Subject: CHEMISTRY, MATHEMATICS & PHYSICS

Paper Code: JEE_Main_Sample Paper –I_Answer File

Part – A – Chemistry

1)

Ans: d

Exp: Sudden jump from $IE_2 \rightarrow IE_3$ is given by elements which attain stable configuration of noble gas after losing 2 electrons.

2)

Ans: c

Exp: NA

3)

Ans: c

Exp: It is nitrogen. It forms NCl_3, N_2O_5 and Mg_3N_2 but NCl_5 does not exist due to absence of vacant d-orbital. Nitrogen exhibits a covalency of three only.

4)

Ans: b

Exp: Factual question.

CH_4 = Tetrahedral Shape,

SeF_4 = Selenium in SeF_4 has an oxidation state of +4. Its shape in the gaseous phase is similar to that of SF_4, having a see-saw shape

5)

Ans: b

Exp: K.E. of emitted photoelectrons depend on frequency of radiation.

6)

Ans: c

Exp: Adding Catalyst increase or decrease the Activation Energy (Ea).

7)

Ans: c

Exp: NA

8)

Ans: a

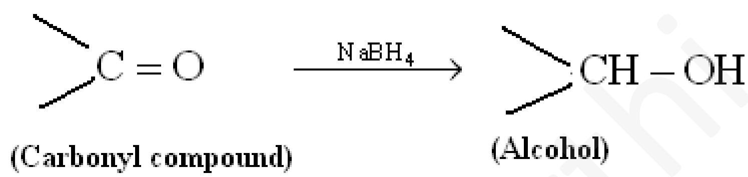
Exp: The number of chiral carbons = 5 (odd number).

It is symmetrical molecule so no. of meso forms

$= 2^{(n/2 - ½)} = 2^{(5/2 - ½)} = 2^2 = 4$

9)

Ans: b

Exp: 

10)

Ans: a

Exp: (A), (C) and (D) decreases down the group but (B) increases down the group with increasing metallic character.

11)

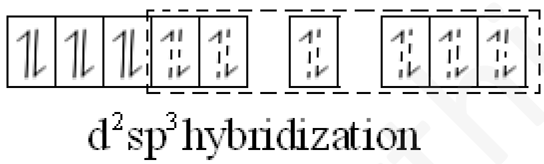
Ans: a

Exp: In $[CuCl]_4^{2-}$ orange compound there is one unpaired electron in d-orbital, therefore, its hybridization is sp^3 where as in $[CuCl_4]^{2-}$, yellow compound, unpaired electron jumps to 5s & thus dsp^2 hybridization.

12)

Ans: d

Exp: Complex is $[Fe(en)_3]^{2+}$; as 'en' is a strong field ligand pairing of electrons will take place

$[Fe(en)_3]^{2+}$: 

Hence, hybridization is d^2sp^3 and complex is diamagnetic. As it has 3 bidentate symmetrical 'en' ligands so it will not show geometrical isomerism.

13)

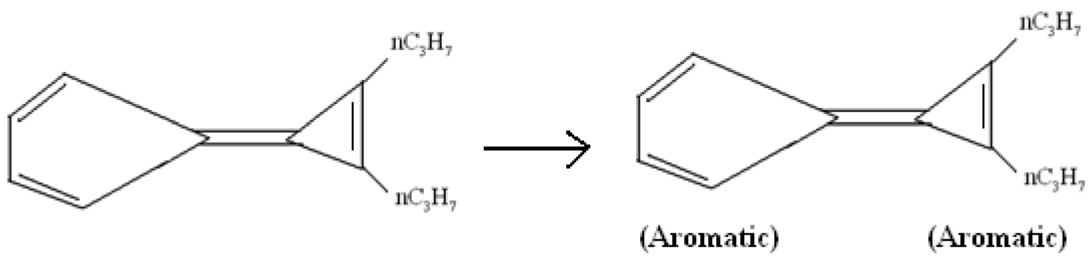
Ans: b

Exp: There are unpaired electrons, others have no unpaired electrons.

14)

Ans: c

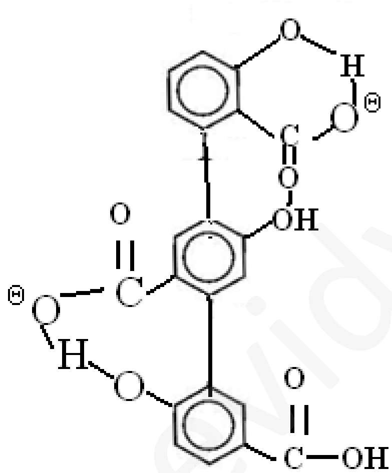
Exp:



15)

Ans: a

Exp:



16)

Ans: b

Exp: NA

17)

Ans: b

Exp: $\Delta H_f(CH_4) = (\Delta H_{comb.}$Of C) + (2 x $\Delta H_{comb.}$ Of H_2) – ($\Delta H_{comb.}$ Of CH_4)

18)

Ans: c

Exp: Increase in pressure favours the melting of ice.

19)

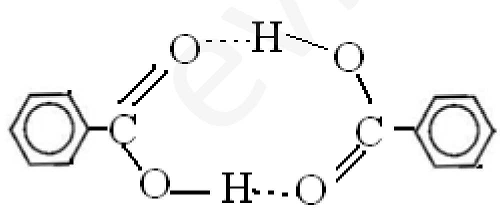
Ans: d

Exp: K_w increases with increase in temperature.

20)

Ans: c

Exp: The benzoic acids forms a dimer due to H-bonding as



21)

Ans: b

Exp: NH_3 is weakly ionising while NH4Cl will be converting into NH_4 + only.

