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Quadratic Equation and In equations (Inequalities) Solutions

SUBJECTIVE PROBLEMS:

Sol 1.

$4^x - 3^{x-1/2} = 3^{x+1/2} - (2^2)^{x}/2$

$\Rightarrow 4^x - 3^x/\sqrt{3} = 3^x\sqrt{3} - 4^x/2$

$\Rightarrow 3/2\ 4^x = 3^x(\sqrt{3} + 1/\sqrt{3}) \Rightarrow 3/2\ 4^x = 3^x\ 4/\sqrt{3}$

$\Rightarrow (4/3)^{x-3/2} = 1 \Rightarrow x - 3/2 = 0$

$\Rightarrow x = 3/2$

Sol 2.

RHS = $(m - 1, n + 1) + x^{m-n-1}(m - 1, n)$

$= (1 - x^{m-1})(1 - x^{m-2}) \ldots\ldots (1 - x^{m-n-1}) / (1 - x)(1 - x^2) \ldots\ldots\ldots (1 - x^{m+1})$

$+ x^{m-n-1}[(1 - x^{m-1})(1 - x^{m-2}) \ldots\ldots\ldots (1 - x^{m-n}) / (1 - x)(1 - x^2) \ldots\ldots (1 - x^n)]$

$= (1 - x^{m-1})(1 - x^{m-2}) \ldots\ldots (1 - x^{m-n}) / (1 - x)(1 - x^2) \ldots\ldots (1 - x^n)$

$[1 - x^{m-n-1}/1 - x^{n+1} + x^{m-n-1}]$

$[1 - x^{m-n-1} + x^{m-n-1} - x^m/1 - x^{n+1}]$

$= (1 - x^m)(1 - x^{m-1}) \ldots\ldots (1 - x^{m-n})/ (1 - x)(1 - x^2) \ldots\ldots (1 - x^n)(1 - x^{n+1})$

$= (m, n + 1)$ = L. H. S.

Sol 3.

$\sqrt{x + 1} = 1 + \sqrt{x - 1}$

Squaring both sides, we get

$x + 1 = 1 + x - 1 + 2\sqrt{x - 1} \Rightarrow 1 = 2\sqrt{x - 1}$.

$\Rightarrow 1 = 4(x - 1)$

$\Rightarrow x = 5/4$

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Sol 4.

Given $a > 0$, so we have to consider two cases: $a \neq 1$ and $a = 1$. Also it is cleat that $x > 0$

And $x \neq 1$, $ax \neq 1$, $a^2x \neq 1$

Case I: If $a > 0, \neq 1$

Then given equation can be simplified as

$2/\log_a x + 1/\log_a x + 3/2 + \log_a x = 0$

Putting $\log_a x = y$, we get

$2(1 + y)(2 + y) + y(2 + y) + 3y(1 + y) = 0$

$\Rightarrow 6y^2 + 11y + 4 = 0 \Rightarrow y = -4/3$ and $-1/2$

$\Rightarrow \log_a x = -4/3$ and $\log_a x = -1/2$

$\Rightarrow x = a^{-4/3}$ and $x = a^{-1/2}$

Case II: If $a = 1$ then equation becomes

$2\log_x 1 + \log_x 1 + 3\log_x 1 = 6\log_x 1 = 0$

Which is true $\forall\, x > 0, \neq 1$

Hence solution is if $a = 1$, $x > 0, \neq 1$

If $a > 0, \neq 1$; $x = a^{-1/2}, a^{-4/3}$

Sol 5.

Let $x = \frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$

$\Rightarrow x^2 = \frac{26-15\sqrt{3}}{50+38+5\sqrt{3}-10\sqrt{76+10\sqrt{3}}}$

$\Rightarrow x^2 = \frac{26-15\sqrt{3}}{88+5\sqrt{3}-10\sqrt{71+1+10\sqrt{3}}}$

$\Rightarrow x^2 = \frac{26-15\sqrt{3}}{88+5\sqrt{3}-10\sqrt{(5\sqrt{3})^2+(1)^2 x\, 5\sqrt{3}\, x\, 1}}$

$\Rightarrow x^2 = \frac{26-15\sqrt{3}}{88+5\sqrt{3}-10\sqrt{(5+\sqrt{3}+1)^2}}$

$\Rightarrow x^2 = \frac{26-15\sqrt{3}}{88+5\sqrt{3}-10(5\sqrt{3}+1)} \Rightarrow x^2 = \frac{26-15\sqrt{3}}{88+5\sqrt{3}-50\sqrt{3}-10} = \frac{26-15\sqrt{3}}{78-45\sqrt{3}} = \frac{26-15\sqrt{3}}{3(26-15\sqrt{3})} = / \frac{1}{3}$, which is rational number.

