



# 13

# PROBABILITY

## KEY CONCEPT INVOLVED

- 1. Conditional Probability** – Let E and F be two events of a random experiment, then, the probability of occurrence of E under the condition that F has already occurred and  $P(F) \neq 0$  is called the conditional probability. It is denoted by  $P(E/F)$

The conditional probability  $P(E/F)$  is given by  $P(E/F) = \frac{P(E \cap F)}{P(F)}$ , When  $P(F) \neq 0$

**Properties of conditional probability –**

- If F be an event of a sample space S of an experiment, then  $P(S/F) = P(F/F) = 1$   
If A and B are any two events of a sample space S and F is an event of S such that  $P(F) \neq 0$ , then
  - $P(A \cup B/F) = P(A/F) + P(B/F) - P(A \cap B/F)$   
If A and B are disjoint event then  $P(A \cup B/F) = P(A/F) + P(B/F)$
  - $P(\bar{E}/F) = 1 - P(E/F)$  or  $P(E'/F) = 1 - P(E/F)$
- 2. Multiplication Theorem On Probability** – Let E and F be two events associated with a sample space S.  $P(E \cap F)$  denotes the probability of the event that both E and F occur, which is given by  $P(E \cap F) = P(E) P(F/E) = P(F) P(E/F)$ , provided  $P(E) \neq 0$  and  $P(F) \neq 0$
  - 3. Independent Event –**
    - Events E and F are independent if  $P(E \cap F) = P(E) \times P(F)$
    - Two events E and F are said to be independent if  $P(E/F) = P(E)$  and  $P(F/E) = P(F)$  provided  $P(E) \neq 0$  and  $P(F) \neq 0$
    - Three events E, F and G are said to be independent or mutually independent if  $P(E \cap F \cap G) = P(E) P(F) P(G)$ .
  - 4. Random Variable** – A random variable is a real valued function whose domain is the sample space of random experiment.
  - 5. Baye's Theorem** – let  $E_1, E_2, \dots, E_n$  be the x events forming a partition of sample space S i.e.  $E_1, E_2, \dots, E_n$  are pairwise disjoint and  $E_1 \cup E_2 \cup \dots \cup E_n = S$  and A is any event of non – zero probability, then  $P(E_i/A) = \frac{P(E_i) P(A/E_i)}{\sum_{j=1}^n P(E_j) P(A/E_j)}$  for any  $i = 1, 2, 3, \dots, n$
  - 6. Bernoulli Trial** – Trials of a random experiment are said to be Bernoulli's trials, if they satisfy the following conditions :
    - The trials should be independent.
    - Each trial has exactly two outcomes ex- success or failure.
    - The probability of success remains the same in each trial.
    - Number of trials is finite.
  - 7. Mean of Random Variable** – let X be a random variable whose possible values are  $x_1, x_2, \dots, x_n$  if  $P_1, P_2, \dots, P_n$  are the corresponding probabilities, then mean of X,

$$\mu = \sum_{i=1}^n x_i p_i = E(X)$$

The mean of a random variables X is also called the expected value of X denoted by E (x).

8. **Variance of a Random Variable** – let X be a random variable with possible values  $x_1, x_2, \dots, x_n$  occur with probabilities are  $p_1, p_2, \dots, p_n$  respectively.

let  $\mu = E(X)$  be the mean of X. The variance of X denoted by  $\text{var}(X)$  or  $\sigma_x^2$  is defined as

$$\text{Var}(X) \text{ or } \sigma_x^2 = \sum_{i=1}^n (x_i - \mu)^2 p_i = E(x_i - \mu)^2 = E(X^2) - [E(X)]^2$$

Standard Deviation,  $\sigma_x = \sqrt{\text{Var}(X)}$

9. **Probability function** – The probability of x success is denoted by  $p(X = x)$  or  $P(x)$  and is given by  $P(x) = {}^n C_x q^{n-x} p^x$ ,  $x = 0, 1, 2, \dots, n$  and  $q = 1 - p$   
The function  $P(x)$  is known as probability function of binomial distribution.

## CONNECTING CONCEPTS

1. **Partition of a sample space** – A set of events  $E_1, E_2, \dots, E_n$  is said to represent a partition of sample S if
- (i)  $E_i \cap E_j = \phi$  if  $i \neq j, i, j = 1, 2, \dots, n$
  - (ii)  $E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n = S$
  - (iii)  $P(E_i) > 0 \forall i = 1, 2, \dots, n$ .

2. **Theorem of total Probability** – let  $\alpha E_1, E_2, \dots, E_n \gamma$  be a partition of sample spaces and each event has a non – zero probability If A be any event associated with S, then

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) + \dots + P(E_n)P(A/E_n)$$

$$P(A) = \sum_{i=1}^n P(E_i)P(A/E_i)$$

3. **A Few Terminologies** –

- (i) **Hypothesis** – When Baye’s theorem is applied the events  $E_1, E_2, \dots, E_n$  are said to be hypothesis x.
- (ii) **Priori Porbability** – The Porbabilites  $P(E_1), P(E_2), \dots, P(E_n)$  are called priori.
- (iii) **Posteriori Porbability** – The conditional probability  $P(E_i/A)$  is known as the posteriori probability of hypothesis  $E_i$  where  $i = 1, 2, 3, \dots, n$

4. **Probability Distribution of a Random Variable** – let real numbers  $x_1, x_2, \dots, x_n$  be the possible value of random variable and  $p_1, p_2, \dots, p_n$  be probability corresponding to each value of the random variable X. Then the probability distribution is

$$\begin{array}{l} X : \quad x_1 \quad \quad \quad x_2 \quad \dots \quad x_n \\ P(X) : \quad p_1 \quad \quad \quad p_2 \quad \dots \quad p_n \end{array}$$

- (i)  $p_i > 0$  (ii) sum of porbabilites  $p_1 + p_2 + \dots + p_n = 1$ .

5. **Binomial Distribution** – Probability distribution of a number of successes, in an experiment consisting of n Bernoulli trials are obtained by Binomial expansion of  $(q + p)_n$ . Such a probability distribution is

$$\begin{array}{l} X : \quad \quad 0 \quad \quad \quad 1 \quad \quad \quad 2 \quad \quad \quad \dots \quad r \quad \quad \quad \dots \quad n \\ P(X) : \quad {}^n C_0 q^n \quad \quad {}^n C_1 q^{n-1} p \quad \quad {}^n C_2 q^{n-2} p^2 \quad \quad \dots \quad {}^n C_r q^{n-r} p^r \quad \quad \dots \quad {}^n C_n p^n \end{array}$$

This probability distribution is called binomial distribution with parameter n and p.

Where, p is the probability of success in each trial and q is the probability of not success in each trial.

$$\therefore p + q = 1, q = 1 - p$$