eVidyarthi
FREE Education

JEE Main - 2015

Set – A, Mathematics

Note: Answers have been highlighted in "Yellow" color and Explanations to answers are given at the end

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/ Black Ball Point Pen, Use of pencil is strictly prohibited,

2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of 90 questions. The maximum marks are **360**

5. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage Each question is allotted **4 (four)** marks for correct response.

6. Candidates will be awarded marks as stated above in instruction NO. 5 for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question, No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

8. Use blue/ Black Ball Point Pen only for writing particulars / marking response on side – 1 and side – 2 of the Answer Sheet. Use of Pencil is strictly prohibited.

9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device , etc, except the admit card inside the examination room/hall.

10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in one page (i.e. Page 39) at the end of the booklet.

11. On completion of the test , the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However , the candidates are allowed to take away this Test Booklet with them.

12. The CODE for this Booklet is **A**. Make sure that CODE printed on Side-2 of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the candidate should immediately report the matter to the invigilator for replacement of both the Test Booklet and the Answer Sheet.

13. Do not fold or make any stray mark on the Answer Sheet.

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Q61. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is:

(1) 219

(2) 256

(3) 275

(4) 510

Q62. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1 z_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a:

(1) straight line parallel to x-axis.

(2) straight line parallel to y-axis.

(3) circle of radius 2.

(4) circle of radius $\sqrt{2}$.

Q63. Let α and ß be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - ß^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to:

(1) 6

(2) – 6

(3) 3

(4) – 3

Q64. $If\ A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to:

(1) (2, – 1)

(2) (– 2, 1)

(3) (2, 1)

(4) (–2, –1)

Q65. The set of all values of λ for which the system of linear equation:

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 \quad = \lambda x_3$

has a non-trivial solution,

(1) is an empty set.

(2) is a singleton.

(3) contains two elements.

(4) contains more than two elements.

Q66. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is:

(1) 216

(2) 192

(3) 120

(4) 72

Q67. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1 - 2\sqrt{x}\right)^{50}$ is:

(1) $\frac{1}{2}(3^{50} + 1)$

(2) $\frac{1}{2}(3^{50})$

(3) $\frac{1}{2}(3^{50} - 1)$

(4) $\frac{1}{2}(2^{50} + 1)$

Q68. If m is the A.M. of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals.

(1) $4\, l^2mn$

(2) $4\, lm^2n$

(3) $4\, lmn^2$

(4) $4\, l^2m^2n^2$

Q69. The sum of first 9 terms of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$ is

(1) 71

(2) 96

(3) 142

(4) 192

Q70. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to:

(1) 4

(2) 3

(3) 2

(4) $\frac{1}{2}$

Q71. If the function,

g$(x) = \begin{cases} k\sqrt{x+1} , & 0 \le x \le 3 \\ mx+2 , & 3 < x \le 5 \end{cases}$ is differentiable, then the value of k + m is:

(1) 2

(2) $\frac{16}{5}$

(3) $\frac{10}{3}$

(4) 4

Q72. The normal to the curve. $x^2 + 2xy - 3y^2 = 0$, at (1, 1):

(1) does not meet the curve again.

(2) meets the curve again in the second quadrant.

(3) meets the curve again in the third quadrant.

(4) meets the curve again in the fourth quadrant.

Q73. Let $f(x)$ be a polynomial of degree four having extreme values at x = 1 and x = 2.

If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then $f(2)$ is equal to:

(1) – 8

(2) – 4

(3) 0

(4) 4

Q74. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals:

(1) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

(2) $(x^4 + 1)^{\frac{1}{4}} + c$

(3) $-(x^4 + 1)^{\frac{1}{4}} + c$

(4) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

Q75. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)} dx$ is equal to:

(1) 2

(2) 4

(3) 1

(4) 6

Q76. The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x \text{ and } y \geq 4x - 1\}$ is:

(1) $\frac{7}{32}$

(2) $\frac{5}{64}$

(3) $\frac{15}{64}$

(4) $\frac{9}{32}$

Q77. Let y(x) be the solution of the differential equation $(x \log x) \frac{dy}{dx} + y = 2x \log x, (x \geq 1)$.

Then y(e) is equal to:

(1) e

(2) 0

(3) 2

(4) 2e

Q78. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is:

(1) 901

(2) 861

(3) 820

(4) 780

Q79. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a:

(1) straight line parallel to x-axis.