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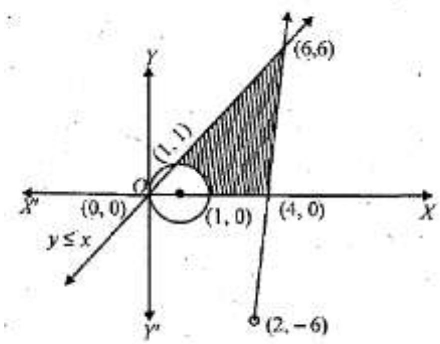
Sol 6.

$x^2 + y^2 = 2x \geq 0 \Rightarrow x^2 - 2x + 1 + y^2 \geq 1$

$\Rightarrow (x - 1)^2 + y^2 \geq 1$ which represents the boundary and exterior region of the circle with Centre at (1, 0)

And radius as 1. For $3x - y \leq 12$, the corresponding equation is $3x - y = 12$; any two points on it can be taken as (4, 0), (2, -6); Also putting (0, 0) in given in equation, $0 \leq 12$ which true.



∴given in equation represents that half plane region of line $3x - y = 12$ which contains origin.

For $y \leq x$, the corresponding equation $y = x$ has any two points on it as (0, 0) and (1, 1). Also putting (2, 1) In the given in equation, we get $1 \leq 2$ which is true, so $y \leq x$ represents that half plane which contains the points (2, 1).$y \geq 0$ represents upper half Cartesian plane.

Combining all we find the solution set as shaded region in the graph.

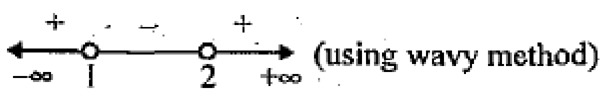
Sol 7.

There are two parts of this question

$(5x - 1) < (x + 1)^2$ and $(x + 1)^2$ and $(x + 1)^2 < (7x - 3)$ Taking first part

$(5x - 1) < (x + 1)^2 \Rightarrow 5x - 1 < x^2 + 2x + 1$

$\Rightarrow x^2 - 3x + 2 > 0 \Rightarrow (x - 1)(x - 2) > 0$

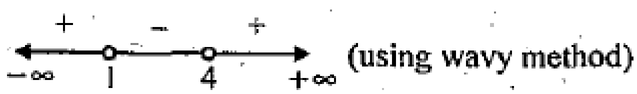


$\Rightarrow x < 1$ or $x > 2$ . . . .(1)

Taking second part

$(x + 1)^2 < (7x - 3) \Rightarrow x^2 - 5x + 4 < 0$

$\Rightarrow (x - 1)(x - 4) < 0$



$\Rightarrow 1 < x < 4$ . . . . (2)

Combining (1) and (2) [taking common solution], we get $2 < x < 4$ but x is an integer therefore x = 3.

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Sol 8.

∵ α, β are the roots of $x^2 + p x + q = 0$

∴ $\alpha + \beta = -p, \quad \alpha\beta = q$

∵ γ, δ are the roots of $x^2 + rx + s = 0$

∴ $\gamma + \delta = -r, \gamma\,\delta = s$

Now, $(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$

$= [\alpha^2 - (\gamma + \delta)\alpha + \gamma\,\delta]\,[\beta^2 - (\gamma + \delta)\beta + \gamma\,\delta]$

$= \alpha^2 + r\alpha + s]\,[\beta^2 + r\beta + s]$ [∵α, β are roots of $x^2 + p x + q = 0$

∴ $\alpha^2 + p\alpha + q = 0$ and $\beta^2 + p\beta + q = 0]$

$= [(r - p)\alpha + (s - q)]\,[(r - p)\beta + (s - q)]$

$= (r - p)^2 \alpha\beta + (r - p)(s - q)(\alpha + \beta) + (s - q)^2$

$= q(r - p)^2 - p(r - p)(s - q) + (s - q)^2$

Now if the equations $x^2 + p x + q = 0$ and $x^2 + r x + s = 0$ have a common root say α, then $\alpha^2 + p\alpha + q = 0$ and $\alpha^2 + r\alpha + s = 0$

$\Rightarrow \alpha^2/ps - q r = \alpha/q - s = 1/r - p$

$\Rightarrow \alpha^2 = ps - q r/r - p$ and $\alpha = q - s/r - p$

$\Rightarrow (q - s)^2 = (r - p)(ps - q r)$ which is the required condition.

