.22

X-rays have a certain minimum wavelength and also the wavelength larger than this minimum value.

Sol.23

As $\mu = current \times area = \frac{q\omega}{2\pi} \times \pi r^2 = \frac{1}{2}\omega q r^2$

Orbital angular momentum

$L = m\omega r^2 = \frac{h}{2\pi} = h$

$\Rightarrow \omega r^2 = \frac{h}{m}$

$\therefore \mu = \frac{1}{2}\frac{qh}{m} = \frac{1.6\times10^{-19}\times1.05\times10^{-34}}{2\times9.1\times10^{-31}} = 9.2 \times 10^{-24} Am^2$

Sol.24

Asper the penetrating power

$P_y < P_\beta < P_a$

Sol.25

$Eg = hv = \frac{hc}{\lambda}$

Given $\lambda = 2480nm = 2480 \times 10^{-9}m = 248 \times 10^{-8}m$

$\Rightarrow Eg = \frac{6.6\times10^{-34}\times3\times10^{8}}{248\times10^{-8}} = 7.984 \times 10\hat{} - 10^{-20}J$

$= \frac{7.984\times10^{-20}}{1.6\times10^{-19}}eV = 0.499eV = 0.5eV$

Sol.26

Power gain $= \frac{a^2R_L}{R_{in}} = \left(\frac{25}{26}\right)^2 \times \frac{800}{200} = 3.69$

Sol.27

Emitter Current, $I_e = \frac{n_e\times e}{t} = \frac{10^{10}\times1.6\times10^{-19}}{10^{-6}} = 1.6 \times 10^{-3}A = 1.6mA$

Sol.28

$[Y] = \left[\frac{\text{x}}{z^2}\right] = \left[\frac{capacitance}{(magneticinduction)}\right]$

$= \frac{[M^{-1}L^{-2}Q^2T^2]}{[M^2Q^{-2}T^{-2}]} = [M^{-3}L^{-2}T^4Q^4]$

Sol.29

A person has to swim perpendicular to the river current in order to cross the river in shortest time.

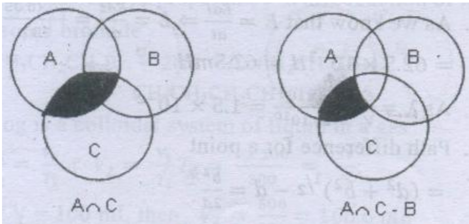
Sol.30

$L = m\frac{\text{V}}{\sqrt{2}}r\perp$

Here $r\perp = h = \frac{v^2\sin^2 45^0}{2g} = \frac{v^2}{4g}$ $or\ L = m\left(\frac{v}{\sqrt{2}}\right)\left(\frac{v^2}{4g}\right) = \frac{mv3}{4\sqrt{2}g}$

MATHEMATICS

Sol.1



Sol.2

When the left most integer is subtracted from its corresponding real number, the result will always range between 0 and 1 (0 included but 1 not included).

Sol.3

Let the given Relation be R. In the Relation R from A→B, x ∈A and y∈ B and (x,y).Hence A is {2,4,6}.

Sol.4

As α is the root of the equation $ax^2 + bx + c = 0$.

Therefore , $a\alpha^2 + b\alpha + c = 0$.

Hence $\frac{1}{a\alpha+b} = -\frac{\alpha}{c}$ (i)

Similarly $\frac{1}{a\beta+b} = -\frac{\beta}{c}$ (ii)

S = Sum of roots $= \left(-\frac{\alpha}{c}\right) + \left(\frac{\beta}{c}\right) = -\frac{(\alpha+\beta)}{c} = \frac{b}{ac}$ (because $\alpha + \beta =- b/a$) Product of roots $= \frac{\alpha\beta}{c^2} = \frac{1}{ac}$

Equation is $x^2 - Sx + P = 0$

$x^2 - \frac{b}{ac}x + \frac{1}{ac} = 0$

$acx^2 - bx + 1 = 0$.