(2) straight line parallel to y-axis.

(3) circle of radius $\sqrt{2}$.

(4) circle of radius $\sqrt{3}$.

Q80. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is:

(1) 1

(2) 2

(3) 3

(4) 4

Q81. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is:

(1) $\frac{27}{4}$

(2) 18

(3) $\frac{27}{2}$

(4) 27

Q82. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is:

(1) $x^2 = y$

(2) $y^2 = x$

(3) $y^2 = 2x$

(4) $x^2 = 2y$

Q83. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is:

(1) $2\sqrt{14}$

(2) 8

(3) $3\sqrt{21}$

(4) 13

Q84. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is:

(1) $2x + 6y + 12z = 13$

(2) $x + 3y + 6z = -7$

(3) $x + 3y + 6z = 7$

(4) $2x + 6y + 12z = -13$

Q85. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is:

(1) $\frac{2\sqrt{2}}{3}$

(2) $\frac{-\sqrt{2}}{3}$

(3) $\frac{2}{3}$

(4) $\frac{-2\sqrt{3}}{3}$

Q86. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is:

(1) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ Note: Question seems to be wrong as per our expert

(2) $55\left(\frac{2}{3}\right)^{10}$

(3) $220\left(\frac{1}{3}\right)^{12}$

(4) $22\left(\frac{1}{3}\right)^{11}$

Q87. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, then the mean of the resultant data, is:

(1) 16.8

(2) 16.0

(3) 15.8

(4) 14.0

Q88. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is:

(1) $\sqrt{3} : 1$

(2) $\sqrt{3} : \sqrt{2}$

(3) $1 : \sqrt{3}$

(4) 2 : 3

Q89. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is:

(1) $\frac{3x-x^3}{1-3x^2}$

(2) $\frac{3x+x^3}{1-3x^2}$

(3) $\frac{3x-x^3}{1+3x^2}$

(4) $\frac{3x+x^3}{1+3x^2}$

Q90. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) $s \wedge \sim r$

(2) $s \wedge (r \wedge \sim s)$

(3) $s \vee (r \vee \sim s)$

(4) $s \vee r$

Answer Key and Explanations

Sol.61 (1)

n(A) = 4

n(B) = 2

$n(A \times B) = n(A) . n(B)$

$= 4 \times 2$

$= 8$

Number of subsets having atleat 3 elements $= {}^8C_3 + {}^8C_4 + {}^8C_5 ...... {}^8C_8$

$= 2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2)$

$= 219$

Sol. 62 (3)

$|z| = 1$

$\left|\frac{z_1 - 2z_2}{2 - z_1 z_2}\right| = 1$

$|z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$

$(z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)\,(2 - \bar{z}_1 z_2)$

$|z_1|^2 - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4|z_2|^2 = 4 - 2z_1 z_2 - 2z_1\bar{z}_2 + |z_1|^2|z_2|^2$

$|z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2|z_2|^2$

$|z_1|^2 - |z_1|^2|z_2|^2 = 4 - 4|z_2|^2$

$|z_1|^2\,(1 - |z_2|^2) = 4\,(1 - |z_2|^2)$

$\Rightarrow |z_1|^2 = 4$

$\Rightarrow |z_1| = 2$

Sol. 63 (3)

$x^2 - 6x - 2 = 0$

$\frac{(\alpha^{10} - ß^{10}) - 2(\alpha^8 - ß^8)}{2(\alpha^9 - ß^9)}$

$\alpha^2 - 6\alpha - 2 = 0$ $\qquad \alpha^{10} - 2\alpha^8 = 6\alpha^9$

$ß^{10} - 2ß^8 = 6ß^9$

$\frac{6\,(\alpha^9-ß^9)}{2(\alpha^9-ß^9)} = 3$

Sol. 64 (4)

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$

$AA^T = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$

$\begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix}$

$a+4+2b=0$

$2a+2-2b=0$

$3a+6=0$

$a=-2$

$-2+4+2b=0$

$2+2b=0$

$b=-1$

$(-2,-1)$

Sol.65 (3)

$(2-\lambda)x_1 - 2x_2 + x_3 = 0$

$2\,x_1 - (3+\lambda)\,x_2 + 2x_3$

$-x_1 + 2x_2 - \lambda x_3 = 0$

$\begin{bmatrix} 2-\lambda & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Delta = [(2-\lambda)\,[\lambda(3+\lambda) - 4] + 2\,(-2\lambda + 2) + 1\,(4 - (3+\lambda)]$