Sol 9.

We know that for sides a, b, c of a Δ

$(a - b)^2 \geq 0$

$\Rightarrow a^2 + b^2 \geq 2ab$ .... (1)

Similarly $b^2 + c^2 \geq 2bc$ .... (2)

$C^2 + a^2 \geq 2ca$ .... (3)

Adding the three in equations, we get

$2(a^2 + b^2 + c^2) \geq 2(ab + b\,c + ca)$

$\Rightarrow a^2 + b^2 + c^2 \geq ab + b\,c + ca$

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Adding 2 (ab + b c + ca) to both sides, we get

$(a + b + c)^2 \geq 3(ab + b c + ca)$

Or $3(ab + b c + ca) \leq (a + b + c)^2$ . . . . (A)

Also $c < a + b$ (triangle inequality)

$\Rightarrow c^2 < ac + b c$ . . . . (4)

Similarly $b^2 < ab + b c$ . . . . (5)

$a^2 < ab + ca$ . . . . (6)

Adding (4), (5) and (6), we get $a^2 + b^2 + c^2 < 2(ab + b c + ca)$

Adding 2 (ab + b c + ca) to both sides, we get

$\Rightarrow (a + b + c)^2 < 4(ab + b c + ca)$ . . . . (B)

Combining (A) and (B), we get

$3(ab + b c + ca) \leq (a + b + c)^2 < 4(ab + b c + ca)$

First two expressions will be equal for $a = b = c$.

Sol 10.

Given that $n^4 < 10^n$ for n for a fixed + ve integer $n \geq 2$.

To prove that $(n + 1)^4$, 10^{n+1}

Proof: Since $n^4 < 10^n \Rightarrow 10n^4 < 10^{n+1}$ . . . . (1)

So it is sufficient to prove that $(n + 1)^4 < 10n^4$

Now $(n + 1/n)^4 = (1 + 1/n)^4 \leq (1 + 1/2)^4$ [$\because n \geq 2$]

$= 81/16 < 10$

$\Rightarrow (n + 1)^4 < 10n^4$ . . . . (2)

From (1) and (2), $(n + 1)^4 < 10^{n+1}$

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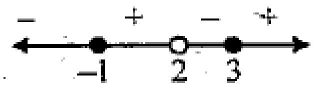
Sol 11.

$Y = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$

y will take all real values if $\frac{(x+1)(x+3)}{x-2} \geq 0$

By wavy method:->



$x \in [-1, 2) \cup [3, \infty)$

[2 is not included as it makes denominator zero, and hence y an undefined number.]

Sol 12.

The given equations are 3x + my – m = 0 and 2x – 5y – 20 = 0 Solving these equations by cross product

method, we get

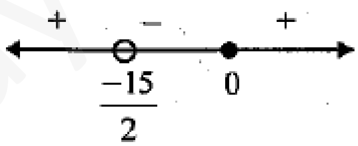
x/-20m – 5m = y/-2m + 60 = 1/-15 – 2m NOTE THIS STEP

⇒ x = 25m/2m + 15, y = 2m – 60/2m + 15

For x > 0 ⇒ 25m/2m + 15 > 0

⇒ m < -15/2 or m > 0 ....(1)

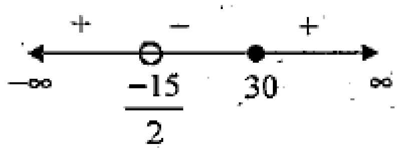
[using wavy method]:->



For y > 0 ⇒ 2(m – 30)/2m + 15 = 0

⇒ m < -15/2 or m >
30 ....(2)

[using wavy method]:->



Combining (1) and (2), we get the common values of m as follows:

m < 15/2 or m > 30 ∴ m ∈ (-∞, -15/2) U (30, ∞)

Sol 13.

The given system is

x + 2y + z = 1 ....(1)

2x – 3y – ω ≥ 0

Multiplying eqn. (1) by (2) and subtracting from (2), we get

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$7y + 2z + \omega = 0 \Rightarrow \omega = -(7y + 2z)$

Now if $y, z > 0$, $\omega < 0$(not possible)

If $y = 0, z = 0$ then $x = 1$ and $\omega = 0$.

$\therefore$The only solution is $x = 1, y = 0, z = 0, \omega = 0$.

Sol 14.