22)

Ans: c

Exp: NA

23)

Ans: c

Exp: Rate is governed by slowest step

$$A + B_2 \xrightarrow{k_1} AB + B$$
$$r = k_1 [A][B_2] \ldots\ldots\ldots(i)$$
$$\text{From } A_2 \overset{k_c}{\rightleftharpoons} A + A$$
$$k_c = \frac{[A]^2}{[A_2]} \quad \ldots\ldots\ldots.(ii)$$
$$[A] = \sqrt{k_c}\,[A_2]^{1/2}$$
$$r = k_1 \sqrt{k_c}\,[A_2]^{1/2}[B_2] \qquad \text{order is } = \frac{1}{2} + 1 = \frac{3}{2}$$

24)

Ans: d

Exp: When all particle along are body diagonal one removed, these 2X atoms from corner are removed, one Y particle removed & 2Z particle removed.

Hence new arrangement, X particle = 1/8 x 6 + ½ x 6 = 15/4; Y particle = 6: Z particle = 3

Hence formula = $X_{15/4}\ Y_3Z_6 = X_{5/4}YZ_2 = X_5Y_4Z_8$

25)

Ans: a

Exp: (A) $H_2S_2O_7$ = +6 Each, $Na_2S_4O_6$ = +5, 0 , $Na_2S_2O_3$ = +4,0

26)

Ans: c

Exp: The hydration energy of the ions of alkaline earth metals (M^{2+}) is nearly 4 or 5 times greater than that of alkali metals because of their small size and increased charge.

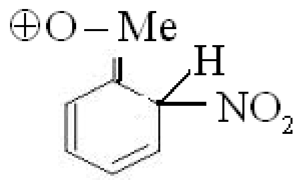
27)

Ans: a

Exp: It is believed that out of two hydrogen, one is associated with each oxygen atom at bond angle of 94.8^o. All the four atoms H-O-O-H do not lie in the same plane.

28)

Ans: b

Exp:  : In this structure, every atom

(except, of course, H) has a stable octet of elections. So it is the most stable.

29)

Ans: c

Exp: Ionization energy of hydrogen atom is about $1310 kJmol^{-1}$. Covalent single bond energies are of the order of a few hundred $kJ\ mol^{-1}$. Molecular translation energy of gasses = 3/2 RT, which is about $3.7\ kJ\ mol^{-1}$ at 300K. Rotational barrier between eclipsed and staggered forms of ethane is about $12kJ\ mol^{-1}$

30)

Ans: c

Exp: SF_6: sp^3d^2 hybridization; molecule is octahedral XeF_6:sp^3 hybridization; molecule is pentagonal [pyramid]

XeF_4: sp^3d^2 hybridization: molecule is square planar I^-_3: sp^3d hybridization: molecule is linear

Part – B - Physics

31) Ans: c

Exp: Initially, $PV = nRT$ (for each vessel)

$P = 1atm$, $T = 300k$, V = vol of each vessel

Finally, $P_1V = n_1R\ 600$, $P_1V = n_2R300$

$n_1/n_2 \cdot 2 = 1$

$2n_1 = n_2$

Total no of moles is constant

$\Rightarrow n_1 + n_2 = 2n$

$3n_1 = 2n \Rightarrow n = 3n_1/2$

We have $PV = nR300$ ....(1)

& $P_1V = n_1R600$ ....(2)

From 1 & 2,

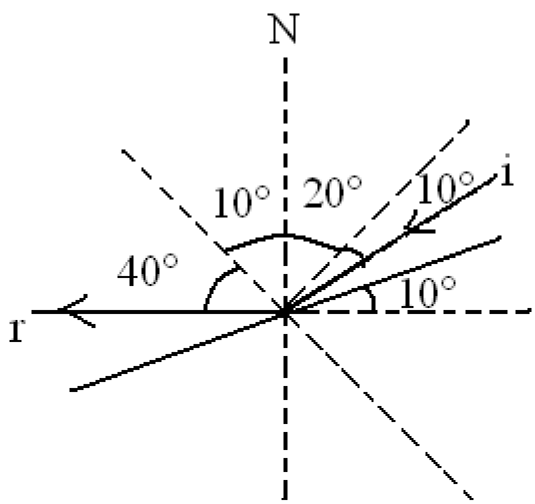
$P_1 x\ 3n_1/2.\ R \times 300 = n_1R\ 600$

$P_1 = 4/3$

32) Ans: c

Exp: Initial angle of reflected ray with the normal = 20^o

Final angle of the reflected ray with the same normal = 50^o



Angle through which reflected ray is rotated = $50^o - 20^o = 30^o$

33) Ans: c

Exp: T.E = KE + PE

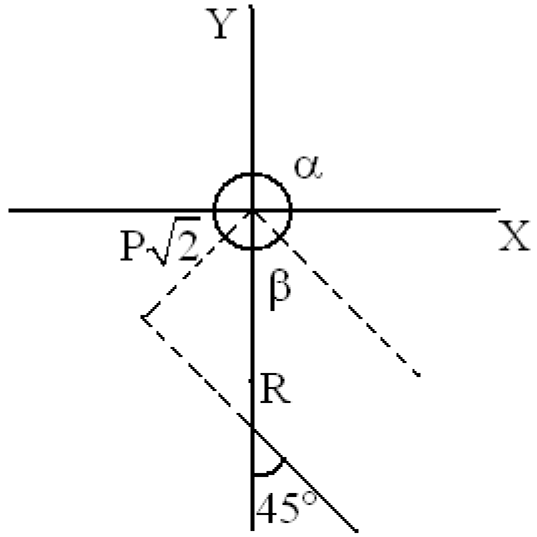
= KE – GMm/R (on surface of earth)