$(2-\lambda)\,[3\lambda+\lambda^2 - 4] + 4 - 4\lambda + 1 - \lambda = 0$

$6\lambda + 2\lambda^2 - 8 - 3\lambda^2 - \lambda^3 + 4\lambda - 4\lambda + 4 + 1 - \lambda$

$5\lambda - \lambda^2 - 3 - \lambda^3 = 0$

$\lambda^3 + \lambda^2 - 5\lambda + 3 = 0$

$(\lambda - 1)\ [\lambda^2 + 2\lambda - 3]$

$[\lambda^2 + 3\lambda - \lambda - 3]$

for more – trivial solutions

$\Delta = 0$

Hence for 2 values

1, -3

Sol. 66 (2)

6,000

$\Rightarrow 3 \times 4 \times 3 \times 2 = 72$

$\underline{\ldots\ldots\ldots\ldots 5! = 120}$

$= 192$

Sol. 67 (1)

$$(1-2\sqrt{x})^{50} = {}^{50}C_0(1)^0(-2\sqrt{x})^{50} - {}^{50}C_1(1)^1\left(2\sqrt{x}\right)^{49}$$

$$(1+2\sqrt{x})^{50} = {}^{50}C_0(1)^0\left(2\sqrt{x}\right)^{50} + \ldots\ldots\ldots\ldots$$

$$\left(1+2\sqrt{x}\right)^{50} + \left(1-2\sqrt{x}\right)^{50} = \left[{}^{50}C_0 + \ldots\ldots\ldots\ldots\right]$$

$$3^{50} + (-1)^{50} = 2 \times \text{Required}.$$

$$\frac{3^{50}+1}{2} = \text{Required}.$$

Sol. 68 (2)

$m = \frac{l+n}{2}$

$l\ G_1\ G_2\ G_3\ n$

$G_1 = rl$

$G_2 = r^2 l$

$G_3 = r^3 l$

$n = r^4 l \qquad \Rightarrow r = \left(\frac{n}{l}\right)^{1/4}$

$\text{Required} = G_1^4 + 2G_2^4 + G_3^4$

$= r^4 l^4 + 2(r^2 l)^4 + (r^3 l)^4$

$= l^4\{r^4 + 2(r^4) + (r^4)^3\}$

$= l^4\left\{\frac{n}{l}+\frac{2n^2}{l^2}+\frac{n^3}{l^3}\right\}$

$= nl^3 + 2n^2l^2 + n^3l.$

$= nl\,\{l^2 + 2nl + n^2\}$

$= nl\,\{l+n\}^2$

$= nl\,(2m)^2 = 4\,lm^2n$

Sol. 69 (2)

$T_r = \frac{1^3+2^3+3^3+\ldots\ldots r^3}{1+3+5+\ldots\ldots(2r-1)}$

$\frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2} = \frac{(r+1)^2}{4}$

$$\sum T_t = \frac{1}{4}\sum_{r=1}^{9}(r+1)^2$$

$$= \frac{1}{4}\left\{2^2+3^2+\ldots\ldots 10^2\right\}$$

$$= \frac{1}{4}\left\{\frac{10}{6}(11)(21)-1\right\}$$

$$= 96$$

Sol. 70 (3)

$$\lim_{x\to\infty}\frac{(1-\cos 2x)(3+\cos x)}{x\left(\frac{\tan 4x}{4x}\right)\times 4x}$$

$$= \lim_{x\to\infty}\frac{2\sin^2 x\,(3+\cos x)}{4x^2}$$

$\Rightarrow \frac{1}{2}\times 4 = 2$

Sol. 71 (1)

$g(x) = \begin{cases} k\sqrt{x+1} & 0 \le x \le 3 \\ mx+2 & 3 < x \le 5 \end{cases}$

$g'(x) = \begin{cases} \frac{k}{2\sqrt{x+1}} & 0 \le x \le 3 \\ m & 3 < x \le 5 \end{cases}$

For differentiability at 3,

$\frac{k}{2\sqrt{3+1}} = m$

k = 4m ..... (1)

Also g(3⁻) = g(3⁺)

$k\sqrt{3+1} = m(3) + 2$

2k = 3m + 2 ..... (2)

From (1) and (2) $m = \frac{2}{5}$ $k = \frac{8}{5}$

k + m = 2

Sol. 72 (4)

