



CHAPTER

11

INTRODUCTION TO THREE  
DIMENSIONAL GEOMETRY

GENERAL KEY CONCEPTS

1. **Distance Formula :** Distance between two points  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$ ,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2. **Section Formula :**

- (i) If a point R divides the line segment joining the points  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$  in the ratio  $m : n$  internally, then

$$R \quad \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}$$

- (ii) If a point R divides the line segment joining the points  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$  in the ratio  $m : n$  externally, then

$$R \quad \frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}$$

3. **Mid-point Formula :** If R be the mid point of the line segment joining the points  $A(x_1, y_1)$  and  $B(x_2, y_2)$ .

$$R \quad \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}$$

4. Centroid of the triangle whose vertices are  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  is

$$\left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$

CONNECTING CONCEPTS

- To locate the position of a point in three dimensional space, we consider a rectangular coordinate system of three mutually perpendicular lines as the coordinate axes. These axes are called x, y and z-axes.
- The three planes determined by the pair of axes are the coordinate planes called XY, YZ and ZX-planes. The three coordinate planes divide the space into eight parts known as octants. The coordinates of a point P in three dimensional geometry is always written in the form of triplet like (x, y, z). Here x, y and z are the distances of the point P from the YZ, ZX and XY-plane. The co-ordinate of a point in three dimensional space are also the distances from the origin of the feet of the perpendicular drawn from the point on the respective co-ordinate axes.

3. The sign of the coordinates of a point is determined by the octant in which the point lies.

Octant Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

4. (i) Any point on x-axis is of the form  $(x, 0, 0)$   
(ii) Any point on y-axis is of the form  $(0, y, 0)$   
(iii) Any point on z-axis is of the form  $(0, 0, y)$
5. The distance of the point  $(x, y, z)$  from the origin is given by  $\sqrt{x^2 + y^2 + z^2}$