At infinity TE = 0, KE = GMm/R = mgR

($g = GM/R^2$)

34) Ans: c

Exp: B = M.I/4πd *sinα + sinβ+



$= M.I/4\pi(R/\sqrt{2}) * \sin(2\pi-(\pi/4)) + \sin 90+$

$= M.I/4\pi R \cdot (\sqrt{2} - 1)$

35) Ans: a

Exp: $P = \sigma \Delta T^4$

$T_1 = 288^oK$, $T_2 = 293K$

36) Ans: c

Exp: For movement of m_1, $kx = \mu m_1 g$

Now, by using work energy theorem

$F_{min}\, x = \mu m_2 g x + ½\, kx^2$

$\Rightarrow F_{min} = \mu m_2 g + ½\, kx$

$F_{min} = \mu g\ (m_2+m_1/2)$

37) Ans: d

Exp: $v_0 = 0 + \alpha \times 2n$

$\alpha = v_0/2n$

for the 2n sec s, displacement

$x_1 = ½\ v_0/2n\ (2n)^2$

for first n sec s,

$x_2 = ½\ v_0/2n\ (n)^2$

Displacement in the last n secs:

$x_1 - x_2 = ½\ v0/2n\ ((2n)^2 - (n)^2)$

$=3v_0n/4$

38) Ans: a

Exp: The distance travelled by the train in 20secs is

$½\ (0.5)\ (20)^2 = 100$

The distance between the two events μ & T is 100m

The observer has to move 100m in 20s in a direction opposite to that of train

39) Ans: b

Exp: $v_x = 4 \times 6 \cos 6t$

$v_y = 4 \times 6 \sin 6t$

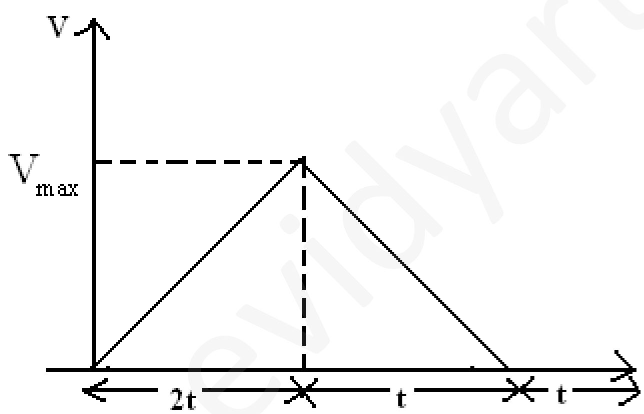
$|v| = \sqrt{(v_x^2 + v_y^2)} = 24 m/s$

Distance traversed = vt = 24 x ¼

= 6m

40) Ans: B

Exp: $v_{max}/2t = 10$



$V_{max} = 20t$

Area of graph

$\frac{1}{2} v_{max} (3t) = 200$

$t = \sqrt{(20/3)}$

total time = 3t = 2√(15) secs

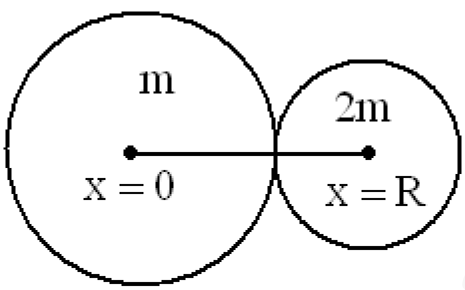
41) Ans: c

Exp: The x-coordinate of all the particles in this case cannot be of the same sign.

42)

Ans: c

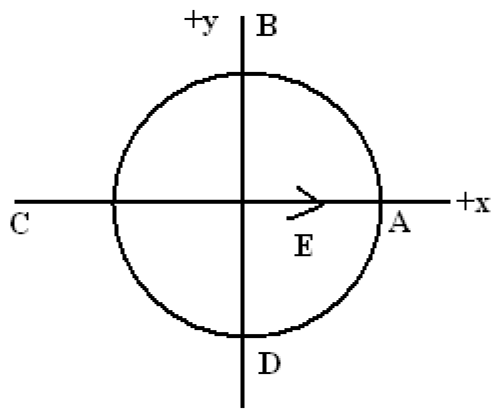
Exp:



Xcom = (2mx3R)/(m+2m) = 2R

COM lies at pt of contact

43) Ans: a



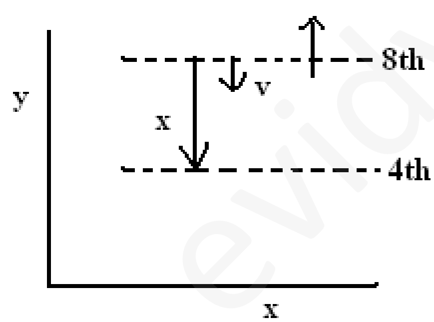
Exp:

The $\vec{E}$ is directed towards the +x axis, the potential has to be minimum at A,

$$V(r) = -\int_{\infty}^{r} \vec{E}.\vec{d}$$

44)

Ans: a



Exp:

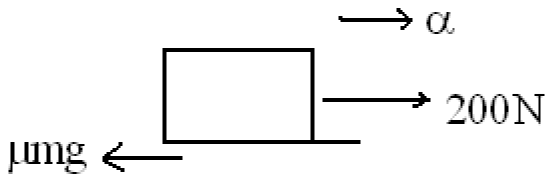
The displacement & velocity are directed in the –ve of direction.

There would be retardation as lift stops at 4^{th} floor, so acceleration is directed opposite to the velocity in the upward direction.