$x^2 + 2xy = 3y^2 = 0$

2x + 2xy' + 2y – 6y y' = 0

(2x – 6y) y' = – (2x + 2y)

$y' = -\frac{(2x+2y)}{2x-6y}$

$$y'|_{(1,1)} = \frac{-4}{-4} = 1$$
$$y'|_{(1,1)}\,(normal) = -1$$

Equation of normal

(y – 1) = – 1 (x – 1)

y – 1 = – x + 1

x + y = 2   x = 2 – y

Solve it with curve

$(2-4)^2 + 2(2-y)y - 3y^2 = 0$

$4 + y^2 - 4y + 4y - 2y^2 - 3y^2 = 0$

$4y^2 = 4$

$y = \pm 11$

x = 1, 3

(1, 1) and (3, –1)

Hence meets the curve again in 4th quadrant

Sol.73 (3)

$$\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]$$

Let f(x) = $ax^4 + bx^3 + cx^2 + dx + e$

$$\lim_{x\to 0}\frac{f(x)}{x^2} = 2$$

$$\frac{ax^4}{x^2}+\frac{bx^3}{x^2}+\frac{cx^2}{x^2}+\frac{dx}{x^2}+\frac{e}{x^2} = 2$$

For limit to exist

d and e = 0

So c = 2

f (x) becomes $ax^4 + bx^3 + 2x^2$

f' (x) = $4ax^3 + 3bx^2 + 4x$

f'(1) = 4a + 3b + h = 0 1

f'(z) = 32a + 12b + 8 = 0 2

8x (1) – (2)

12b + 24 = 0

b = - 2

a = ½

f(x) $\frac{x^4}{2}$ $-2x^3 + 2x^2 \Rightarrow$ f (z) = 8 – 16 + 8

= 0

Sol. 74 (4)

$$\int\frac{dx}{x^2\left(x^4+1\right)^{3/4}}$$

$$\int\frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$$

$$1/x^4 = t$$

$$dt = -4\frac{1}{x^5}dx$$

$$\int\frac{-x^5}{4x^5\left(1+t\right)^{3/4}}dt$$

$$-\int \frac{1}{4(1+t)^{3/4}} dt$$

$$-\frac{(1+t)}{4\times\frac{1}{4}}+c=-\left(1+\frac{1}{x^4}\right)^{1/4}+c$$

Sol. 75 (3)

$$I-\int_2^4 \frac{\log x^2}{\log x^2+\log(x-6)^2} \qquad ...(1)$$

Using property $\int_a^b +(x)\,da=\int_a^b +(a+b-x)$

$$I=\int_2^4 \frac{\log(6-x)^2}{\log(6-x)^2+\log(6-x-6)^2}$$

$$=\int_2^4 \frac{\log(6-x)^2}{\log(6-x)+\log x^2} \qquad ...(2)$$

from (1) and (2)

$$21=\int_2^4 1\,dx$$

$$21=x|_2^4=2$$

$$I=1$$

Sol. 76 (4)

$x=\frac{1}{2}$ $\qquad$ $x=\frac{1}{8}$

$y^2=2x$

when $x=\frac{1}{8}$ $\qquad$ $y^2=2\left(\frac{1}{8}\right)_4$

$y=\frac{1}{2}$

$y=4x-1$

when $x=\frac{1}{8} y=4\left(\frac{1}{8}\right)-1$

$y=\frac{1}{2}-1$

$y=-\frac{1}{2}$

$$\int_{-1/2}^{+1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dx$$

$$\frac{1}{4}\left(\frac{y^2}{2}+y\right)_{-1/2}^{+1}-\frac{1}{2}\left(\frac{y^3}{3}\right)_{-1/2}^{+1}$$

$$\frac{1}{4}\left[\left(\frac{1}{2}+1\right)-\left(\frac{1}{8}-\frac{1}{2}\right)\right]-\frac{1}{2}\left\{\frac{1}{3}+\frac{1}{24}\right\}$$

$$\frac{1}{4}\left\{\frac{3}{2}+\frac{3}{8}\right\}-\frac{1}{2}\left\{\frac{9}{24}\right\}$$

$$\Rightarrow\frac{3}{4}\left(\frac{5}{8}\right)-\frac{9}{48}=\frac{15}{-32}-\frac{9}{48}=\frac{9}{32}$$

