CHAPTER – 3

LINEAR EQUATIONS IN TWO VARIABLES

An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that a and b are not both zero, is called a linear equation in two variables.

Important points to Note:

S.no	Points
1	A linear equation in two variable has infinite solutions
2	The graph of every linear equation in two variable is a straight line
3	$x = 0$ is the equation of the y-axis and $y = 0$ is the equation of the x-axis
4	The graph $x=a$ is a line parallel to y -axis.
5	The graph $y=b$ is a line parallel to x -axis
6	An equation of the type $y = mx$ represents a line passing through the origin.
7	Every point on the graph of a linear equation in two variables is a solution of the linear equation. Moreover, every solution of the linear equation is a point on the graph

S.no	Type of equation	Mathematical representation	Solutions
1	Linear equation in one Variable	$ax+b=0$, $a \neq 0$ a and b are real number	One solution
2	Linear equation in two Variable	$ax+by+c=0$, $a \neq 0$ and $b \neq 0$ a, b and c are real number	Infinite solution possible
3	Linear equation in three Variable	$ax+by+cz+d=0$, $a \neq 0$, $b \neq 0$ and $c \neq 0$	Infinite solution possible
		a, b, c, d are real number	

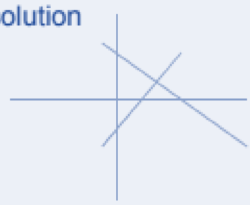
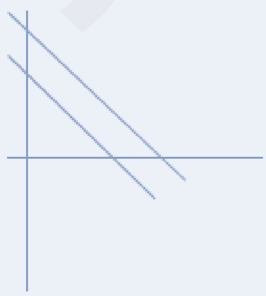
Simultaneous pair of linear equation:

A pair of linear equation in two variables

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Graphically it is represented by two straight lines on Cartesian plane.

Simultaneous pair of Linear equation	Condition	Graphical representation	Algebraic interpretation
$a_1x+b_1y+c_1=0$ **$a_2x+b_2y+c_2=0$** **Example** **$x-4y+14=0$** **$3x+2y-14=0$**	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines. The intersecting point coordinate is the only solution 	One unique solution only.
$a_1x+b_1y+c_1=0$ **$a_2x+b_2y+c_2=0$**	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines. The any coordinate on the line is the solution.	Infinite solution.
Example **$2x+4y=16$** **$3x+6y=24$**			
$a_1x+b_1y+c_1=0$ **$a_2x+b_2y+c_2=0$** **Example** **$2x+4y=6$** **$4x+8y=18$**	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel Lines 	No solution

The graphical solution can be obtained by drawing the lines on the Cartesian plane.

<u>Algebraic Solution of system of Linear equation:</u>

S.no	Type of method	Working of method
1	Method of elimination by substitution	1) Suppose the equation are $a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$ 2) Find the value of variable of either x or y in other variable term in first equation 3) Substitute the value of that variable in second equation
		4) Now this is a linear equation in one variable. Find the value of the variable 5) Substitute this value in first equation and get the second variable
2	Method of elimination by equating the coefficients	1) Suppose the equation are $a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$ 2) Find the LCM of a_1 and a_2 .Let it k. 3) Multiple the first equation by the value k/a_1 4) Multiple the first equation by the value k/a_2 4) Subtract the equation obtained. This way one variable will be eliminated and we can solve to get the value of variable y 5) Substitute this value in first equation and get the second variable
3	Cross Multiplication method	1) Suppose the equation are $a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$ 2) This can be written as $\frac{x}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$ 3) This can be written as

$$\frac{x}{b_1c_2-b_2c_1} = \frac{-y}{a_1c_2-a_2c_1} = \frac{1}{a_1b_2-a_2b_1}$$

4) Value of x and y can be find using the

x => first and last expression

y=> second and last expression