45)

Ans: c

Exp:



$\alpha = 4/2 = 2 m/s^2$

$200 - \mu \times 30 \times 10 = 30 \times 2$

$\mu = 14/20 = 7/15 = 0.47$

46)

Ans: b

Exp: PR = d

$\therefore$ PO = d secθ

and CO = PO cos 2θ = d sec θ 2 θ

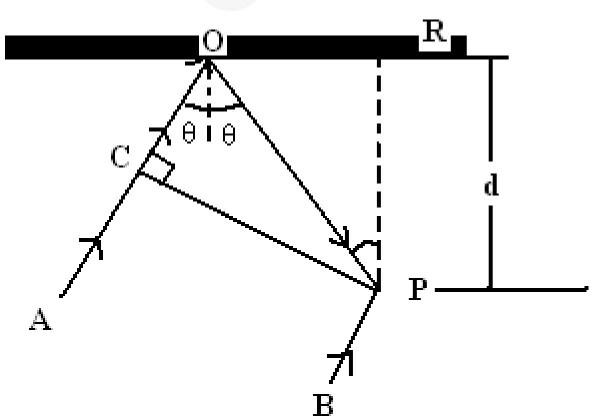
path difference between the two rays is,

Δx = PO + OC = (d sec θ + d sec θ cos 2 θ)

Phase difference between the two rays is

$\Delta \phi = \pi$ (one is reflected, while another is direct)

Therefore, condition of construction interference should be



$\Delta x = \lambda/2, 3\lambda/2$ .....

Or d $\sec\theta (1 + \cos \theta) = \lambda/2$

Or $(d/\cos\theta) (2\cos^2\theta) = \lambda/2$

Or $\cos\theta = \lambda/4d$

47)

Ans: B

Exp: Heat released by 5kg of water when its temperature falls from 20^oC to 0^oC is,

$Q1 = mc\Delta\theta = (5)(103) (20-0) = 105cal$

When 2kg ice at -20oC comes to a temperature of 0^oC, it takes energy

$Q2 = mc\Delta\theta = (2) (500) (20) = 0.2 \times 105cal$

The remaining heat

Q = Q1 – Q2 = 0.8 x 105cal will melt a mass m of the ice,

Where,

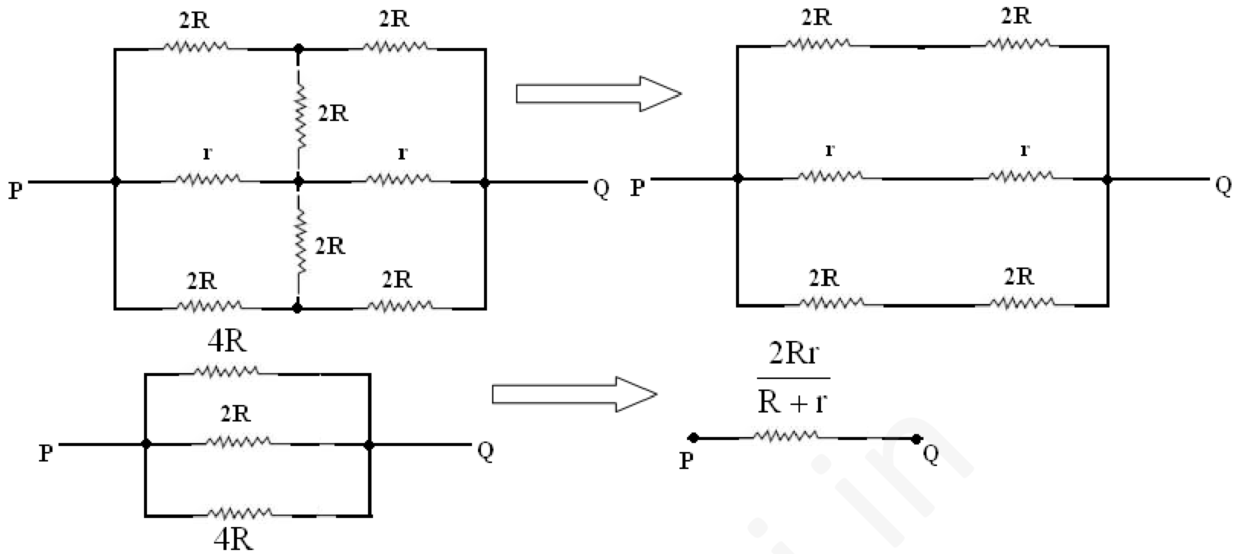
m = Q/L = 0.8x105/80x103 = 1kg

total mass of water=5+1=6kg

48)

Ans: a

Exp: The circuit can be redrawn as follows:



49)

Ans: b

Exp: All the three plates will produce electric field at P along negative z – axis.

Hence,

$$\vec{E}_p = \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right]\left(-\hat{k}\right)$$

$$= -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

∴Correct answer is (b).

50)

Ans: b

Exp: Net electrostatic energy of the configuration will be

$$U = K\left[\frac{q.q}{a} + \frac{Q.q}{\sqrt{2a}} + \frac{Q.q}{a}\right] \text{Here, } K = \frac{1}{4\pi\varepsilon_0}$$

$$\text{Putting } U = 0 \text{ we get, } Q = \frac{-2q}{2+\sqrt{2}}$$

51)

Ans: d

$$\frac{dN}{dt} = 50 - \frac{N}{0.5}$$

Exp: $$\int_0^N \frac{dN}{50-2N} = \int_0^t dt$$

$$N = \left(100\left(1-e^{-t/2}\right)\right) = 25$$

$$t = 2\ln(4/3)$$

52) Ans: a

Exp: Path difference = (μ - 1) t = nλ;

For minimum t, n = 1;

∴ t = 2λ

53)

Ans : C

Exp: B.A’+B+A.B’+A

= B.(A’+1)+A.(B’+1)