$e^{\sin x} - e^{-\sin x} - 4 = 0$

Let $e^{\sin x} = y$ then $e^{-\sin x} = 1/y$

$\therefore$ Equation becomes, $y - 1/y - 4 = 0$

$\Rightarrow y^2 - 4y - 1 = 0$

$\Rightarrow y = 2 + \sqrt{5}, 2 - \sqrt{5}$

But y is real +ve number,

$\therefore y \neq 2 - \sqrt{5} \Rightarrow y = 2 + \sqrt{5}$

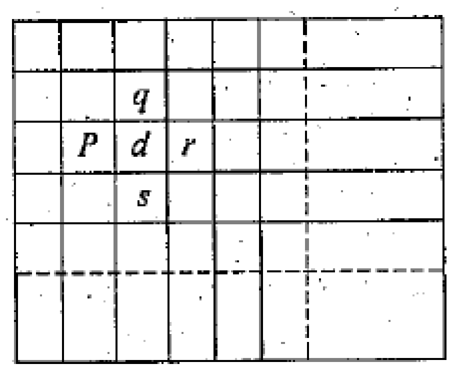
$\Rightarrow e^{\sin x} = 2 + \sqrt{5} \Rightarrow \sin x = \log_e (2 + \sqrt{5})$

But $2 + \sqrt{5} > e \Rightarrow \log_e (2 + \sqrt{5}) > \log_e e$

$\Rightarrow \log_e (2 + \sqrt{5}) > 1$ Hence, $\sin x > 1$

Which is not possible $\therefore$Given equation has no real solution.

Sol 15.



For any square can be at most 4, neighbours squares.

Let for a square having largest number d, p, q, r, s be written then

According to the question,

$p + q + r + s = 4d$

$\Rightarrow (d - p) + (d - q) + (d - r) + (d - s) = 0$

Sum of four +ve numbers can be zero only if these are zero individually

$\therefore d - p = 0 = d - q = d - r = d - s$

$\Rightarrow p = q = r = s = d$

$\Rightarrow$ all the numbers written are same. Hence Proved.

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Sol 16.

Let α, β be the roots of eq. $ax^2 + bx + c = 0$

According to the question,

$B = \alpha^n$

Also $\alpha + \beta = -b/a$; $\alpha\beta = c/a$

$\alpha\beta = c/a \Rightarrow \alpha.\ \alpha^{n} = c/a \Rightarrow \alpha = (c/a)^{1/n+1}$

then $\alpha + \beta = -\,b/a \Rightarrow \alpha + \alpha^n = -b/a$

Or $(c/a)^{1/n+1} + (c/a)^{n/n+1} = -b/a$

$\Rightarrow a\,.(c/a)^{1/n+1} + a\,.(c/a)^{n/n+1} + b = 0$

$\Rightarrow a^{n/n+1}\, c^{1/n+1} + a^{1/n+1}\, c^{n/n+1} + b = 0$

$\Rightarrow (a^n c)^{1/n+1} + (can)^{1/n+1} + b = 0$ Hence proved.

Sol 17.

$x^2 - 3x + 2 > 0$, $x^2 - 3x - 4 \le 0$

$\Rightarrow (x - 1)(x - 2) > 0$ and $(x - 4)(x + 1) < 0$

$\Rightarrow x \in (-\infty, 1) \cup (2, \infty)$ and $x \in [-1, 4]$

$\therefore$ Common solution is $[-1, 1) \cup (2, 4)]$

Sol 18.

The given equation is

$\left(5 + 2\sqrt{6}\right)^{x^2-3} + \left(5 - 2\sqrt{6}\right)^{x^2-3} = 10$ .... (1)

Let $\left(5 + 2\sqrt{6}\right)^{x^2-3} = y$ .... (2)

Then $\left(5 - 2\sqrt{6}\right)^{x^2-3} = \left(\frac{(5-2\sqrt{6})(5+2\sqrt{6})}{5+2\sqrt{6}}\right)^{x^2-3}$

$= \left(\frac{25-24}{5+2\sqrt{6}}\right)^{x^2-3} = \left(\frac{1}{5+2\sqrt{6}}\right)^{x^2-3} = 1/y$ (Using (2))

$\therefore$The given equation (1) be becomes $y + 1/y = 10$

$\Rightarrow y^2 - 10y + 1 = 0$

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$\Rightarrow y = 10 \pm \sqrt{100\text{-}4}/2 = 10 \pm 4\sqrt{6}/2$

$\Rightarrow y - 5 + 2\sqrt{6}$ or $5 - 2\sqrt{6}$

Consider $y = 5 + 2\sqrt{6}$

$\Rightarrow \left(5 + 2\sqrt{6}\right)^{x^2-3} = (5 + 2\sqrt{6})$

$\Rightarrow x^2 - 3 = 1 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

Again consider

$y = 5 - 2\sqrt{6} = 1/5 + 2\sqrt{6} = (5 + 2\sqrt{6})^{-1}$

$\Rightarrow \left(5 + 2\sqrt{6}\right)^{x^2-3} = (5 + 2\sqrt{6})^{-1} \Rightarrow x^2 - 3 = -1$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm \sqrt{2}$ Hence the solution are 2, -2, $\sqrt{2}$, $-\sqrt{2}$.

