CHAPTER 11

INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

GENERAL KEY CONCEPTS

1. **Distance Formula :** Distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2. **Section Formula :**

(i) If a point R divides the line segment joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio m : n internally, then

$$R \quad \frac{mx_2 \quad nx_1}{m \quad n}, \frac{my_2 \quad ny_1}{m \quad n}, \frac{mz_2 \quad nz_1}{m \quad n}$$

(ii) If a point R divides the line segment joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio m : n externally, then

$$R \quad \frac{mx_2 \quad nx_1}{m \quad n}, \frac{my_2 \quad ny_1}{m \quad n}, \frac{mz_2 \quad nz_1}{m \quad n}$$

3. **Mid-point Formula :** If R be the mid point of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$.

$$R \quad \frac{x_1 \quad x_2}{2}, \frac{y_1 \quad y_2}{2}, \frac{z_1 \quad z_2}{2}$$

4. Centroid of the triangle whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

CONNECTING CONCEPTS

1. To locate the position of a point in three dimensional space, we consider a rectangular coordinate system of three mutually perpendicular lines as the coordinate axes. These axes are called x, y and z-axes.

2. The three planes determined by the pair of axes are the coordinate planes called XY, YZ and ZX-planes. The three coordinate planes divide the space into eight parts known as octants.

The coordinates of a point P in three dimensional geometry is always written in the form of triplet like (x, y, z). Here x, y and z are the distances of the point P from the YZ, ZX and XY-plane.

The co-ordinate of a point in three dimensional space are also the distances from the origin of the feet of the perpendicular drawn from the point on the respective co-ordinate axes.

3. The sign of the coordinates of a point is determined by the octant in which the point lies.

Octant / Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

4. (i) Any point on x-axis is of the form (x, 0, 0)

(ii) Any point on y-axis is of the form (0, y, 0)

(iii) Any point on z-axis is of the form (0, 0, y)

5. The distance of the point (x, y, z) from the origin is given by $\sqrt{x^2 + y^2 + z^2}$