=B.1+A.1=A+B

54)

Ans: d

$$eV_0 = \frac{hc}{\lambda_0} - W_0 \text{ and } eV' = \frac{hc}{2\lambda_0} - W_0$$

Exp: Subtracting them, we have

$$e(V_0 - V') = \frac{hc}{\lambda_0}\left[1 - \frac{1}{2}\right] = \frac{hc}{2\lambda_0} \text{ or } V' = V_0 - \frac{hc}{2e\lambda_0}$$

55)

Ans: D

Exp: Tension in the string b/w B and C=T,

Tension in the string attached to A and B=T_1

For block C: T=Mg, For block B: T-u100g-u(100+140)g-2T_1=0

For block C: T_1=u100g, hence,

Mg-100ug-240ug-200ug=0

M=u540=162kg

56)

Ans: D

$$\int \vec{E}.\vec{dl} = \left|\frac{d\phi}{dt}\right|$$

$$= S\left|\frac{dB}{dt}\right|$$

$$\text{or} \qquad E(2\pi r) = \pi a^2 \left|\frac{dB}{dt}\right| \text{for } r \geq a$$

$$\therefore \qquad E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

$$\therefore \qquad \text{Induced electric field} \propto \frac{1}{r}$$

For $\qquad r \leq a$

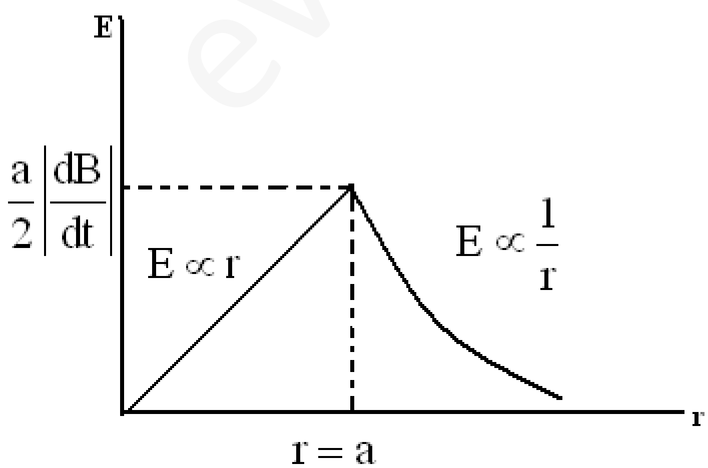
$$E(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right|$$

Exp:

$$\text{or} \qquad E = \frac{r}{2}\left|\frac{dB}{dt}\right| \text{ or } E \propto r$$

$$\text{At} \qquad r = a, E = \frac{a}{2}\left|\frac{dB}{dt}\right|$$

Therefore, variation of E with r (distance from centre) will be as follows:

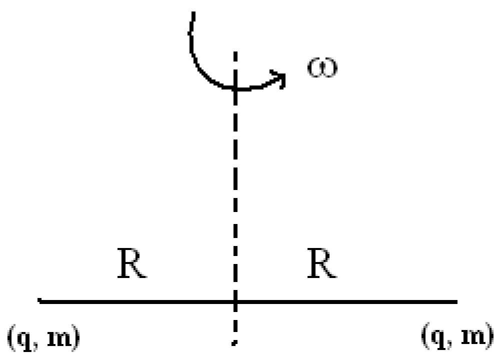


57)

Ans: a

Exp: Current, i = (frequency) (charge)

$$= \left(\frac{\omega}{2\pi}\right)(2q) = \frac{q\omega}{\pi}$$



Magnetic moment,

$$M = (i)(A) = \left(\frac{q\omega}{\pi}\right)(\pi R^2) = (q\omega R^2)$$

Angular momentum,

$$L = 2I\omega = 2(mR^2)\omega$$

$$\therefore \frac{M}{L} = \frac{q\omega R^2}{2(mR^2)\omega} = \frac{q}{2m}$$

58)

Ans: a

Exp:

$$U = eV = eV_0 \ln\left(\frac{r}{r_0}\right)$$

$$|F| = \left|-\frac{dU}{dr}\right| = \frac{eV_0}{r}$$

This force will provide the necessary centripetal force.

Hence, $\frac{mv^2}{r} = \frac{eV_0}{r}$

or $v = \sqrt{\frac{eV_0}{m}}$ $\quad\ldots(i)$

Moreover, $mvr = \frac{nh}{2\pi}$ $\quad\ldots\ldots(ii)$

Dividing Eq. (ii) by (i), we have

$mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_0}}$ or $r_n \propto n$

59)

Ans: B

Exp: The apparent freq. of sound coming from the police car and from the stationary siren observed by the motorcyclist will be same.

(330-v)/(330-22) *176= (330+v)/330 *165

v is approx. 22m/s

60)

Ans: a

$$I_{remaining} = I_{whole} - I_{removed}$$

Exp: or $$I = \frac{1}{2}(9M)(R)^2 - \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + m\left(\frac{2R}{3}\right)^2\right] \quad ....(i)$$

Here, $$m = \frac{9M}{\pi R^2} x\pi\left(\frac{R}{3}\right)^2 = M$$

Substituting in Eq. (i), we have I = $4MR^2$

∴correct answer is (a)

Part – C – Mathematics

61) Ans: c

Exp: Let $f(x) = x^{12} - x^9 + x^4 - x + 1$

$= (x^9 + x)(x^3 - 1) + 1 > 0, \forall x \geq 1$ ....(i)

Again, $f(x) = x^4(x^8 + 1) - x(x^8 + 1) + 1$

$= x(x^8 + 1)(x^3 - 1) + 1 > 0, \forall x \leq 0$ ....(ii)

Next $f(x) = x^{12} + x^4(1 - x^5) + (1 - x) > 0, \forall\, 0 < x < 1$
.........(iii)

Combining Eqs. (i), (ii), (iii) we get

$X \in (-\infty, \infty)$

62) Ans: b

Exp: $6! - 2!\,5! = 480$

63) Ans: b

Exp: $2x+y = 5$

$x + 3y = 5$

$x - 2y = 0$

Solving eqs. (i) and (ii), we get

$x = 2$ and $y = 1$

Which is satisfied Eq. (iii).