Sol 19.

The given equation is,

$X^2 - 2a\ |x - a| - 3a^2 = 0$

Here two cases are possible

Case I: $x - a > 0$ then $|x - a| = x - a$

∴ Eq. Becomes

$X^2 - 2a(x - a) - 3a^2 = 0$

Or $x^2 - 2ax - a^2 = 0 \Rightarrow x = 2a \pm \sqrt{4a^2 + 4a^2}/2$

$\Rightarrow x = a \pm a\sqrt{2}$

Case II: $x - a < 0$ then $|x - a| = -(x - a)$

∴ Eq. becomes

$X^2 + 2a(x - a) - 3a^2 = 0$

Or $x^2 + 2ax - 5a^2 = 0$

$\Rightarrow x = -2a \pm \sqrt{4a^2 + 20a^2}/2 \Rightarrow x = -2a \pm 2a\sqrt{6}/2$

$x = -a \pm a\sqrt{6}$ Thus the solution set is $\{a \pm a\sqrt{2}, -a \pm a\sqrt{6}\}$

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Sol 20.

We are given $2x/2x^2 + 5x + 2 > 1/x + 1$

$\Rightarrow 2x/2x^2 + 5x + 2 - 1/x + 1 > 0$

$\Rightarrow 2x^2 + 2x - 2x^2 - 5x - 2/ (2x^2 + 5x + 2) (x + 1) > 0$

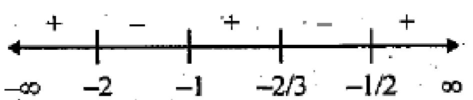
$\Rightarrow -3x -2/ (2x + 1) (x + 1) (x + 2) > 0$

$\Rightarrow (3x + 2)/ (x + 1) (x + 2) (2x + 1) < 0$

$\Rightarrow (3x + 2) (x + 1) (x + 2) (2x + 1)/ (x + 1)^2 (x + 2)^2 (2x + 1)^2 < 0$

$\Rightarrow (3x + 2) (x + 1) (x + 2) (2x + 1) < 0$ .... (1)

NOTE THIS STEP: Critical pts. Are $x = -2/3, -1, -2, -1/2$ on number line



Clearly Inequality (1) holds for,

$x \in (-2, -1) \cup (-2/3, -1/2)$ [as $x \neq -2, -1, -2/3, -1/2$]

Sol 21.

We are given that α_1, α_2 are the roots of

$ax^2 + bx + c = 0$

$\therefore \alpha_1 + \alpha_2 = -b/a; \alpha_1 \alpha_2 = c/a$ .... (1)

And β_1, β_2 are the roots of $px^2 + qx + r = 0$

$\therefore \beta_1 + \beta_2 = -q/p; \beta_1 \beta_2 = r/p$ .... (2)

The system of equations, $\alpha_1 y + \alpha_2 z = 0$

And $\beta_1 y + \beta_2 z = 0$ has a non trivial solution.

$\therefore$we must have $|\alpha_1 \beta_1 \ \alpha_2 \beta_2| = 0$

NOTE THIS STEP:

$\Rightarrow \alpha_1 \beta_2 - \alpha_2 \beta_1 = 0 \Rightarrow \alpha_1/ \alpha_2 = \beta_1/\beta_2$

By componendo and dividend, we get

$\alpha_1 + \alpha_2/ \alpha_1 - \alpha_2 = \beta_1 + \beta_2/ \beta_1 - \beta_2$

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$\Rightarrow$ $(\alpha_1 + \alpha_2)(\beta_1 - \beta_2) = (\alpha_1 - \alpha_2)(\beta_1 + \beta_2)$

$\Rightarrow (\alpha_1 + \alpha_2)^2 [(\beta_1 - \beta_2)^2 - 4\beta_1 \beta_2]$

$= [(\alpha_1 + \alpha_2)^2 - 4\alpha_1\alpha_2](\beta_1 + \beta_2)^2$

Using equations (1) and (2) we get

$b^2/a^2 [q^2/p^2 - 4r/p] = q^2/p^2 [b^2/a^2 - 4c/a]$

$\Rightarrow b^2 q^2/a^2 p^2 - 4b^2r/a^2r = q^2 b^2/q^2 a^2 - 4cq^2/ap^2 \Rightarrow -4b^2 r/a^2 p = -4cq^2/ap^2 \Rightarrow b^2r/a = sq^2/p \Rightarrow b^2/q^2 = ac/p\, r$ Hence Proved

<u>**Sol 22.**</u>

The Given equation is,

$|x^2 + 4x + 3| + 2x + 5 = 0$

Now there can be two cases.

