13 PROBABILITY

KEY CONCEPT INVOLVED

1. **Conditional Probability** – Let E and F be two events of a random experiment, then, the probability of occurance of E under the condition that F has alredy occured and P (F) $\neq$ 0 is called the conditional probability. It is denoted by P (E/F)

   The conditional probability P (E/F) is given by $P(E/F) = \frac{P(E \cap F)}{P(F)}$, When $P(F) \neq 0$

   Properties of conditional probability –
   (i) If F be an event of a sample space s of an experiment, then P (S/F) = P (F/F) = 1
   If A and B are any two events of a sample space S and F is an event of s such that P (F) $\neq$ 0, then
   (ii) $P(A \cup B/F) = P(A/F) + P(B/F) - P(A \cap B/F)$
   IF A and B are disjoint event then $P(A \cup B/F) = P(A/F) + P(B/F)$
   (iii) $P(\bar{E}/F) = 1 - P(E/F)$ or $P(E'/F) = 1 - P(E/F)$

2. **Multiplication Theorem On Probability** – Let E and F be two events associated with a sample space S. $P(E \cap F)$ denotes the probability of the event that both E and F occur, which is given by $P(E \cap F) = P(E)\,P(F/E) = P(F)\,P(E/F)$, provided $P(E) \neq 0$ and $P(F) \neq 0$

3. **Independent Event** –
   (i) Events E and F are independent if $P(E \cap F) = P(E) \times P(F)$
   (ii) Two events E and F are said to be independent if P (E/F) = P (E) and P (F/E) = P (F) provided $P(E) \neq 0$ and $P(F) \neq 0$
   (iii) Three events E, F and G are said to be independent or mutually independent if
   $P(E \cap F \cap G) = P(E)\,P(F)\,P(G)$.

4. **Random Variable** – A random variable is a real valued function whose domain is the sample space of random experiment.

5. **Baye's Theorem** – let $E_1, E_2, \cdots\cdots, E_n$ be the x events forming a partition of sample space S i.e. $E_1, E_2, \cdots\cdots, E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \cdots\cdots \cup E_n = S$ and A is any event of non – zero porbability, then $P(E_i/A) = \frac{P(E_i)\ P(A/E_i)}{\sum_{j=1}^{n} P(E_j)P(A/E_j)}$ for any i = 1, 2, 3, ..., n

6. **Bernoulli Trial** – Trials of a random experiment are said to be Bernoulli's trials, if they satisfy the following conditions :
   (i) The trials should be independent.
   (ii) Each trial has exactly two outcomes ex- success or falilure.
   (iii) The probability of success remains the same in each trial.
   (iv) Number of trials is finite.

7. **Mean of Random Variable** – let X be a random variable whose possible values are $x_1, x_2, \cdots, x_n$ if $P_1, P_2, \cdots, P_n$ are the corresponding probabilities, then mean of X,

$$\mu = \sum_{i=1}^{n} x_i p_i = E(X)$$

The mean of a random variables X is also called the expected value of X denoted by E (x).

8. Variance of a Random Variable – let X be a random variable with possible values $x_1\ x_2, \cdots\cdots\ x_n$ occur with probabilities are $p_1, p_2, \cdots\cdots\ p_n$ respectively.

let $\mu = E(X)$ be the mean of X. The variance of X denoted by var (X) or σ_x^2 is defined as

$$\text{Var}(X) \text{ or } \sigma_x^2 = \sum_{i=1}^{n} (x_i - \mu)^2 p_i \qquad = E(x_i - \mu)^2 \qquad = E(X^2) - [E(X)]^2$$

Standard Deviation, $\sigma_x = \sqrt{\text{Var}(X)}$

9. Probability function – The probability of x success is denoted by p (X = x) or P(x) and is given by $P(x) = {}^nC_x\, q^{n-x} p^x$, x = 0, 1, 2, ·······, n and q = 1 – P

The function P (x) is known as probability function of binomial distribution.

CONNECTING CONCEPTS

1. Partition of a sample space – A set of events $E_1, E_2, \cdots\cdots\ E_n$ is said to represent a partition of sample S if

(i) $E_i \cap F_j = \phi$ if $i \neq j$, i, j = 1, 2,·······, n

(ii) $E_1 \cup E_2 \cup E_3 \cup \cdots\cdots, \cup E_n = S$

(iii) $P(E_i) > 0\ \forall\ i = 1,2, \cdots\cdots, n.$

2. Theorem of total Probability – let $\alpha\ E_1, E_2, \cdots\cdots, E_n\ \gamma$ be a partition of sample spaces and each event has a non – zero probability If A be any event associated with S, then

$$P(A) = P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2) + P(E_3)\, P(A/E_3) + \cdots\cdots + P(E_n)\, P(A/E_n)$$

$$P(A) = \sum_{i=1}^{n} P(E_i)\, P(A/E_i)$$

3. A Few Terminologies –

(i) **Hypothesis** – When Baye's theorem is applied the events $E_1, E_2, \cdots\cdots\cdots\cdots, E_n$ are said to be hypothesis x.

(ii) **Priori Porbability** – The Porbabilites $P(E_1), P(E_2), \cdots\cdots\cdots\cdots, P(E_n)$ are called priori.

(iii) **Posteriori Porbabililty** – The conditional probability $P(E_i/A)$ is known as the posteriori probability of hypothesis E_i where i = 1, 2, 3, ..., n

4. Probability Distribution of a Random Variable – let real numbers $x_1, x_2, \cdots\cdots\cdots\cdots, x_n$ be the possible value of random variable and $p_1, p_2, \cdots\cdots\cdots\cdots, p_n$ be probability corresponding to each value of the random variable X. Then the probability distribution is

X :	x_1	x_2	.... x_n
P(X) :	p_1	p_2	.... p_n.

(i) $p_i > 0$ (ii) sum of porbabilites $p_1 + p_2 + \cdots\cdots\cdots\cdots + p_n = 1.$

5. Binomial Distribution – Probability distribution of a number of successes, in an experiment consisting of n Bernoulli trials are obtanied by Binomial expansioin of $(q + p)_n$. Such a probability distribution is

X :	0	1	2	.... r	.... n
P(X) :	${}^nC_0\, q^n$	${}^nC_1\, q^{n-1} P$	${}^nC_2\, q^{n-2} P^2$	${}^nC_r\, q^{n-r} P^r$	${}^nC_n\ P^n$

This probability distribution is called binomial distribution with parameter n and p.

Where, p is the probability of success in each trial and q is the probability of not sucess in each trial.

$\therefore \quad p + q = 1,\ q = 1 - p$