64) Ans: a

Exp: $i^i = (e^{i\pi/2})^i$

$= e^{-\pi/2}$

65) Ans: b

Exp: $\because |z_1 - z_2| = |z_1 - (z_2 - 3 - 4i) - (3+4i)|$

$\geq |z_1| - |z_2 - 3 - 4i| - |3+4i|$

$= 12-5-5=2$

$\therefore |z_1 - z_2| \geq 2$

66) Ans: d

Exp: In 10jumps = 10m

11^{th} jump = 10 + 2 = 12m (not slip)

67) Ans: c

Exp: $32 = 2^5$

$(32)^{32} = (2^5)^{32} = 2^{16}$:

$=$ $(3-1)^{16}$:

$=$ $3m+1,\ m \in I_+$

Now, $(32)^{32^{32}} = 32^{3m+1} = 2^{5(3m+1)} = 2^{15m+5}$

$\therefore$ $32^{32^{32}} = 2^{3(5m+1)}.2^2$

$= 4.(8)^{5m+1}$

$= 4.(7+1)^{5m+1}$

$= 4.[7n + 1],\ n \in I_+$

$= 28n+4$

$\therefore$ Remainder = 4

68) Ans: d

Exp: Probability of showing even number in a throw = 3/6 = ½

$\therefore$Required probability

$= {}^{2n+1}C_1 \cdot \frac{1}{2} (1/2)^{2n} + {}^{2n+1}C_3 (1/2)^3(1/2)^{2n-2} + \ldots\ldots. + {}^{2n+1}C_{2n+1}(1/2)^{2n+1}$

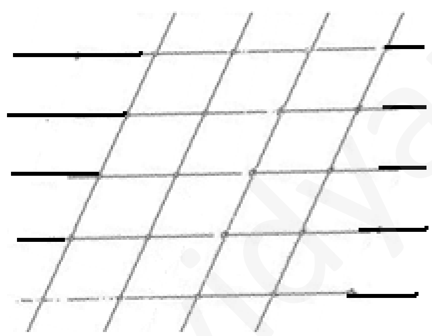
$=(1/2)^{2n+1}[{}^{2n+1}C_1 + {}^{2n+1}C_3 + \ldots\ldots\ldots + {}^{2n+1}C_{2n+1}]$

$= 1/2^{2n+1} \times 2^{2n+1-1} = \frac{1}{2}$

69) Ans: b

Exp: Number of parallelograms = ${}^5C_2 \times {}^4C_2$

= 60



70) Ans: b

Exp: Let A = $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This is nilponent matrix of index 2.

71) Ans: a

Exp: Total ways = $^{12}C_2 = 66$

E_3 = the even of the sum being 3

E_6 = the even of the sum being 6

E_9 = the even of the sum being 9

E_{12} = the even of the sum being 12

E_{15} = the even of the sum being 15

E_{18} = the even of the sum being 18

E_{21} = the even of the sum being 21

(sum maximum = 11 + 12 = 23)

$$\begin{bmatrix} (\text{Which is not divisible by } 3) \\ (\text{Heighest number divisible by } 3 \text{ is } 21) \\ \therefore n(E_3) = 1,\ n(E_6) = 2, n(E_9) = 4, \end{bmatrix}$$

$$n(E_{12}) = 5, n(E_{15}) = 5, n(E_{18}) = 3, n(E_{21}) = 3$$

$$\therefore \quad \text{Required probability} = \frac{\sum_{i=1}^{7} n(E_{3i})}{66}$$

$$= \frac{22}{66} = \frac{1}{3}$$

72) Ans: b

Exp: $4\cos^2\theta - 2\sqrt{2}\cos\theta - 1 = 0$

$\cos\theta = 2\sqrt{2} \pm \sqrt{(8+16)}/8 = \sqrt{2} \pm \sqrt{6}/4$

$\cos\theta = \sqrt{6}+\sqrt{2}/4 \quad \Rightarrow \theta = \pi/12;\ 2\pi - \pi/12 = 23\pi/12$

$\cos\theta = -\sqrt{6} - \sqrt{2}/4$

$\cos\theta = \cos(\pi - 5\pi/12);\ \cos(\pi+5\pi/12)$

$\Theta = 7\pi/12;\ 17\pi/12$

73) Ans: c

Exp: $E = 4+5(\omega)^{334} + 3(\omega)^{365} = 4+5\omega + 3\omega^2$

$= 1+2\omega+3(1+\omega+\omega^2) = 1+(-1+i\sqrt{3}) = i\sqrt{3}$

74) Ans: b

Exp: Any line passing through the intersection of the given lines is

x+3y+4+λ(3x+y+4) = 0 ....(i)

Or, (1+3λ)x+(3+λ)y+4(1+λ) = 0

The slope of the line m = - 1+3λ/3+λ

As the line is equally inclined with the axes,

m = tan45° or tan135° = ±1

∴ - 1+3λ/3+λ = ±1, ⇒λ =±1

∴The required lines are (putting λ = - 1, 1 in (i))

x+ 3y + 4 ±(3x+y+4) = 0

Or, x + y + 2 = 0 and x – y = 0

75) Ans: b

Exp:

$$\left(x^2+y^2\right)\sin^2\alpha=\left(x\cos\beta-y\sin\beta\right)^2$$
$$\Rightarrow x^2\left(\sin^2\alpha-\cos^2\beta\right)+2xy\sin\beta\cos\beta+y^2\left(\sin^2\alpha-\sin^2\alpha\right)=0 \quad ....(i)$$