Case I: $x^2 + 4x + 3 \geq 0 \Rightarrow (x + 1)(x + 3) \geq 0$

$\Rightarrow x \in (-\infty, -3] \cup [-1, \infty)$ .... (i)

Then given equation becomes,

$\Rightarrow x^2 + 6x + 8 = 0$

$\Rightarrow (x + 4)(x + 2) = 0 \Rightarrow x = -4, -2$

But x = -2 does not satisfy (i), hence rejected

$\therefore x = -4$ is the sol.

Case II: $x^2 + 4x + 3 < 0$

$\Rightarrow (x + 1)(x + 3) < 0$

$\Rightarrow x \in (-3, -1)$ .... (ii)

Then given equation becomes,

$-(x^2 + 4x + 3) + 2x + 5 = 0$

$\Rightarrow -x^2 - 2x + 2 = 0 \Rightarrow x^2 + 2x - 2 = 0$

$\Rightarrow x = -2 \pm\sqrt{4 + 8}/2 \Rightarrow x = -1 + \sqrt{3}, -1 - \sqrt{3}$

Out of which $x = -1 - \sqrt{3}$ is sol. Combining the two cases we get the solutions of given equation as $x = -4, -1 - \sqrt{3}$.

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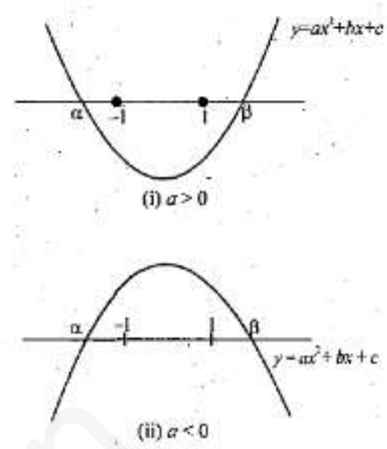
Sol 23.

Given that for a, b, c ∈ R, $ax^2 + bx + c = 0$ has two real roots α and ß, where α < -1 and ß > 1. There may be two cases depending upon value of a, as shown below.



In each of cases (i) and (ii) of (-1) < 0 and of (1) < 0

⇒ a (a – b + c) < 0 and a (a + b + c) < 0

Dividing by a^2 (>0), we get

1 – b/a + c/a < 0

. . . . (1)

And 1 + b/a + c/a < 0 . . . . (2)

Combining (1) and (2) we get

$1 + \left|\frac{b}{a}\right| + \frac{c}{a} < 0$ or $1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$ Hence Proved.

Sol 24.

The given equation is,

$2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$

On the basis of absolute values involved here (|y| and | $2^{y-1} - 1$), there are two critical pts 0 and l.

So we shall consider three cases, when y lies in three different intervals namely (- ∞, 0), [0, 1], (1,∞)

Case I: y∈ (- ∞, 0) then

$|y| = -y$ and $|2^{y-1} - 1| = 1 - 2^{y-1}$

∴The given equation becomes

$2^{-y} - 1 + 2^{y-1} = 2^{y-1} + 1$

⇒ $2^{-y} = 2$ ⇒ y = -1 ∈ (- ∞, 0)

Case II: y ∈ [0, 1]

If y = 0 we get 1 - |1/2 – 1| = 1/2 + 1

1 – 1/2 = 1/2 + 1 (not satisfied)

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$\therefore y = 0$ is not a sol ^{n}

If $y = 1$ we get $2 - |2^0 - 1| = 2^0 + 1$

$\Rightarrow \quad 2 = 2$ (satisfied)

$\therefore y = 1$ is a sol ^{n}

If $y \in (0, 1)$ then $|y| = y$ and $|2^{y-1} - 1| = 1 - 2^{y-1}$

$\therefore$the eq. ^{n} becomes

$2^y - 1 + 2^{y-1} + 1 \Rightarrow 2^y = 2$

$\Rightarrow y = 1 \notin (0, 1)$

$\therefore y = 1$ is the only sol ^{n} in this case.