Sol. 77 (2) Note: Insufficient information as per our expert

$$(\mathrm{x}\log \mathrm{x})\frac{dy}{dx}+y=2x\log x$$

$$\frac{dy}{dx}+\left(\frac{1}{x\log x}\right)y=2$$

$$IF=e^{\int pdx}$$

$$=e^{\int\frac{1}{x\log x}dx}$$

$$=e\log\log x$$

$$=\log x$$

$$y\times IF=\int IF\times Q\,dx$$

$$y\log x=\int 2\log x\ \ dx$$

$$y\log x=2x(\log \mathrm{x}-1)+\mathrm{c}$$

$$\mathrm{y}(\mathrm{c})=\frac{2e(\log \mathrm{e}-1)+\mathrm{c}}{\log e}$$

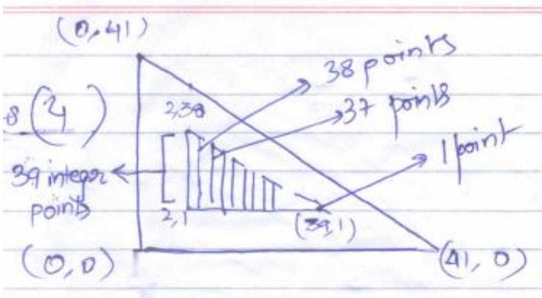
$$y(e)=\frac{c}{\log e}=c\ \ (\text{c is not given so assume c to be 0})$$

Sol. 78 (4)

Numbers of points interior of triangle = 39 + 38 + 37 = H

$=\frac{39\times 40}{2}$

= 780



Sol. 79 (3)

(2x – 3y + 4) + k(a – 2y + 3) = 0

This is an eqn. of formally of straight lines.

So, fixed point $\left.\begin{matrix} 2x - 3y + 4 = 0 \\ x - 2y + 3 = 0 \end{matrix}\right\} solving$

So fixed point $\frac{y=+2 \text{ and } x=+1}{(1,2)}$

Locus will be circle

With radius = $\sqrt{(2-1)^2 + (3-2)^2}$

$= \sqrt{1^2 + 1^2}$

$= \sqrt{2}$

Sol. 80 (3)

Given circles are

$(x-2)^2 + (y-3)^2 = 5^2$

$(x+3)^2 + (y+9)^2 = 8^2$

Distance between centers $= \sqrt{(3+2)^2 + (9+3)^2}$

$= \sqrt{25 + 144}$

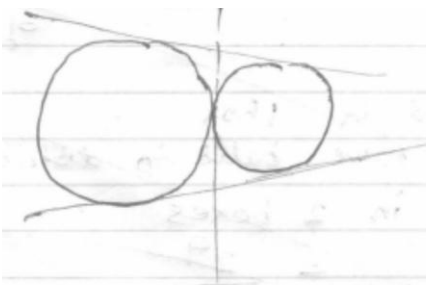
$= \sqrt{169} = 13$

Sum of radius $= 8 + 5 = 13$

which means circles are touching externally

Hence 3 common tangents.

Sol. 81 (4)



Tangent at $\left(2, \frac{5}{3}\right)$

$\frac{2x}{9} + \frac{5 \times y}{3 \times} = 1$

$\frac{2x}{9} + \frac{y}{3} = 1$

$6x + y = 9$

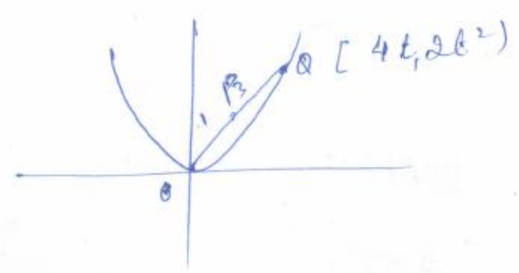
$\frac{x}{9/2} + \frac{4}{9} = 0 \rightarrow l$

Area of Δ formed $= \frac{27}{4}$

by l with axis

Now due to symmetry area of quad will be 4 times i.e 27 sq units.