Let the angle between the lines representing by (1) is θ

$$\therefore \tan\theta = 2\left|\frac{\sqrt{h^2-ab}}{a+b}\right|$$

$$= 2\frac{\sqrt{\sin^2\beta\cos^2\beta-\left(\sin^2\alpha-\cos^2\beta\right)\left(\sin^2\alpha-\sin^2\beta\right)}}{|\sin^2\alpha-\cos^2\beta+\sin^2\alpha-\sin^2\beta}$$

$$= 2\frac{\sqrt{\left\{\sin^2\beta\cos^2\beta-\sin^4\alpha+\sin^2\alpha\sin^2\beta+\sin^2\alpha\cos^2\beta-\sin^2\beta\cos^2\beta\right\}}}{|\left(2\sin^2\alpha-1\right)|}$$

$$= 2\frac{\sqrt{\sin\alpha\left(1-\sin^2\alpha\right)}}{|-\cos 2\alpha|} = \frac{2\sin\alpha\cos\alpha}{|-\cos 2\alpha|} = \tan 2\alpha$$

$$\Rightarrow \theta = 2\alpha$$

76) Ans: d

Exp: Equation of circle is $x^2+y^2+2gx+2fy+c=0$

$(1, t) \Rightarrow 1 + t^2+2g+2ft+c=0$ ....(i)

$(t, t) \Rightarrow t^2+t^2+2gt+2ft+c=0$..........(ii)

$(t, 1) \Rightarrow 1+t^2+2gt+2f+c=0$ ....(iii)

Subtracting (ii) from (i),

$1+2g - t^2-2gt = 0$

$\Rightarrow 1-t^2+2g(1-t)=0$

$\Rightarrow(1-t)(1+t+2g) = 0$

$\Rightarrow t = 1$

$\therefore$one point (t, t)

$\therefore$ passes through (1, 1)

77) Ans: c

Exp:

$$I\int \sec^2\theta(\sec\theta+\tan\theta)\bullet(\sec\theta+\tan\theta)d\theta$$

$$\text{Put } \sec\theta+\tan\theta = y \qquad \text{..........}(1)$$

$$\therefore \sec\theta(\tan\theta+\sec\theta)d\theta = dy$$

$$\text{Now}, \sec\theta-\tan\theta = \frac{1}{y} \qquad \text{.......}(2)$$

$$\text{From } (1) \text{ and } (2), 2\sec\theta = y+\frac{1}{y}$$

$$\therefore I = \frac{1}{2}\int y\left(y+\frac{1}{y}\right)dy = \frac{1}{2}\left[\frac{y^3}{3}+y\right]+C$$

$$=\frac{1}{2}\left[\frac{(\sec\theta+\tan\theta)^3}{3}+(\sec\theta+\tan\theta)\right]+C$$

$$=\frac{(\sec\theta+\tan\theta)}{6}\left[(\sec\theta+\tan\theta)^2+3\right]+C$$

78) Ans: d

Exp:

Given, $V = \pi r^2 h$

Differentiating both sides

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + 2r\frac{dr}{dt}h\right) = \pi r\left(r\frac{dh}{dt} + 2h\frac{dr}{dt}\right)$$

$$\frac{dr}{dt} = \frac{1}{10} \quad \text{and} \quad \frac{dh}{dt} = -\frac{2}{10}$$

$$\frac{dV}{dt} = \pi r\left(r\left(-\frac{2}{10}\right) + 2h\left(\frac{1}{10}\right)\right) = \frac{\pi r}{5}(-r+h)$$

Thus, when $r = 2$ and $h = 3$,

$$\frac{dV}{dt} = \frac{\pi(2)}{5}(-2+3) = \frac{2\pi}{5}$$

79) Ans: b

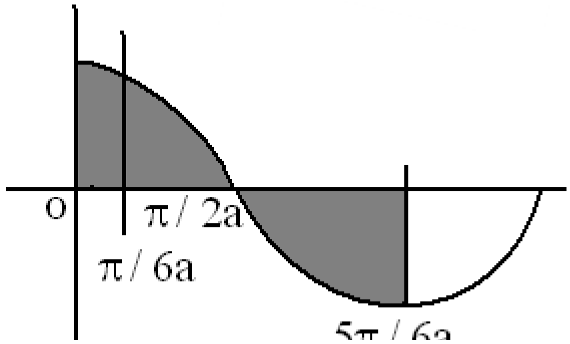
Exp:

$$\cos ax = 0 \text{ if } ax = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

$$x = \frac{\pi}{2a} \text{ or } \frac{3\pi}{2a}$$

$$A_1 = \int_{\frac{\pi}{6a}}^{\frac{\pi}{2a}} \cos ax\,dx$$

$$= \frac{1}{a}\int_{\pi/6}^{\pi/2} \cos t\,dt = \frac{1}{a}[\sin t]_{\pi/6}^{\pi/2} = \frac{1}{2a}$$



$$\text{Similarly } A_2 = \left|\int_{\frac{\pi}{2a}}^{\frac{\pi}{6a}} \cos axdx\right| = \left|-\frac{1}{2a}\right| = \frac{1}{2a}$$

$$\therefore \text{Total area} = \frac{1}{a} > 3 \qquad \therefore \qquad 0 < a < \frac{1}{3}$$