Case III: $y \in (1, \infty)$

Then $|y| = 1, |2^{y-1} - 1| = 2^{y-1} - 1$

The given eq. ^{n} becomes, $2^y - 2^{y-1} + 1 = 2^{y-1} + 1$

$\Rightarrow 2^y - 2^y = 0$

Which is satisfied for all real values of y but $y \in (1, \infty)$

$\therefore (1, \infty)$ is the sol ^{n} in this case.

Combining all the cases, we get the sol ^{n} as $y \in \{1\} \cup [1, \infty]$

Sol 25.

$a^2 = p^2 + s^2, b^2 = (1 - p)^2 + q^2$

$c^2 = (1 - q)^2 + (1 - r)^2, a^2 = r^2 + (1 - s)^2$

$\therefore a^2 + b^2 + c^2 + d^2 = \{p^2 + (1 + p)^2\} + \{q^2 - (1 - q)^2\} + \{r^2\} + (1 - r)^2\} + \{s^2 + (1 - s)^2\}$

Where p, q, r, s all vary in the interval [0, 1].

Now consider the function

$y^2 = x^2 + (1 - x)^2, 0 \leq x \leq 1,$

$2y\, d\, y/dx = 2x - 2(1 - x) = 0$

$\Rightarrow x = 1/2$ which $d^2\, y/dx^2 = 4$ i.e. +ve

Hence y is minimum at $x = 1/2$ and it's minimum

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Value is $1/4 + 1/4 = 1/2$

Clearly value is maximum at the end pts which is 1.

$\therefore$ Minimum value of $a^2 + b^2 + c^2 + d^2 = 1/2 + 1/2 + 1/2 + 1/2 = 2$

And maximum value is $1 + 1 + 1 + 1 = 4$. Hence proved.

ALTERNATE SOLUTION:

$x^2 + y^2 \leq (x + y)^2 \leq 1$ if $x + y = 1$

Here $x = p, y = 1 - p \therefore x + y = 1$

$\therefore a^2 + b^2 + c^2 + d^2 \leq 1 + 1 + 1 + 1 = 4$

Again $x^2 + y^2 = (x + y)^2 - 2xy = 1 - 2xy$

NOTE THIS STEP: $\therefore$ Minimum of $(x^2 + y^2) = 1 - 2$(maximum of xy).

Now we know that product of two quantities xy is maximum when the quantities are equal provided their sum is constant.

Here $x + y = p + 1 - p = 1 =$ constant.

$\therefore$xy is maximum when $x/1 = y/1 = x + y/2 = 1/2$

$\therefore x = 1/2, y = 1/2$

Minimum of $x^2 + y^2 = 1 - 2. 1/2. 1/2 = 1 - 1/2 = 1/2$

$\therefore$ Minimum value of

$a^2 + b^2 + c^2 + d^2 = 1/2 + 1/2 + 1/2 + 1/2 = 2$

$\therefore 2 \leq a^2 + b^2 + c^2 + d^2 \leq 4$.

Sol 26.

We know that,

$(\alpha - \beta)^2 = [(\alpha + \delta) - (\beta + \delta)]^2$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\alpha + \delta + \beta + \delta)^2 - 4(\alpha + \delta)(\beta + \delta)$

$\Rightarrow b^2/a^2 - 4c/a = B^2/A^2 - 4C/A \Rightarrow 4ac - b^2/a^2 = 4AC - B^2/A^2$

[Hence $\alpha + \beta = -b/a$, $\alpha\beta = c/a$

$(\alpha + \delta)(\beta + \delta) = -B/A$ and $(\alpha + \delta)(\beta + \delta) = C/A$] Hence proved.

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Sol 27.

$\alpha + \beta = -b/a, \alpha\beta = c/a$

Roots if the equation $a^3 x^3 + abcx + c^3 = 0$ are

$x = -abc \pm \sqrt{(abc)^2 - 4a^3 c^3}/2a^3$

$= (-b/a)(c/a) \pm \sqrt{(b/a)^2 (c/a)^2 - 4(c/a)^3} / 2$

$(\alpha + \beta)\ (\alpha\beta) \pm \sqrt{(\alpha + \beta)^2 (\alpha\beta)^2 - 4(\alpha\beta)^3} / 2$

$= (\alpha\beta)\ ((\alpha + \beta) \pm \sqrt{(\alpha - \beta)^2} / 2$

$= (\alpha\beta)\ ((\alpha + \beta) \pm (\alpha - \beta)/2 = \alpha^2\beta, \alpha\beta^2$

Let γ and δ be the required roots. Then

$\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$.