Sol.5

$x = \sqrt{a+x}\ or x^2 = a + x$

$\Rightarrow x^2 - x - a = 0$

$\Rightarrow x = \frac{1 \pm \sqrt{1+4a}}{2}$

Sol.6

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$[C_1 \to C_1 + C_2]$

$$\Rightarrow \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 2 & 1+\cos^2\theta & 4\sin 4\theta \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$[R_2 \to R_2 - R_1, R_3 \to R_3 - R_1]$

$$\Rightarrow \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow 2 + 4\sin 4\theta = 0$

$\Rightarrow \sin 4\theta = -\frac{1}{2}$

$\Rightarrow 4\theta = \frac{7\pi}{24}, \frac{11\pi}{24}$

Sol.7

For non-zero solution $\Delta=0$

$$\Rightarrow \begin{vmatrix} 1 & -K & -1 \\ K & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix}$$

On solving, we get $K^2 - 1 = 0 \Rightarrow K = \pm 1$

Sol.8

We have total number of persons = 3 girls + 9 boys = 12

The total number of numbered seats = 2 × 3 + 4 × = 14 So, the total number of ways in which 12 persons can be seated on 14 seats = number of arrangements of 14 seats ny talking 12 at a time = ${}^{14}P_{12}$. Three girls can be seated together in a back row on adjacent seats in the following way:

1, 2, 3 or 2, 3, 4 of first van and 1, 2, 3 or 2, 3, 4 of second van.

In each way the three girls can interchange among themselves in 3! ways.So, the total number of ways in which three girls can be seated together in a back row on adjacent seats = 4×3!

Now , 9 boys are to be seated on remaining 11 seats, which can be done in ${}^{11}P_9$ ways. Hence, by the fundamental principle of counting, the total number of seating arrangement is ${}^{11}P_9 \times 4 \times 3! = {}^{11}P_9 \times 4!$

Sol.9

Arrangement of n things in circle irrespective of the direction = $\frac{(n-1)!}{2} = \frac{4!}{2} = 12$

Sol.10

$$\left(\frac{a}{a+x}\right)^{\frac{1}{2}} + \left(\frac{a}{a-x}\right)^{\frac{1}{2}} = \left(\frac{a+x}{a}\right)^{-\frac{1}{2}} + \left(\frac{a-x}{a}\right)^{-\frac{1}{2}}$$

$$= \left(1+\frac{x}{a}\right)^{-\frac{1}{2}} + \left(1-\frac{x}{a}\right)^{-\frac{1}{2}}$$

$$= \left[1 + \left(-\frac{1}{2}\right)\left(\frac{x}{a}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2.1}\left(\frac{x}{a}\right)^2 + \cdots\right]$$

$$+ \left[1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{a}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2.1}\left(-\frac{x}{2}\right)^2 + \cdots\right]$$

$$= 2 + \frac{3x^2}{4a^2} + \cdots$$

Sol.11

Put x = 1 in the expansion of $(1 - 3x + 10x^2)^n$,

We get

$(1 - 3 + 10(1)^2)^n = a$

$\Rightarrow a = (1 - 3 + 10)^n = 8^n$

$\Rightarrow a = 2^{3n}$ (i)

Put x = 1 in the expansion of $(1 + x^2)^n$, we get $(1 + 1)n = b$

$\Rightarrow b = 2^n$ (ii)

From (i) and (ii), we get $a = b^3$

Sol.12

Let T_n be the nth term of the series 3 + 10 + 17 + …

Therefore $T_n = 3 + (n - 1)\,7 = 7n - 4$

Let $T_n{}'$ be the nth term of the series 63 + 65 + 67 + …….

Therefore $T_n{}' \Rightarrow 63 + (n - 2)\,2 = 2n + 61$

Now $T_n = T_n{}' \Rightarrow 7n - 4 = 2n + 61 \Rightarrow n = 13$

Sol.13

Given that a, b, c are in A. P, and (b – a), (c – b), a are in G.P, therefore

we get 2b = a + c and $(c - b)^2 = (b - a)\,a$

$\Rightarrow (b - a)^2 = (b - a)\,a$ (using c – b = a – b)

$\Rightarrow (b - a) = a$

$\Rightarrow b = 2a$

$\Rightarrow c = 3a$ (using 2b = a + c).

Therefore, a : b : c = 1 : 2 : 3

Sol.14

We have $\lim_{x\to 0} \frac{(1-x)^n - 1}{x}$

On applying the L 'Hospital rule, we get

$\lim_{x\to 0} \frac{-n(1-x)^{n-1}}{1} = -n$

Hence , option (c) is correct .