80) Ans: b

Exp: $y = \sin^4 \pi x$ intersects the x-axis at

$x = 0, x = 1$

The curve, $y = \log x$ also passes through (1, 0)

Required area =

$$\left|\int_0^1 \sin^4 \pi xdx\right| + \left|\int_0^1 \log_e xdx\right| = \int_0^\pi \sin^4\theta \frac{d\theta}{\pi} + \left|\left[x\log_e x - x\right]_0^1\right|$$

$$= \frac{2}{\pi}\int_0^{\pi/2} \sin^4\theta d\theta + 1 = \frac{2}{\pi} x \frac{3}{4} x \frac{1}{2} x \frac{\pi}{2} + 1 = \frac{11}{8}$$

81) Ans: b

Exp: $x^2 + (y-r)^2 = r^2$ ....(i)

$\therefore x + (y-r)\, dy/dx = 0, \therefore (r-y)\, dy/dx = x$

$\therefore r = y + x/(dy/dx)$

Put it in (i), we get $(x^2 - y^2)\, dy/dx - 2xy = 0$

82) Ans: c

Exp: $(1+y^2)\, dx + (1+x^2)dy = 0$

$\Rightarrow dx/1\text{-}x^2 + dy/1+y^2 = 0$

On integration, we get

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}C$

$\Rightarrow x+y/1-xy = C$ $\Rightarrow x+ y = C(1-xy)$

83) Ans: b

Exp: Let a, ar, ar^2, ...

a + ar = 12 ....(1)

$ar^2 + ar^3 = 48$ ....(2)

Dividing (2) by (1), we have

$ar^2(1+r)/a(r) = 4$

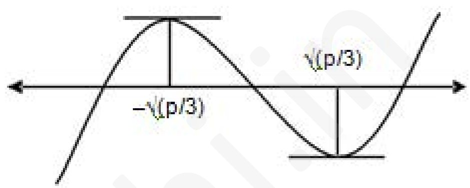
$\Rightarrow r^2 = 4$ if $r \neq -1$

$\therefore r = -2$

Also, $a = -12$ (using (1)).

84) Ans: b

Exp: Let $f(x) = x^3 - px + q$



Now for

Maxima/minima $f'(x) = 0$

$\Rightarrow 3x^2 - p = 0$

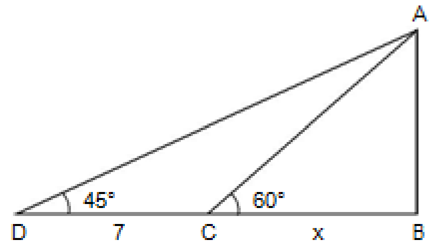
$\Rightarrow x^2 = P_3$

$\therefore x = \pm\sqrt{(p/3)}$

85) Ans: b

Exp: BD = AB = 7+x

Also $AB = x \tan 60° = x\sqrt{3}$



∴x√3 = 7+x

X = 7/√3-1

AB = 7√3/2(√3+1)

86) Ans: c

Exp: A = {4, 5, 6}, B = {1, 2, 3, 4}.

Obviously P (A∪B) = 1

87) Ans: c

Exp: Let (h, k) be the coordinates of the midpoint of a chord which subtends a right angle at the origin. Then equation of the chord is

$$kx + ky - 4 = h^2 + k^2 - 4 (u \sin g \ T = S')$$

$$or \quad hx + ky = h^2 + k^2$$

The combined equation of the pair of lines joining the origin to the points of intersection of $x^2 + y^2 = 4 \ and \ hx + ky = h^2 + k^2 \ is$

$$x^2 + y^2 - 4\left(\frac{hx + ky}{h^2 + k^2}\right)^2 = 0$$

Lines given by the above equations of the pair of lines joining the origin to therefore coeff. of $\mathrm{x}^2 + \mathrm{coeff.of\ y}^2 = 0$

$$\Rightarrow 2(h^2 + k^2) - (4h^2 + 4k^2) = 0 \Rightarrow h^2 + k^2 = 2$$

$$\therefore Locus\ of\ (h,k)\, is\ \ x^2 + y^2 = 2$$

88) Ans: d

Exp:

$$\text{Mean of } a, b, 8, 5, 10 \text{ is } 6$$

$$\Rightarrow \frac{a + b + 8 + 5 + 10}{5} = 6$$

$$\Rightarrow a + b = 7 \qquad \text{..........}(1)$$

$$\therefore \text{Variance} = \sum \frac{(X - A)^2}{1}$$

$$= \frac{(a-6)^2 + (b-6)^2 + 4 + 1 + 16}{5} = 6.8$$

$$\Rightarrow a^2 + b^2 = 25$$

$$a^2 + (7 - a)^2 = 25\ (\text{Using}\,(1))$$

$$\Rightarrow a^2 - 7a + 12 = 0$$

$$\therefore a = 4, 3 \text{ and } b = 3, 4$$

89) Ans: a

Exp:

$$(x-h)^2+(y-2)^2=25 \qquad ....(1)$$

$$\Rightarrow 2(x-h)+2(y-2)\frac{dy}{dx}=0$$

$$\Rightarrow (x-h)=-(y-2)\frac{dy}{dx}$$

substituting in (1), we have

$$(y-2)^2\frac{dy}{dx}2+(y-2)^2=25$$

$$(y-2)2y'2=25-(y-2)^2$$

90) Ans: a

Exp:

$$S_1\equiv(6,5);S_2\equiv(-4,5),e=5/4$$

$$S_1S_2=10\Rightarrow 2ae=10\Rightarrow a=4$$

$$\text{and } b^2=a^2(e^2-1)=16\left(\frac{25}{16}-1\right)=9$$

Centre of the hyperbola is