Sol. 82 (4)



$\frac{OP}{PQ} = 1 : 3$

$h = \frac{4t}{4}\ t = h$

$k = \frac{2t}{4}\ t^2 = 2k$

$\therefore h^2 = 2k$

$x^2 = 2y$

Sol. 83 (4)

$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = k$

$x = 3k + 2$

$y = 4k - 1$

$z = 12k + 2$

$\therefore x + z - y = 16$

$3k + 2 + 12k + 2 - 4k + 1 = 16$

$11k = 11$

$k = 1$

$\therefore (x, y, z) = (5, 3, 14)$

Distance

$= \sqrt{16+9+144}$

$= \sqrt{144+25}$

$\sqrt{169} = 13$

Sol. 84 (3)

plane parallel to $x + 3y + 6z = 1$

will be $x + 3y + 6z = k$

and as the lines lieon the plane

so it must pass through common point

i e (1,0,1)

$\therefore 1 + 0 + 6 = k$

$k = 7$

$\therefore x + 3y + 6z = 7$

Sol. 85 (1)

$(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$

$(\vec{b}\cdot\vec{c})\,\vec{a} = (\vec{a}\cdot\vec{c})\vec{b} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$

$\left(\vec{b}\cdot\vec{c} - \frac{1}{3}|\vec{b}||\vec{c}|\right)\vec{a} = (\vec{a}\cdot\vec{c})\vec{b}$

$\left(|\vec{b}||\vec{c}|\cos\theta - \frac{1}{3}|\vec{b}|\,|\vec{c}|\right)\vec{a} = (\vec{a}\cdot\vec{c})\vec{b}$

Since $\vec{a}$ and $\vec{b}$ non collinear so $\vec{a} \neq \lambda\vec{b}$ non zero

i.e. $|\vec{b}|\,|\vec{c}|\cos\theta - \frac{1}{3}|\vec{b}|\,|\vec{c}| = 0$ (one of the possible values)

$\cos\theta = \frac{1}{3}$

$\sin\theta = \frac{2\sqrt{2}}{3}$

Sol.86 Note: Question seems to be wrong as per our expert

Sol.87 (4)

Sum of 16 observations $= 16 \times 16$

$= 256$

Sum of remaining 15 observations $= 240$

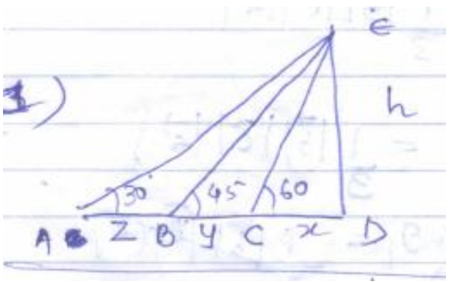
New sum of 18 observations $= 240 + 3 + 4 + 5$

$= 252$

Mean of 18 observations $= \frac{\mathbf{252}}{\mathbf{18}}$

$= 14$

Sol.88 (1)



dϵ is tower

$\tan 60 = \frac{h}{x}$

$x = \frac{h}{\sqrt{3}}$ $\quad$ ..... (1)

$\tan 45 = \frac{h}{x+y}$

$x + y = h$ $\quad$ .... (2)

$\tan 30 = \frac{h}{x+y+z}$

$x + y + z = \sqrt{3}h$ $\quad$ ...... (3)

To find $\frac{AB}{BC} = \frac{Z}{Y}$

From eq. (2) – (1)

$y = h\left(1 - \frac{1}{\sqrt{3}}\right)$ $\quad$ ... (4)

From eq. (3) – (2)

$z = \cdot h\left(\sqrt{3} - 1\right)$ $\quad$ ..... (5)

From eq. (5) ÷ (4)

$\frac{z}{y} = \frac{h(\sqrt{3}-1)}{h\left(1-\frac{1}{\sqrt{3}}\right)} = \sqrt{3}$

Sol. 89 (1)

$|x| < \frac{1}{\sqrt{3}}$

$\frac{-1}{\sqrt{3}} \leq x \leq \frac{1}{\sqrt{3}}$

$\therefore -\frac{\pi}{3} \leq \tan^{-1} x \leq \frac{\pi}{3}$

and $2\tan^{-1} x = \tan^{-1}\left\{\frac{2x}{1-x^2}\right\}$

$= \tan^{-1} x + 2\tan^{-1} x$

$\tan^{-1} x = 3\tan^{-1} x$

$y = \tan[3\tan^{-1} x]$

$\therefore y = \frac{3x-x^3}{1-3x^2}$

Sol. 90 (4)

r	s	~r	~r ∧s	~s	~s ∨(~r∧s)	Negation
T	T	F	F	F	F	T
T	F	F	T	T	T	F
F	T	T	T	F	T	F
F	F	T	F	T	T	F