Sol. 15

$Rf'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$

$\lim_{h\to 0} \frac{|2+h|-|2|}{2}$

$\lim_{h\to 0} \frac{2+h-2}{h} = \lim_{h\to 0} \frac{h}{h} = 1$

$Lf'(2) = \lim_{h\to 0} \frac{f(2-h)-f(2)}{-h}$

$= \lim_{h\to 0} \frac{|2-h|-|2|}{-h}$

$= \lim_{h\to 0} \frac{2-h-2}{-h} = \lim_{h\to 0} \frac{h}{h} = 1$

Hence $Lf,(2) = Rf'(2) = 1$

Therefore $f'(2) = 1$

Sol.16

We have $y - e^{xy} + x = 0$ (i)

Hence, on differentiating (i) , we get

$y' - e^{xy}(xy' + y) + 1 = 0$

or $y' = \frac{ye^{xy}-1}{1-xe^{xy}}$

At any point the vertical tangent would have its slope as infinite. Now equating the denominator with 0, we get $1 - xe^{xy} = 0$ (ii)

(1, 0) satisfies (ii)

Therefore only option (d) is correct.

Sol.17

$I = \int e^{\log(x^{-1})dx} = \int x^{-1}dx = \int \frac{1}{x}dx = \log|x|$

Sol.18

$I = \frac{1}{2}\int^{\frac{\pi}{2}}(1 - cos2x)dx = \frac{1}{2}\left[x - \frac{sin2x}{x}\right]^{\frac{\pi}{2}} = \frac{\pi}{4}$

Sol.19

$\frac{dr}{dt} = -rt \Rightarrow \frac{dr}{r} = -tdt \Rightarrow \log r = -\frac{t^2}{2} + c$ (i)

Putting $t = 0$ and $r = r_0$ in (i), we get $c = \log r_0$

$\Rightarrow \log r = -\frac{t^2}{2} + \log r_0$

Hence , $r = r_0\left(e^{-t^2/2}\right)$

Sol.20

$\frac{dy}{dx} + \left(\frac{1-y^2}{1-x^2}\right)^{1/2} = 0$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$

$\Rightarrow \sin^{-1} y + \sin^{-1} x = \sin^{-1} c$

$\Rightarrow \sin^{-1}\left[y\sqrt{1-x^2} + x\sqrt{1-y^2}\right] = \sin^{-1} c \Rightarrow \left[y\sqrt{1-x^2} + x\sqrt{1-y^2}\right] = c$

Sol.21

$\left(x\sqrt{1+x^2}\right)dx + \left(y\sqrt{1+x^2}\right)dy = 0$

$\Rightarrow \frac{\text{xdx}}{\sqrt{1+x^2}} + \frac{\text{ydy}}{\sqrt{1+y^2}} = 0$

$\Rightarrow \sqrt{1+x^2} + \sqrt{1+y^2} = c$

Sol.22 we know that y = mx + c touches y^2 = 4ax, iff c =a/m and m ≠ 0

Therefore the line y = 2x + c touches the curve $y^2 = 16$ x only if c =4/2 = 2

Sol.23

We know that if α, β, γ are the directional angles with x-axis, y-axis and z-axis, then

$\cos^2 a + \cos^2 \beta + \cos^2 \gamma = 1$

$\Rightarrow (1 - \sin^2 \alpha) + (1 - \sin^2 \beta) + (1 - \sin^2 \gamma) = 1$

$\Rightarrow \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

Sol.26

The total number of ways of choosing two numbers out of 1, 2, 3…….., 30 is $^{30}C_2 = 435$

⇒ Exhaustive number of cases = 435

Since $a^2 - b^2$ ios divisible by 3 if either a and b both are divisible by 3 or none of a and b is divisible by 3.

Thus , the favourable number of cases

$= {}^{10}C_2 + {}^{20}C_2 = 235$

Hence, the required probability = $\frac{235}{435} = \frac{47}{87}$

Sol.29

$b^2 = a^2(e_1^2 - 1) and\ a^2 = b^2(e_2^2 - 1)$

$\Rightarrow b^2 = b^2(e_2^2 - 1)(e_1^2 - 1)$

$\Rightarrow 1 = (e_2^2 - 1)(e_1^2 - 1)$

$\Rightarrow {e_1}^2\ {e_2}^2 - {e_1}^2 - {e_2}^2 = 0$

$\Rightarrow \frac{1}{{e_1}^2} + \frac{1}{{e_2}^2} = 1$

Sol.30

The slope of the given tangent at any point (x, y) is $\frac{dy}{dx} = \frac{2x}{4-y^2}$

For a vertical tangent the slope must be infinite.

Therefore $\frac{2x}{4-y^2} = \infty$

$\Rightarrow 4 - y^2 = 0$

$\Rightarrow y = \pm 2$

$\Rightarrow x = \pm \frac{4}{\sqrt{3}}$