ALTERNATE SOLUTION:

$ax^2 + bx + c = 0$ has roots α and β. (given)

$\Rightarrow \alpha + \beta = -b/a$ and $\alpha\beta = c/a$

Now, $a^3 x^2 + abcx + c^3 = 0$

Divides the equation by c^2, we get

$a^3/c^2\, x^2 + abcx/c^2 + c^3/c^2 = 0$, $a\,(ax/c)^2 + b\,(ax/c) + c = 0$

$\Rightarrow ax/c = \alpha, \beta$ are the roots

$\Rightarrow x = c/a\,\alpha, c/a\,\beta$ are the roots

$\Rightarrow x = \alpha\beta\alpha, \alpha\beta\beta$ are the roots

$\Rightarrow x\ \alpha^2\beta, \alpha\beta^2$ are the roots

ALTERNATE SOLUTION:

Divide the equation by a^3, we get

$x^2 + b/a.\ c/a.\ x + (c/a)^3 = 0$

$\Rightarrow x^2 - (\alpha + \beta)(\alpha\beta)x + (\alpha\beta)^3 = 0 \Rightarrow x^2 - \alpha^2\beta x - \alpha\beta^2 x + (\alpha\beta)^3 = 0$

$\Rightarrow x(x - \alpha^2\beta) - \alpha\beta^2(x - \alpha^2\beta) = 0 \Rightarrow (x - \alpha^2\beta)(x - \alpha\beta^2) = 0$

$\Rightarrow x = \alpha^2\beta, \alpha\beta^2$ which is the required answer.

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Sol 28.

The given equation is,

$x^2 + (a - b) x + (1 - a - b) = 0$

a, b ∈ R For the eq.^{n} to have unequal real roots ∀ b D > 0

$\Rightarrow (a - b)^2 - 4(1 - a - b) > 0$

$\Rightarrow a^2 + b^2 - 2ab - 4 + 4a + 4b > 0$

$\Rightarrow b^2 + b(4 - 2a) + a^2 + 4a - 4 > 0$

Which is a quadratic expression in b, and it will be true ∀ b ∈ R if discriminant of above of above eq.^{n} less than zero.

i.e., $(4 - 2a)^2 - 4(a^2 + 4a - 4) < 0$

$\Rightarrow (2 - a)^2 - (a^2 + 4a - 4) < 0$

$\Rightarrow 4 - 4a + a^2 - a^2 - 4a + a < 0$

$\Rightarrow -8a + 8 < 0 \Rightarrow a > 1$

Sol 29.

Given that a, b, c are positive real numbers. To prove that $(a + 1)^7 (b + 1)^7 (c + 1)^7 > 7^7 a^4 b^4 c^4$

Consider L. H. S. $= (1 + 7)^7 . (1 + b)^7 . (1 + c)^7$

$= [(1 + a)(1 + b)(1 + c)]^7$

$[1 + a + b + c + ab + bc + ca + abc]^7 > [a + b + c + ab + bc + ca + abc]^7$ ....(1)

Now we know that AM ≥ GM using it for + ve no's a, b, c, ab, bc, ca and abc, we get

$\Rightarrow (a + b + c + ab + bc + ca + abc)^7 \geq 7^7 (a^4 b^4 c^4)$ a

From (1) and (2), we get

$[(1 + a)(1 + b)(1 + c)]^7 > 7^7 a^4 b^4 c^4$

Hence proved.

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Sol 30.

Roots of $x^2 - 10cx - 11d = 0$ are a and b

$\Rightarrow a + b = 10c$ and $ab = -11d$

Similarly c and d are the roots of $x^2 - 10ax - 11b = 0$

$\Rightarrow c + d = 10a$ and $cd = -11b$

$\Rightarrow a + b + c + d = 10(a + c)$ and $abcd = 121\ bd$

$\Rightarrow b + d = 9(a + c)$ and $ac = 121$

Also we have $a^2 - 10\ ac - 11d = 0$ and $c^2 - 10ac - 11b = 0$

$\Rightarrow a^2 + c^2 - 20ac - 11(b + d) = 0$

$\Rightarrow (a + c)^2 - 22 \times 121 - 99\ (a + c) = 0$

$\Rightarrow a + c = 121$ or $- 22$

For $a + c = -22$, we get $a = c$

$\therefore$ rejecting this value we have $a + c = 121$

$\therefore a + b + c + d = 10\ (a + c)$

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