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LINEAR PROGRAMMING

KEY CONCEPT INVOLVED

- 1. Linear Programming Problems Problems which concern with finding the minimum or maximum value of a linear function Z (called objective function) of several variables (say x and y), subject to certain conditions that the variables are non-negative and satisfy a set of linear inequalities (called linear constraints) are known as linear programming problems.
- 2. Objective function A linear function z = ax + by, where a and b are constants, which has to be maximised or minimised according to a set of given conditions, is called a linear objective function.
- 3. Decision Variables In the objective function z = ax + by, the variables x, y are said to be decision variables.
- **4.** Constraints The restrictions in the form of inequalities on the variables of a linear programming problem are called constraints. The condition $x \ge 0$, $y \ge 0$ are known as non negative restrictions.
- 5. Feasible Region The common region determined by all the constraints including non–negative constraints $x,y \ge 0$ of linear programming problem is known as feasible region (or solution region) If we shad c the region according to the given constraints, then the shaded areas is the feasible region which is the common area of the regions drawn under the given constraints.
- **6. Feasible Solution** Each points within and on the boundary of the feasible region represents feasible solution of constraints.
 - In the feasible region there are infinitely many points which satisfy the given condition.
- **7. Optimal Solution** Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.
- 8. Theorem 1 Let R be the feasible region (convex polygon) for a linear programming problem and let Z = ax + by be the objective function. When Z has an optimal value (maximum or minimum), where the varialdes x and y are subject to constraints described by linear inequalities, the optimal value must occur at a corner point of the feasible region.
- **9.** Theorem 2 Let R be the feasible region for a linear programming problem, and let Z = ax + by be the objective function. If R is bounded them the objective function Z has both maximum and minimum value on R and each of these occurs at a corner point of R.
- 10. Different Types of Linear Programming Problem
 - (i) Manufacturing Problems In such problem, we determine the number of units of different products which should to produced and sold by a firm when each product requires a fixed man power required, machines hours, labour hour per unit product needed were house space per unit of the output etc., in order to make maximise profit.
 - (ii) **Diet Problem** We determine the amount of different types of constituents or nutrients which should be included in a diet so as to minimise the cost of the desired diet such that it contains a certain minimum amount of each contituent/nutrients.
 - (iii) **Transportation Problems** In these prodblems, we determine a transportation schedule in order to find the cheapest way of transporting a product from plants/foutories situated at different locations to different markets.

CONNECTING CONCEPTS

- 1. **Formulation of LPP** Formulation of LPP means converting verbal description of the given problem into mathematical form in terms of objective function, constraints and non negative restriction:
 - (i) Identification of the decision variables whose value is to be determined.
 - (ii) Formation of an objective function as a linear function of the decision varibles.
 - (iii) Identification of the set of constraints or restrictions.Express them as linear inequation with appropriate sign of equality or inequality.
 - (iv) Mention the non negative restriction for the decision varibles.

2. Solve The LPP –

- (i) First of all formulate the given problem in terms of mathematical constraints and an objective function.
- (ii) The constraints would be inequations which shall be plotted and relevant area shall be shaded.
- (iii) The corner points of common shaded area shall be identified and the coordinates corresponding to these points shall be substitued in the objective function.
- (iv) The coordinates of one corner point which maximize or minimize the objective function shall be optimal solution of the given problem.
 - If feasible region is unbounded, then a maximum or a minimum value of the objective function may not exist. However, if it exists, it must occur at a corner point of feasible region